Polish Academy of Sciences
Institute of Philosophy and Sociology
Warsaw, Poland

Challenging Partial, Approximate and Partial Approximate Measurement Invariance

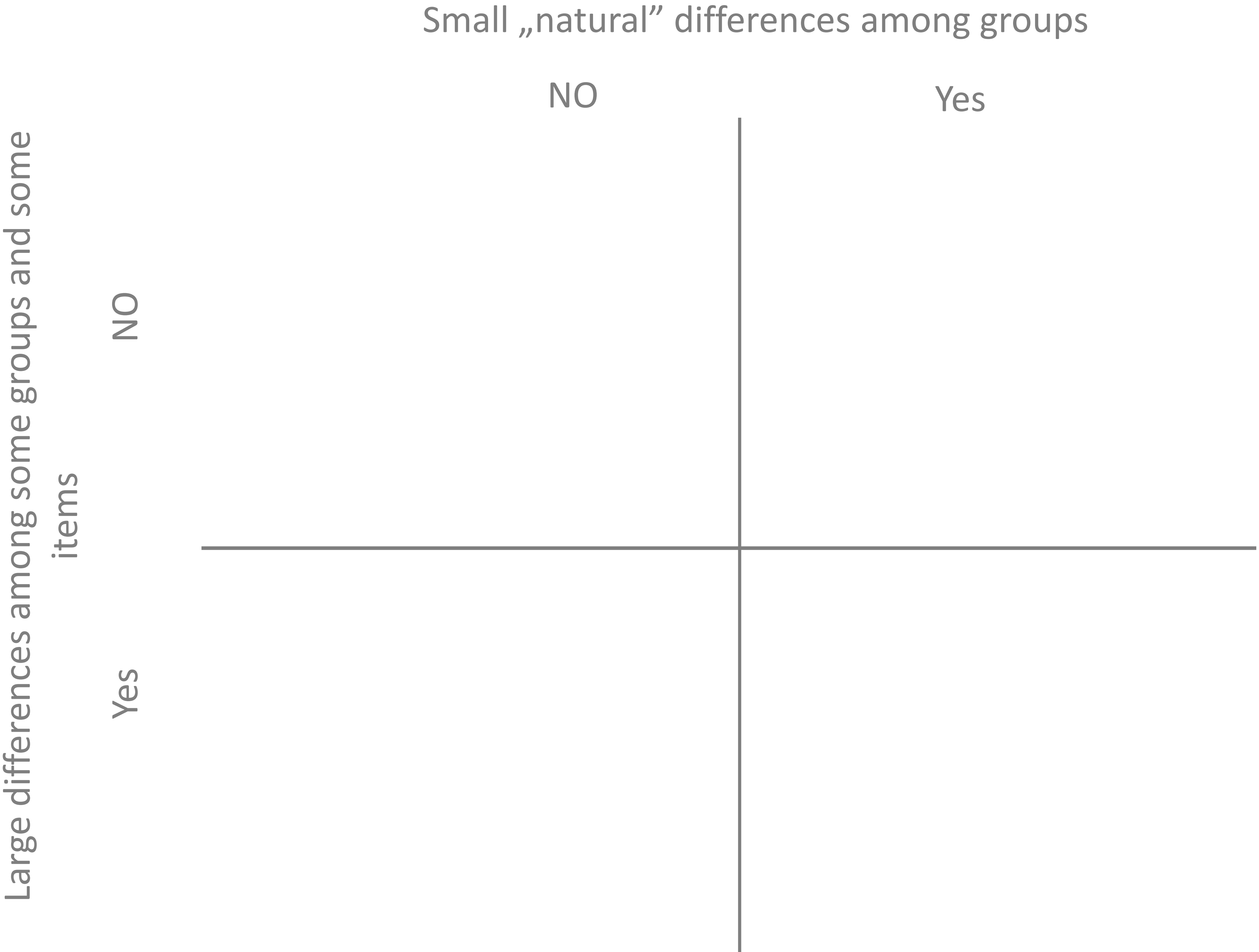
Artur Pokropek, Polish Academy of Science and EC Joint Research Centre
Peter Schmidt, University of Giessen
Eldad Davidov, University of Cologne and University of Zurich

This presentation has been prepared under the Scales Comparability in Large Scale Cross-country Surveys Project, which is funded by the Polish National Science Centre, as part of the grant competition Sonata 8 (UMO-2014/15/D/HS6/04934).

