FACULTY OF MATHEMATICS AND PHYSICS
Charles University

Institute of Mathematics
Czech Academy of Sciences

Nonlocal nonlinear

Workshop I in Prague

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Charles University

UNIVERSITÄT BIELEFELD
Faculty of Mathematics

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Sun-Sig Byun (Seoul)

Calderón-Zygmund estimates for non-uniformly elliptic equations with matrix weights

This talk is concerned with an optimal Calderón-Zygmund estimate for weak solutions to non-uniformly elliptic equations with matrix weights. Reasonable regularity assumptions on the matrix weight associated with the gradient are investigated along with optimal conditions on the nonlinearity.

Cristiana De Filippis (Parma)

μ-ellipticity and nonautonomous integrals

μ-ellipticity describes certain degenerate forms of ellipticity, typical of convex integrals at linear, or nearly linear growth such as the area integral, or the iterated logarithmic model. The regularity of solutions to autonomous or totally differentiable problems is classical after Bombieri and De Giorgi and Miranda, Ladyzhenskaya and Ural'tseva and Frehse and Seregin. The anisotropic case is a later achievement of Bildhauer, Fuchs and Mingione, Beck and Schmidt and Gmeineder and Kristensen, that provided a complete partial and full regularity theory. However, all the approaches developed so far break down in presence of nondifferentiable ingredients. In particular, Schauder theory for certain significant anisotropic, nonautonomous functionals with Hölder continuous coefficients was only recently obtained by myself and Mingione. I will give an overview of the latest progress on the validity of Schauder theory for anisotropic problems whose growth is arbitrarily close to linear within the maximal nonuniformity range, and discuss sharp results and deep insights on more general nonautonomous area type integrals. From recent, joint work with Filomena De Filippis (Parma), Giuseppe Mingione (Parma), and Mirco Piccinini (Pisa).

Lars Diening (Bielefeld)

Fractional Sobolev spaces with degenerate weights

We study fractional Sobolev spaces with weights of Muckenhoupt type and their related non-local equations. We show Hölder regularity and Harnack inequality of the solutions. This is a joint project with Linus Behn, Jihoon Ok and Julian Rolfes.

Hongjie Dong (Brown University)

Degenerate elliptic and parabolic equations with distributional boundary data

I will first review some recent results on elliptic and parabolic equations which are either degenerate or singular near the boundary of the domain, with homogeneous Dirichlet or conormal boundary condition. Then I will discuss a recent work with Bekarys Bekmaganbetov on equations with (nonhomogeneous) distributional boundary data.

Franz Gmeineder (Konstanz)

Higher gradient integrability for BD-minimizers

In this talk, I give an update on recent progress on the gradient integrability of L^p- and bounded BD-minimizers. When considering convex linear growth functionals depending on the symmetric gradients, the usual coercivity space is BD, the functions of bounded deformation. By a basic obstruction, colloquially termed Ornstein's non-inequality, this space is strictly larger than BV, the functions of bounded variation. Yet, as will be discussed in this talk, suitably elliptic variational problems of linear growth then come with essentially the same Sobolev regularity as in the full gradient BV-case. Based on joint work with Lisa Beck & Ferdinand Eitler (Augsburg).

Daniel Hauer (Cottbus)