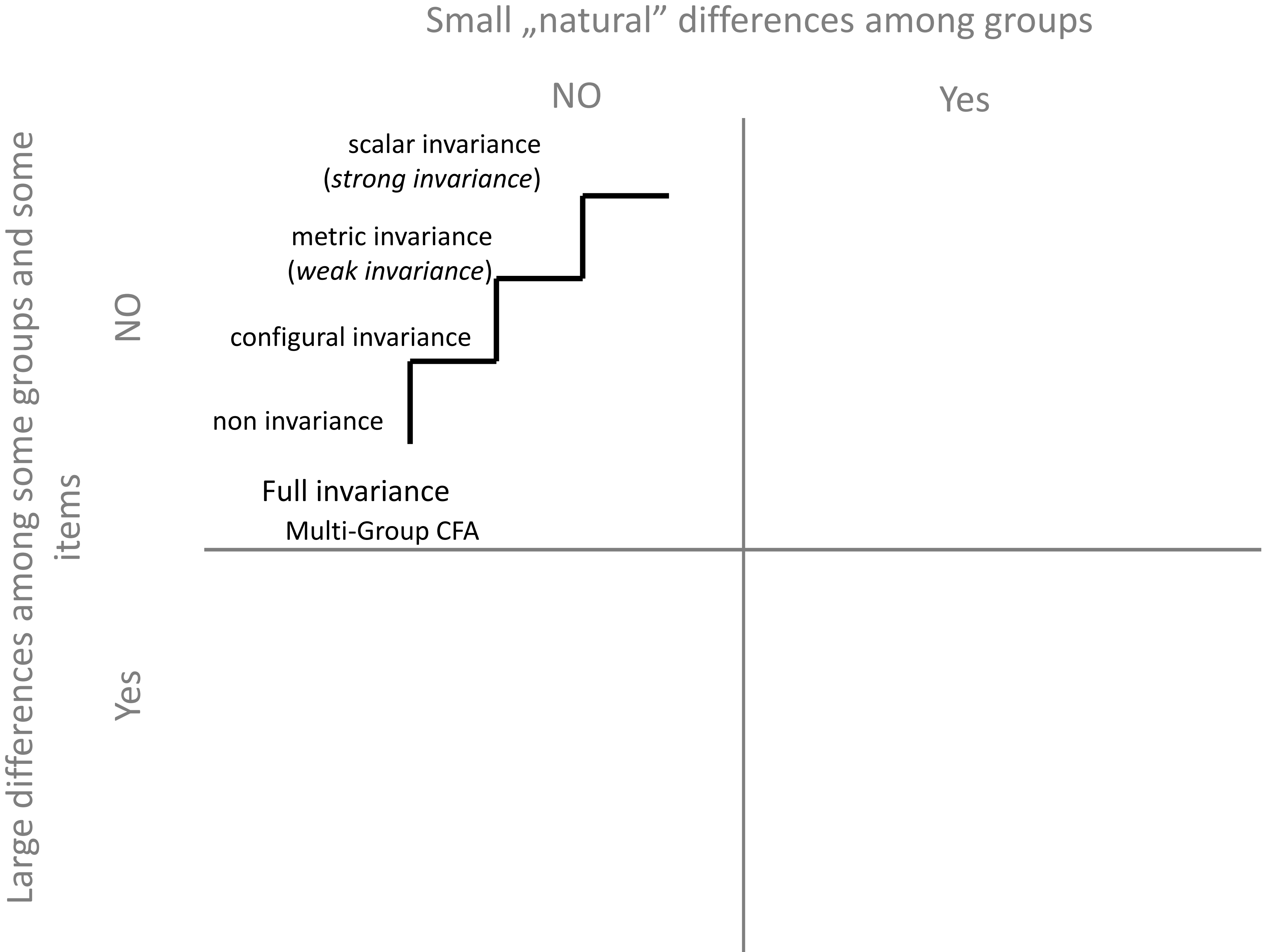
NATIONAL SCIENCE CENTRE
POLAND

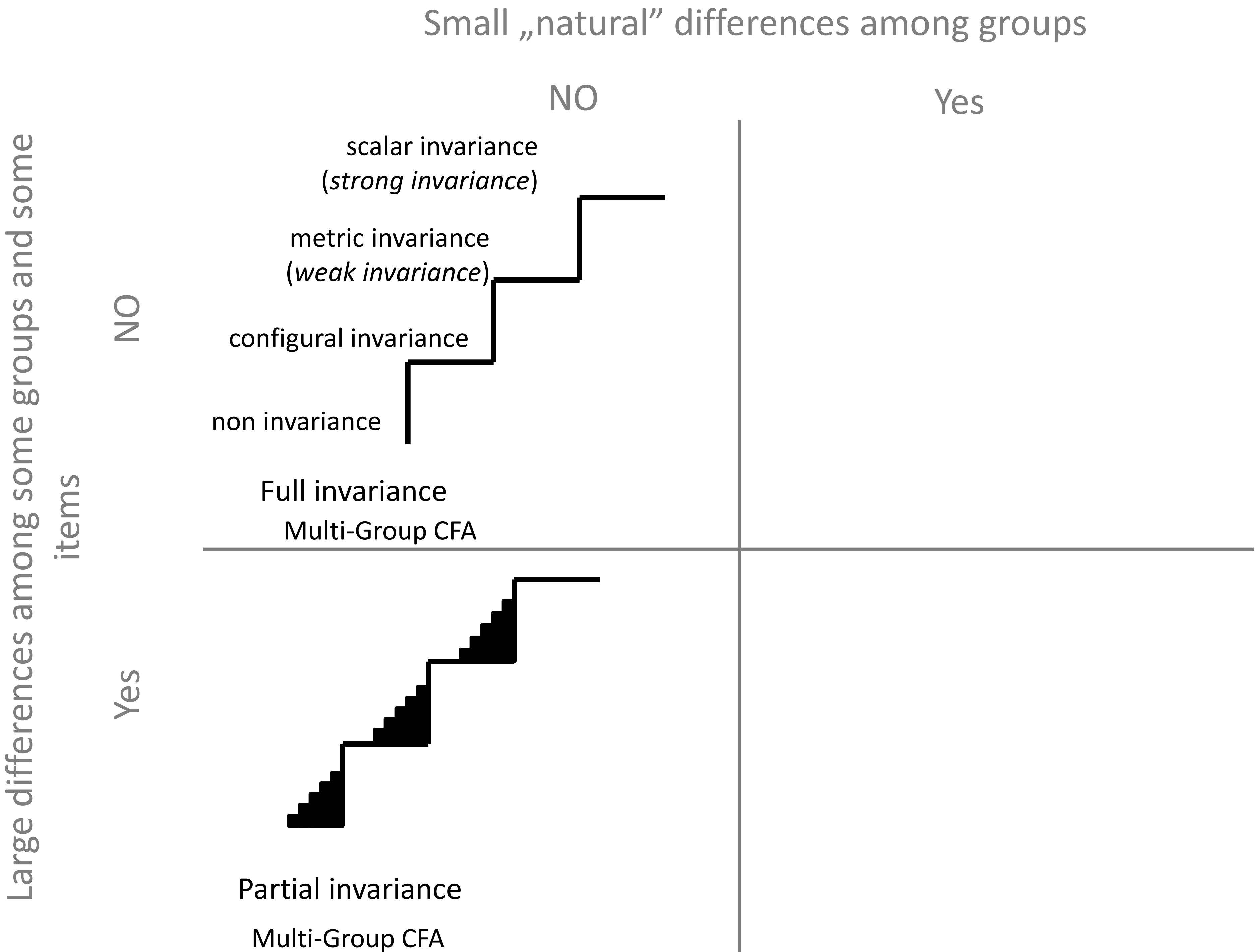
Outline

- Background (empirical approaches for testing comparability)
- Aim of the study
- Design of simulations
- Results of simulation study
- Conclusions
- Limitations and further work

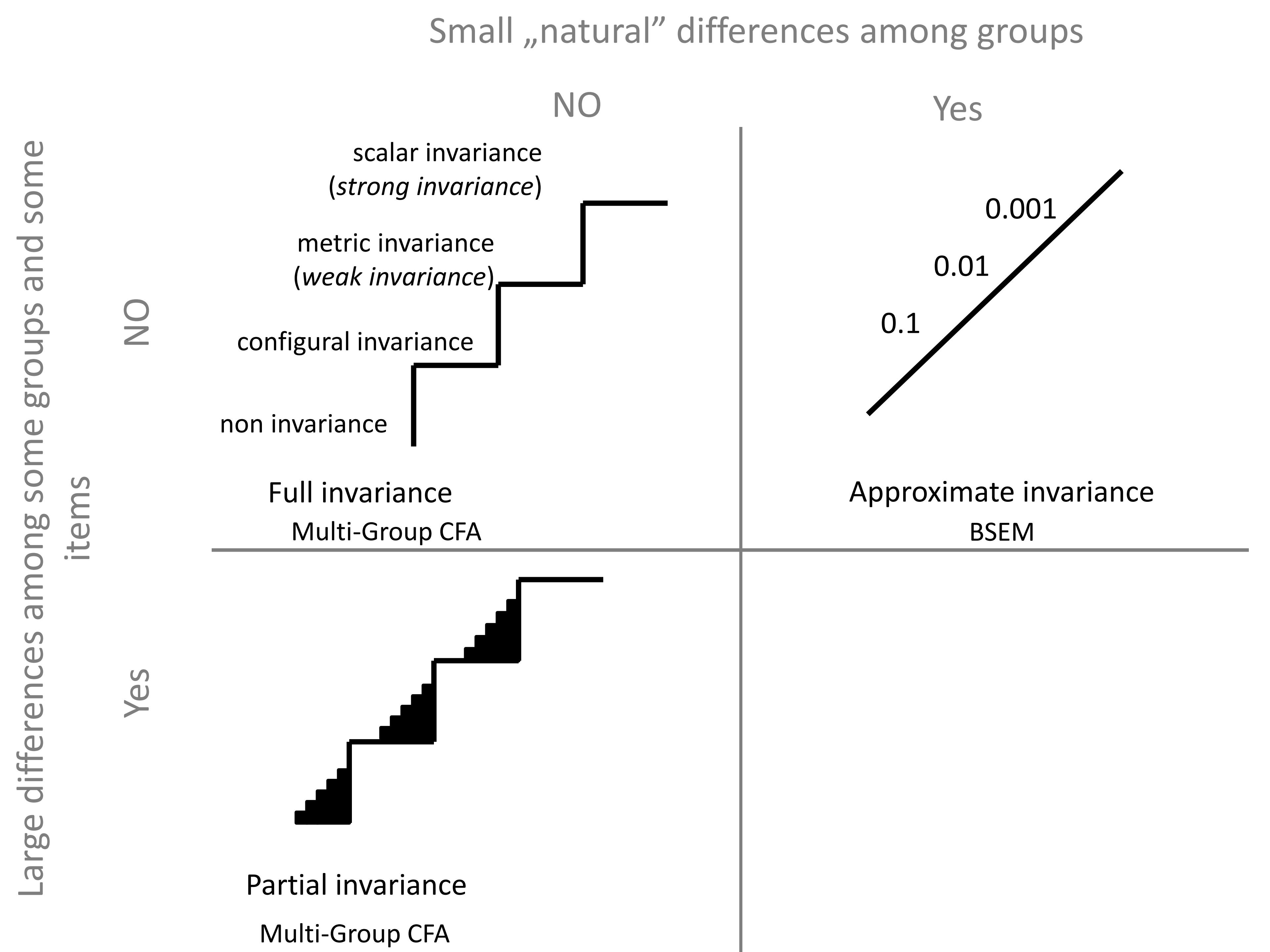


Small „natural” differences among groups
NO
Yes
Large differences among some groups and some items
NO
Yes