An extension problem for the logarithmic Laplacian

Motivated by the fact that for positive s tending to zero the fractional Laplacian of order s converges to the identity and for s tending to 1 to the (negative) local Laplacian, Chen and Weth [Comm. PDE 44 (11), 2019] introduced the logarithmic Laplacian as the first variation of the fractional Laplacian at $s = 0$. In particular, they showed that the logarithmic Laplacian admits an integral representation and can, alternatively, be defined via the Fourier-transform with a logarithmic symbol. The logarithmic Laplacian turned out to be an important tool in various mathematical problems; for instance, to determine the asymptotic behavior as the order s tends to zero of the eigenvalues of the fractional Laplacian equipped with Dirichlet boundary conditions (see, e.g., [Feulefack, Jarohs, Weth, J. Fourier Anal. Appl. 28(2), no. 18, 2022]), in the study of the logarithmic Sobolev inequality on the unit sphere [Frank, König, Tang, Adv. Math. 375, 2020], or in the geometric context of the 0-fractional perimeter, see [De Luca, Novaga, Ponsiglione, ANN SCUOLA NORM-SCI 22(4), 2021]. Caffarelli and Silvestre [Comm. Part. Diff. Eq. 32(7-9), (2007)] showed that for every sufficiently regular u, the values of the fractional Laplacian at u can be obtained by the co-normal derivative of an s-harmonic function wu on the half-space (by adding one more space dimension) with Dirichlet boundary data u. This extension problem represents the important link between an integro-differential operator (the nonlocal fractional Laplacian) and a local 2nd-order differential operator. This property has been used frequently in the past in many problems governed by the fractional Laplacian. In this talk, I will present an extension problem for the logarithmic Laplacian, which shows that this nonlocal integro-differential operator can be linked with a local Poisson problem on the (upper) half-space, or alternatively (after reflection) in a space of one more dimension. As an application of this extension property, I show that the logarithmic Laplacian admits a unique continuous property. The results presented here were obtained in joint work with Huyuan Chen (Jiangxi Normal University, China) and Tobias Weth (Goethe-Universität Frankfurt, Germany)

Alex Kaltenbach (Berlin)

Finite element approximations of a simplified model for smart fluids

In this talk, finite element approximations of a simplified model for smart fluids are discussed. Smart fluids are characterised by the property that the power-law index is not a fixed constant, but variable dependent. The most common examples for smart fluids are electro-rheological fluids and chemically reacting fluids. More precisely, in this talk, a priori error estimates for a finite element approximation of the $p(x)$-Navier–Stokes equations and the $p(t, x)$-Stokes equations, which are prototypical for the class of smart fluids, are derived assuming appropriate fractional regularity properties of the velocity vector field and the kinematic pressure. Finally, numerical experiments are presented that confirm the optimality of the error decay rates.

Moritz Kassmann (Bielefeld)

The parabolic Harnack inequality for nonlocal operators

The parabolic Harnack inequality for nonlocal operators is an important research topic in the field of stochastic analysis, potential theory and analysis. I review the history of the last 20 years and compare the approaches of stochastic analysis and partial differential equation techniques. I then present the results of two recent papers, which were written in collaboration with Marvin Weidner. In [1] we develop a purely analytical approach for the parabolic Harnack inequality under optimal conditions for the solutions. In [2] we show why the Harnack inequality for kinetic equations such as the fractional Kolmogorov equation fails by presenting a counterexample.

[1] M. Kassmann and M. Weidner, The parabolic Harnack inequality for nonlocal equations, to appear in Duke Math. J., see also arXiv:2303.05975

[2] M. Kassmann and M. Weidner, The Harnack inequality fails for nonlocal kinetic equations, to appear in Adv. Math., see also arXiv:2405.05223

Naian Liao (Salzburg)

Time-insensitive nonlocal parabolic Harnack estimates

I will present new Harnack-type estimates for nonlocal diffusions that defy the waiting-time phenomenon. The talk is based on a joint work with Marvin Weidner.

Giuseppe Mingione (Parma)

Partial regularity in nonlocal problems

The theory of partial regularity for elliptic systems replaces the classical De Giorgi-Nash-Moser one for scalar equations asserting that solutions are regular outside a negligible closed subset called the singular set. Eventually, Hausdorff dimension estimates on such a set can be given. The singular set is in general non-empty. The theory is classical, started by Giusti and Miranda and Morrey, in turn relying on De Giorgi's seminal ideas for minimal surfaces. I will present a few results aimed at extending the classical, local partial regularity theory to nonlinear integrodifferential systems and to provide a few basic, general tools in order to prove so called epsilon-regularity theorems in general non-local settings. From recent, joint work with Cristiana De Filippis (Parma) and Simon Nowak (Bielefeld)