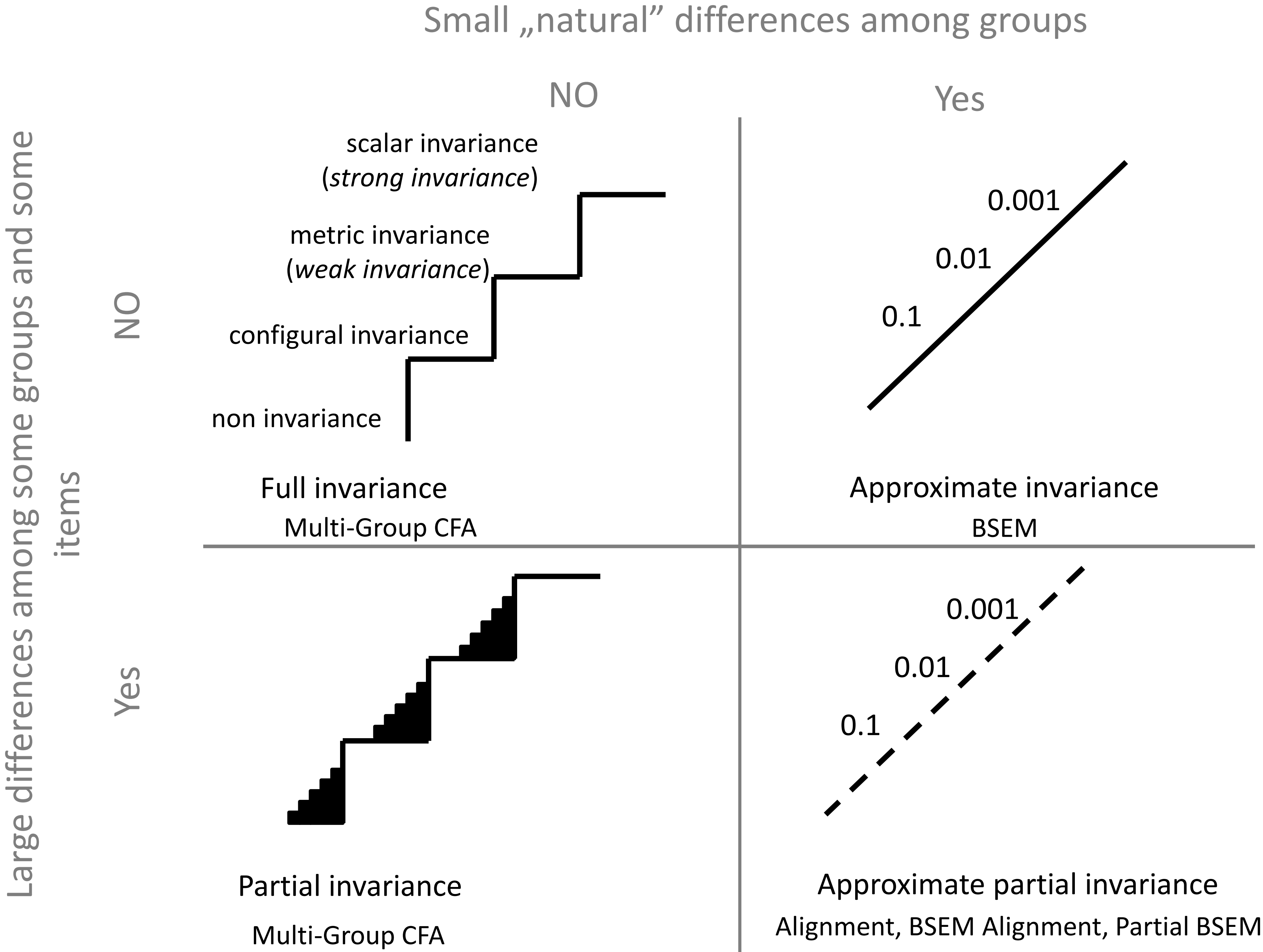

Small „natural” differences among groups
NO
Yes
Large differences among some groups and some items
NO
Yes
scalar invariance
(*strong invariance*)
metric invariance
(*weak invariance*)
configural invariance
non invariance
Full invariance
Multi-Group CFA



Small „natural" differences among groups
NO
Yes
Large differences among some groups and some items
NO
Yes
scalar invariance
(strong invariance)
metric invariance
(weak invariance)
configural invariance
non invariance
Full invariance
Multi-Group CFA
Partial invariance
Multi-Group CFA



Small „natural” differences among groups
NO
Yes
Large differences among some groups and some items
NO
Yes
scalar invariance
(*strong invariance*)
metric invariance
(*weak invariance*)
configural invariance
non invariance
0.001
0.01
0.1
Full invariance
Multi-Group CFA
Approximate invariance
BSEM
Partial invariance
Multi-Group CFA



Small „natural” differences among groups
NO
Yes
Large differences among some groups and some items
NO
Yes
scalar invariance
(*strong invariance*)
metric invariance
(*weak invariance*)
configural invariance
non invariance
Full invariance
Multi-Group CFA
0.001
0.01
0.1
Approximate invariance
BSEM
Partial invariance
Multi-Group CFA
0.001
0.01
0.1
Approximate partial invariance
Alignment, BSEM Alignment, Partial BSEM

Two general questions

- **Under what conditions deviations from strict invariance allow for conducting meaningful comparisons of model parameters (latent means and unstandardized regression coefficients across groups)**
  - Scalar, metric, configural invariance
  - Partial invariance
  - Approximate invariance
  - Partial approximate invariance
- **How good are the tools for detecting different types of measurement invariance**
  - Scalar, metric, configural invariance
  - Partial invariance
  - Approximate invariance
  - Partial approximate invariance

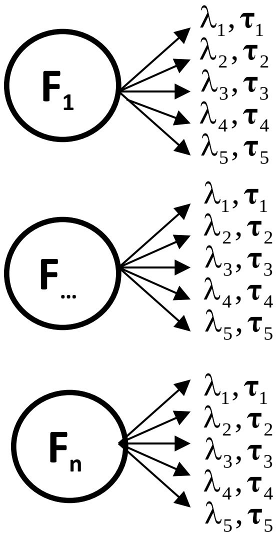
Two general questions

- **Under what conditions deviations from strict invariance allow for conducting meaningful comparisons of model parameters (latent means and unstandardized regression coefficients across groups)**
  - Scalar, metric, configural invariance ← well studied
  - Partial invariance
  - Approximate invariance ← limited number of studies
  - Partial approximate invariance
- **How good are the tools for detecting different types of measurement invariance**
  - Scalar, metric, configural invariance ← well studied
  - Partial invariance
  - Approximate invariance ← some pieces are missing
  - Partial approximate invariance

Aim of the study

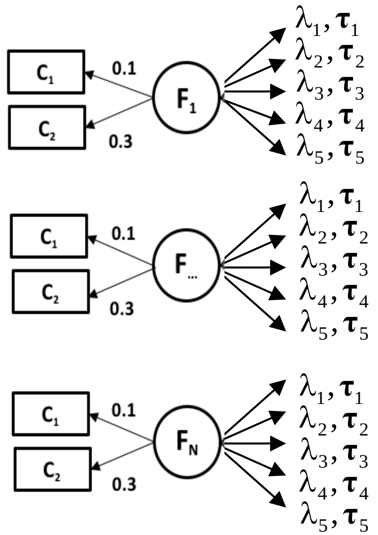
- **Under what conditions deviations from strict invariance allow for conducting meaningful comparisons of model parameters (latent means and unstandardized regression coefficients across groups)**
  - scalar, metric, configural invariance ← well studied
  - Partial invariance
  - Approximate invariance ← limited number of studies
  - Partial approximate invariance
- **How good are the tools for detecting different types of measurement invariance**
  - scalar, metric, configural invariance
  - Partial invariance invariance ← well studied
  - Approximate invariance ← some pieces are missing
  - Partial approximate invariance