Luboš Pick (Prague)

Fractional Orlicz-Sobolev spaces - subcritical case

This is the first part of a two-lecture series (the second one will be delivered by Lenka) in which we will survey some recent results on fractional Orlicz-Sobolev spaces, motivated by some nonlocal problems, and obtained in a recent series of projects jointly with Angela Alberico, Andrea Cianchi and Lenka Slavíková. In this first part, we will introduce fractional Orlicz-Sobolev spaces, explain the role of the threshold between the so-called subcritical and supercritical cases, and then focus on the former. In particular, we will point out sharp embeddings of fractional Orlicz-Sobolev spaces into various function spaces containing unbounded functions. Particular attention will be paid to Orlicz spaces, Orlicz-Lorentz spaces and rearrangement-invariant spaces. Embeddings will be considered both on the entire ambient Euclidean space and its bounded subdomains. Compactness of embeddings will be treated, too.

Abner J. Salgado (Tennessee)

A Semi-Analytic Diagonalization FEM for the Spectral Fractional Laplacian

We present a technique for approximating solutions to the spectral fractional Laplacian, which is based on the Caffarelli-Silvestre extension and diagonalization. Our scheme uses the analytic solution to the associated eigenvalue problem in the extended dimension. We show its relation to a quadrature scheme. Numerical examples demonstrate the performance of the method.

Yannick Sire (Johns Hopkins University)

Geometric measure of nodal, critical and singular sets for solutions of degenerate equations

I will describe recent results on the "size" of various sets associated to solutions of some elliptic PDEs whose coefficients are degenerate or singular. In the case of eigenfunctions of the associated second order operators, these estimates are related to some famous conjectures by Yau (formulated in a more classical setting). Degenerate PDEs appear in a lot of different contexts like conical spaces, realizations of Dirichlet-to-Neumann maps, Poincare-Einstein manifolds in conformal geometry, etc... I will describe several strategies to get these estimates, leading for some of them to sharp bounds. Along the way, I will also describe some eigenfunction and cluster estimates, which are very much related to this topic.

Lenka Slavikova (Prague)

Fractional Orlicz-Sobolev spaces - supercritical case

In this talk, I will give a description of those fractional Orlicz-Sobolev spaces that are continuously embedded into the space $L^\infty(\mathbb{R}^n)$ of essentially bounded functions, and I will determine the optimal modulus of continuity of functions from these spaces. I will also characterize embeddings of fractional Orlicz-Sobolev spaces into Campanato type spaces. This is a joint work with Angela Alberico, Andrea Cianchi and Luboš Pick.

Lyoubomira Softova (Salerno)

Boundedness of the solutions of a kind of nonlinear parabolic systems

We obtain essential boundedness of the weak solutins to the Cauchy-Dirichlet problem for quasilinear parabolic systems. The nonlinear terms are given by Carathéodory functions and support controlled growth conditions. Our result is proved by assuming additional componentwise coercivity of the system and appropriate componentwise control of the lower-order terms.

Bianca Stroffolini (Naples)

A journey across problems with general growth

I will review some old and recent results on elliptic and parabolic problems with general growth. Next, I will present a new proof of partial regularity for double phase systems based on general growth techniques.

Marvin Weidner (Barcelona)

Boundary regularity for nonlocal equations

There are significant differences between local and nonlocal problems when it comes to the boundary behavior of solutions. For instance, it is a well known fact that s-harmonic functions (i.e. solutions to nonlocal elliptic equations governed by the fractional Laplacian) are in general not better than C^s up to the boundary. As a consequence, in recent years there has been a huge interest in the boundary behavior of solutions to nonlocal equations. By now, the boundary regularity is well understood for the fractional Laplacian, however several questions remained open for equations driven by more general nonlocal operators. In this talk I will report on two recent results regarding the boundary regularity theory for nonlocal elliptic equations driven by general integro-differential operators that were obtained in collaboration with Xavier Ros-Oton (Barcelona) and Minhyun Kim (Seoul).

Organizing committee

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