Design of simulations



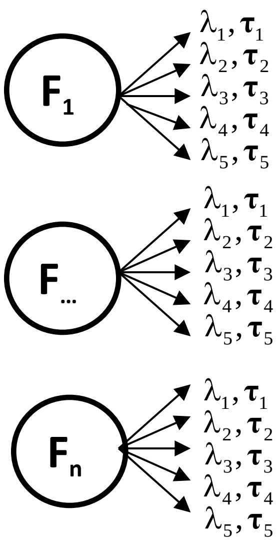
True Model

Design of simulations



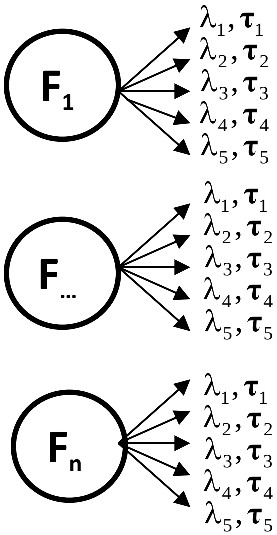
True Model

Design of simulations

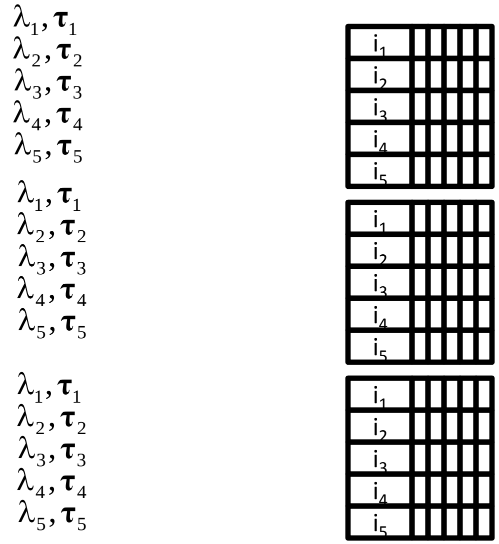


True Model

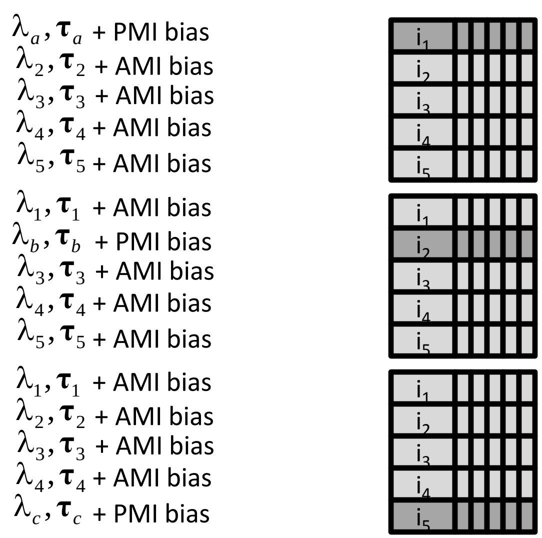
Design of simulations



True Model

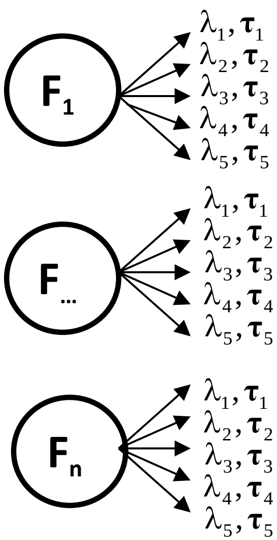


Invariance holds

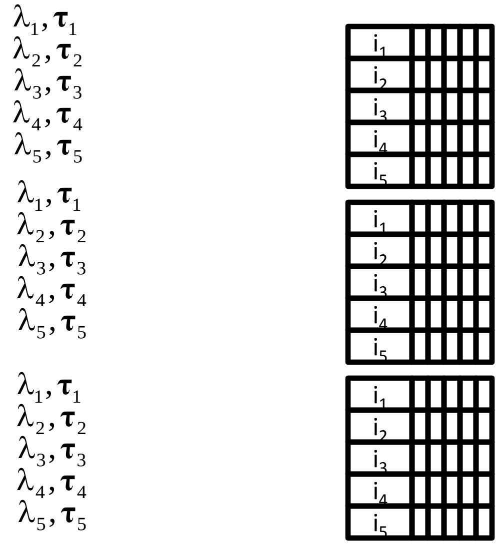


Non-invariance
(partial or/and aprox.)

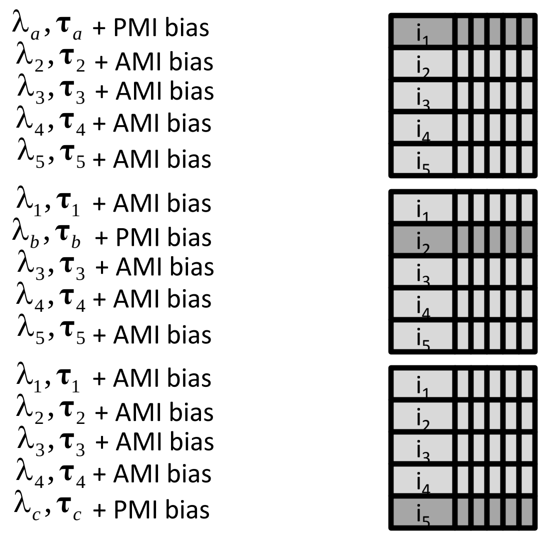
Design of simulations



True Model



Invariance holds



Non-invariance
(partial or/and aprox.)

1. tMG-CFA - Multigroup CFA model using data without non-invariane

2. MG-CFA - Multigroup CFA scalar mode   that ignores MI
3. PMG-CFA - Partial invariance multi-group confirmatory factor analysis (Byrne et al., 1989).
4. MG-BSEM - Multi-group Bayesian SEM
5. PMG-BSEM - Partial invariance Multi-group Bayesian SEM (Muthén & Asparouhov, 2013).
6. AMG-CFA - MG-CFA with alignment optimization (fixed) (Asparouhov & Muthén, 2014; Muthen & Asparouhov, 2014)

Design of simulations (details I)

1. We sampled means and standard deviations for each group from normal distributions **N(0,0.3)** and **N(1,0.1)** respectively.
2. Three random variables were generated for each group: F - representing a latent trait and two variables representing two criterion variables (C_1 and C_2). The C variables were generated from standard normal distributions in such a way so that the regression coefficient $C_1 \leftarrow F$ was set to **0.3** in each group while $C_2 \leftarrow F$ was set to **0.1** in each group.
3. We generated parameters for each item.
   a. Factor loadings from a uniform distribution **[0.5, 0.8]**
   b. Intercepts from a uniform distribution **[-0.15 and 0.15]**
4. Factor indicators were randomly generated according to the sampled item parameters.
5. The continuous factor indicators were discretized into five categories while using for the thresholds the values **–1.30, –0.47, +0.47, +1.30** (a similar approach picking up these thresholds was used in Sass, Schmitt, & Marsh (2014).

Design of simulations (details II)

6. For our simulation settings these parameters resulted in scales with relatively high reliabilities (Cronbach's alpha):
   - **0.75 for the 3-item scale,**
   - **0.80 for the 4-item scale,**
   - **0.85 for the 5-item scale**.
7. We added a non-invariance bias in specific affected groups.
8. Items were independently sampled from each group. In each replication, random assignment of non-invariance was repeated so that no particular pattern of non-invariance was present (in each replication different items were sampled to be non-invariant).
9. Bias was added to the selected items. We added a bias of **+0.2 or -0.2** both for factor loadings and intercepts. The sign of the bias was determined randomly, independently for each item and for each item parameter.
10. Approximate MI was added to rest of the items if it was considered in the condition (**0.000, 0.001, 0.005, 0.010, 0.025, 0.050)**

Simulation conditions for this study

- Number of Groups: **24**
- Sample Size: **1500**
- Number of items per scales: **3, 4, 5**
- Number of non-invariant items per group (full non-invariance): **1,2,3,4**
- Groups affected by fully non-invariant items: **25%, 50%, 75%, 100%**
- Approximate MI, prior dif. variance: **0.000, 0.001, 0.005, 0.010, 0.050**
- **400** replications

Simulation conditions for this study

Ranking recovery and/or precise means recovery practically impossible (regression path coefficients OK)

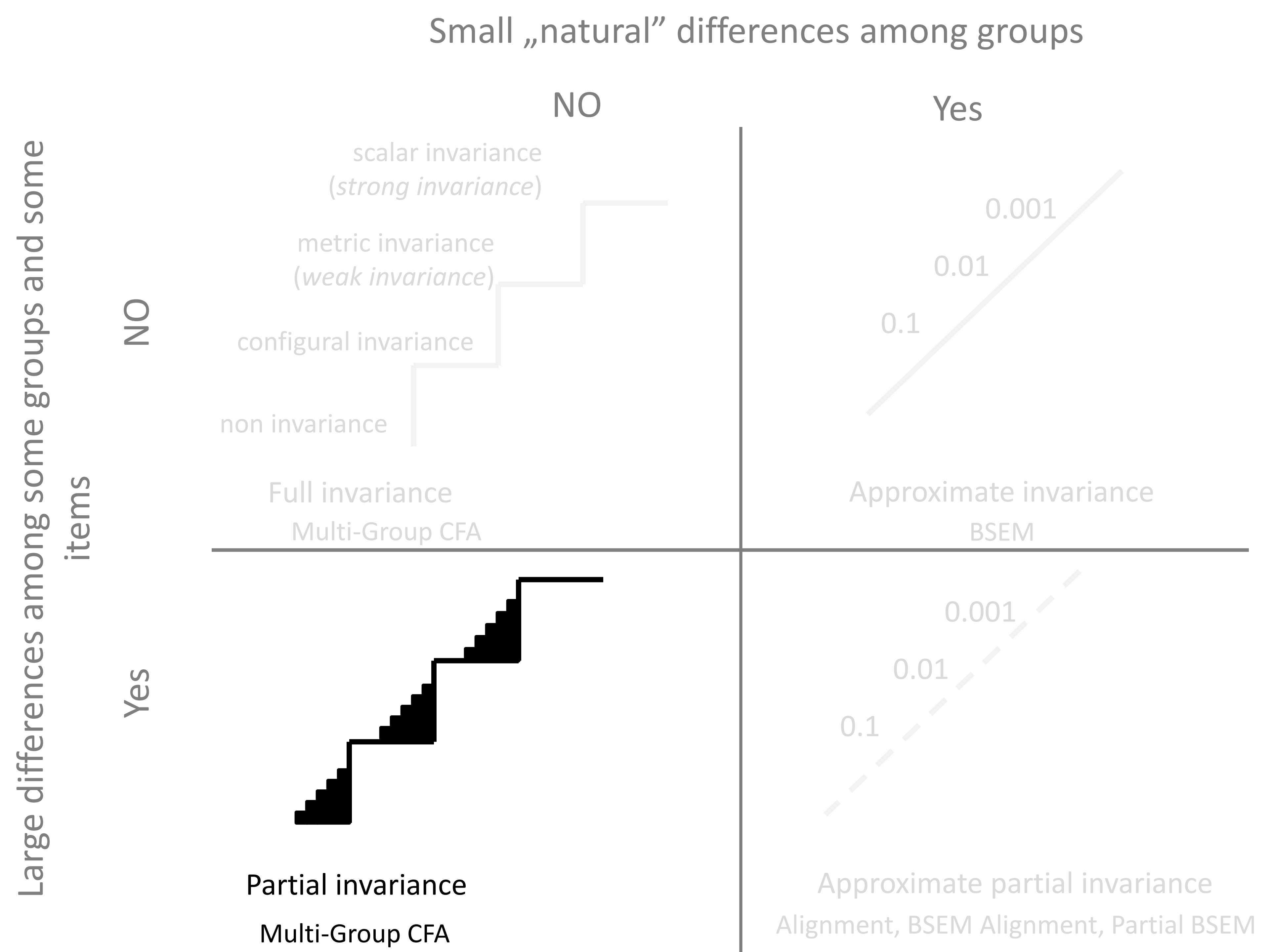
- Number of Groups: **24**
- Sample Size: **1500**
- Number of items per scales: ~~**3**~~**, 4, 5**
- Number of non-invariant items per group (full non-invariance): **1,2,3,4**
- Groups affected by fully non-invariant items: **25%, 50%, 75%, 100%**
- Approximate MI, prior dif. variance: **0.000, 0.001, 0.005, 0.010, 0.050**
- **400** replications

Simulation conditions for this study

- Number of Groups: **24**
- Sample Size: **1500**
- Number of items per scales: **5**
- Number of non-invariant items per group (full non-invariance): **1,2,3,4**
- Groups affected by fully non-invariant items: **25%, 50%, 75%, 100%**
- Approximate MI, prior dif. variance: **0.000, 0.001, 0.005, 0.010, 0.050**
- **400** replications

Evaluation criteria

- **Means Correlations** - according to Muthén and Asparouhov's (2014) and Muthén and Asparouhov's (2013) recommendation, a correlation between the true value of the means and their estimates of **at least 0.98 (and preferably 0.99)** indicates a reasonably good recovery of the mean rankings.
- **95% CI Coverage** - the coverage of the true means by the 95% coefficient intervals (CI) generated using standard errors of the estimated means. **(90%-100%)**
- **Avarage estimates** - For estimates of path coefficients, we are looking at average estimate over the groups and replications for bias detection **(0.29-0.31)**
- **RMSE (Root Mean Squared Error)** – for overall accuracy **(<0.06)**



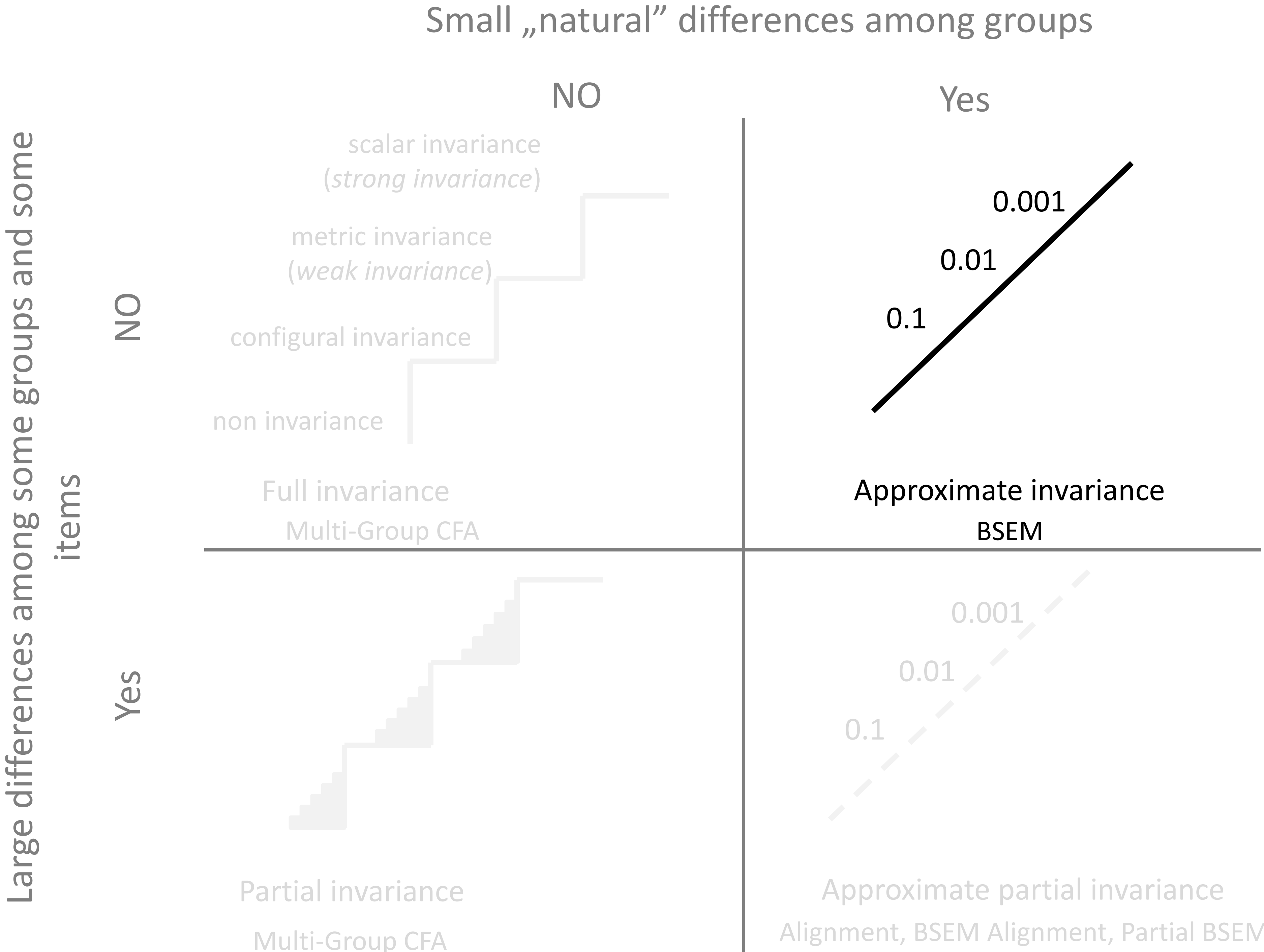
Small „natural" differences among groups
NO
Yes
Large differences among some groups and some items
NO
Yes
scalar invariance
(*strong invariance*)
metric invariance
(*weak invariance*)
configural invariance
non invariance
Full invariance
Multi-Group CFA
0.001
0.01
0.1
Approximate invariance
BSEM
0.001
0.01
0.1
Partial invariance
Multi-Group CFA
Approximate partial invariance
Alignment, BSEM Alignment, Partial BSEM

Recovery of group means for partial MI situation

Groups affect.	Non-invariant	MG-CFA (no MI)			MG-CFA			PMG-CFA			MG-BSEM			PMG-BSEM			AMG-CFA		
		Cor	RMSE	95C	Cor	RMSE	95C	Cor	RMSE	95C	Cor	RMSE	95C	Cor	RMSE	95C	Cor	RMSE	95C
25%	1	**0.994**	**0.040**	**0.948**	**0.989**	**0.049**	0.889	**0.993**	**0.041**	**0.944**	**0.990**	0.063	0.795	**0.994**	**0.039**	**0.950**	**0.993**	**0.044**	**0.928**
	2	**0.994**	**0.040**	**0.948**	**0.984**	**0.057**	0.855	**0.993**	**0.041**	**0.940**	**0.985**	0.084	0.682	**0.994**	**0.039**	**0.948**	**0.989**	**0.051**	0.883
	3	**0.994**	**0.040**	**0.948**	0.979	0.064	0.835	**0.993**	**0.042**	**0.940**	**0.981**	0.110	0.579	**0.993**	**0.039**	**0.950**	0.976	0.069	0.820
	4	**0.994**	**0.040**	**0.948**	0.971	0.073	0.810	**0.991**	**0.045**	**0.935**	0.976	0.132	0.508	**0.992**	**0.042**	**0.956**	0.951	0.096	0.756
50%	1	**0.994**	**0.040**	**0.948**	**0.985**	**0.056**	0.835	**0.993**	**0.041**	**0.942**	**0.985**	0.084	0.675	**0.994**	**0.038**	**0.958**	**0.989**	**0.048**	**0.905**
	2	**0.994**	**0.040**	**0.948**	0.975	0.069	0.764	**0.993**	**0.041**	**0.940**	0.977	0.122	0.545	**0.994**	**0.038**	**0.955**	**0.984**	0.061	0.816
	3	**0.994**	**0.040**	**0.948**	0.964	0.083	0.720	**0.992**	**0.043**	**0.939**	0.968	0.177	0.411	**0.993**	**0.041**	**0.953**	0.962	0.091	0.674
	4	**0.994**	**0.040**	**0.948**	0.953	0.095	0.677	**0.988**	**0.050**	**0.921**	0.960	0.230	0.342	**0.989**	**0.049**	**0.953**	0.928	0.121	0.574
75%	1	**0.994**	**0.040**	**0.948**	**0.981**	0.062	0.782	**0.993**	**0.041**	**0.941**	**0.981**	0.101	0.598	**0.994**	**0.038**	**0.953**	**0.991**	**0.050**	0.881
	2	**0.994**	**0.040**	**0.948**	0.966	0.080	0.677	**0.993**	**0.042**	**0.937**	0.968	0.168	0.433	**0.993**	**0.039**	**0.955**	0.976	0.076	0.703
	3	**0.994**	**0.040**	**0.948**	0.951	0.097	0.607	**0.991**	**0.046**	**0.926**	0.956	0.262	0.313	**0.991**	**0.044**	**0.947**	0.948	0.109	0.524
	4	**0.994**	**0.040**	**0.948**	0.937	0.113	0.542	**0.985**	**0.055**	**0.905**	0.946	0.347	0.262	**0.985**	**0.057**	**0.944**	0.919	0.136	0.419
100%	1	**0.994**	**0.040**	**0.948**	0.977	0.089	0.573	**0.993**	**0.041**	**0.944**	0.978	0.135	0.470	**0.994**	**0.039**	**0.959**	**0.989**	0.061	0.786
	2	**0.994**	**0.040**	**0.948**	0.958	0.118	0.460	**0.992**	**0.045**	**0.931**	0.961	0.240	0.323	**0.992**	**0.044**	**0.950**	0.965	0.110	0.480
	3	**0.994**	**0.040**	**0.948**	0.940	0.145	0.381	**0.989**	**0.051**	**0.917**	0.945	0.368	0.237	**0.990**	**0.053**	**0.938**	0.937	0.141	0.373
	4	**0.994**	**0.040**	**0.948**	0.922	0.171	0.326	NA	NA	NA	0.932	0.529	0.190	NA	NA	NA	0.923	0.153	0.365

Recovery of path coefficients for for partial MI situation

Groups affect.	Non-invariant	MG-CFA (no MI)			MG-CFA			PMG-CFA			MG-BSEM			PMG-BSEM			AMG-CFA		
		mean	RMSE	95C	mean	RMSE	95C	mean	RMSE	95C	mean	RMSE	95C	mean	RMSE	95C	mean	RMSE	95C
25%	1	0.303	0.007	0.915	0.302	0.007	0.909	0.302	0.007	0.917	0.288	0.014	0.894	0.298	0.010	0.933	0.306	0.010	0.931
	2	0.302	0.007	0.918	0.303	0.007	0.897	0.303	0.007	0.919	0.280	0.020	0.842	0.301	0.009	0.938	0.306	0.010	0.921
	3	0.303	0.008	0.914	0.302	0.007	0.886	0.303	0.007	0.914	0.271	0.030	0.749	0.303	0.009	0.941	0.305	0.009	0.901
	4	0.303	0.007	0.919	0.302	0.007	0.871	0.303	0.007	0.915	0.262	0.038	0.644	0.305	0.009	0.939	0.306	0.010	0.847
50%	1	0.303	0.007	0.923	0.303	0.007	0.901	0.303	0.007	0.917	0.280	0.020	0.845	0.302	0.009	0.937	0.306	0.010	0.927
	2	0.303	0.007	0.918	0.302	0.007	0.882	0.302	0.007	0.916	0.262	0.038	0.641	0.304	0.009	0.935	0.305	0.009	0.910
	3	0.302	0.007	0.916	0.302	0.008	0.856	0.303	0.007	0.914	0.246	0.054	0.411	0.308	0.010	0.936	0.304	0.009	0.867
	4	0.303	0.007	0.918	0.302	0.009	0.829	0.302	0.007	0.913	0.229	0.071	0.216	0.312	0.013	0.932	0.304	0.012	0.795
75%	1	0.303	0.008	0.912	0.302	0.007	0.893	0.302	0.007	0.917	0.271	0.029	0.758	0.302	0.009	0.938	0.305	0.009	0.925
	2	0.302	0.007	0.925	0.301	0.008	0.863	0.302	0.007	0.922	0.246	0.054	0.409	0.307	0.010	0.937	0.304	0.009	0.898
	3	0.302	0.007	0.915	0.303	0.009	0.814	0.303	0.007	0.911	0.224	0.076	0.180	0.313	0.014	0.925	0.304	0.010	0.832
	4	0.303	0.007	0.915	0.302	0.009	0.785	0.303	0.007	0.908	0.202	0.098	0.064	0.320	0.020	0.909	0.302	0.012	0.762
100%	1	0.302	0.007	0.918	0.306	0.017	0.853	0.303	0.007	0.922	0.263	0.037	0.644	0.305	0.010	0.937	0.307	0.012	0.916
	2	0.302	0.007	0.917	0.309	0.024	0.768	0.303	0.008	0.909	0.231	0.069	0.265	0.314	0.015	0.918	0.308	0.019	0.817
	3	0.303	0.007	0.910	0.312	0.029	0.715	0.301	0.008	0.915	0.203	0.097	0.092	0.326	0.027	0.884	0.308	0.026	0.717
	4	0.305	0.005	0.908	0.318	0.035	0.636	NA	NA	NA	0.219	0.081	0.167	NA	NA	NA	0.311	0.030	0.665


Small „natural" differences among groups
NO
Yes
Large differences among some groups and some items
NO
Yes
scalar invariance
(*strong invariance*)
metric invariance
(*weak invariance*)
configural invariance
non invariance
Full invariance
Multi-Group CFA
0.001
0.01
0.1
Approximate invariance
BSEM
Partial invariance
Multi-Group CFA
0.001
0.01
0.1
Approximate partial invariance
Alignment, BSEM Alignment, Partial BSEM

Recovery of group means for AMI situation

AMI	MG-CFA			MG-BSEM			AMG-CFA		
	Cor	RMSE	95C	Cor	RMSE	95C	Cor	RMSE	95C
0.001	**0.993**	**0.044**	**0.921**	**0.993**	**0.043**	**0.928**	**0.991**	**0.045**	**0.923**
0.005	**0.988**	**0.058**	0.796	**0.989**	0.064	**0.915**	**0.988**	**0.057**	0.807
0.010	**0.983**	0.071	0.711	**0.984**	0.086	**0.908**	**0.983**	0.070	0.699
0.050	0.939	0.140	0.404	0.939	0.186	**0.907**	0.935	0.134	0.398

Recovery of path coefficients for AMI situation

AMI	MG-CFA			MG-BSEM			AMG-CFA		
	mean	RMSE	95C	mean	RMSE	95C	mean	RMSE	95C
0.001	**0.303**	**0.033**	**0.913**	**0.296**	**0.032**	**0.924**	**0.308**	**0.032**	**0.919**
0.005	**0.303**	**0.036**	0.886	**0.304**	**0.034**	**0.937**	**0.307**	**0.034**	**0.903**
0.010	**0.305**	**0.038**	0.867	0.313	**0.036**	**0.941**	**0.309**	**0.036**	0.883
0.050	0.311	**0.055**	0.716	0.338	**0.057**	**0.907**	**0.308**	**0.051**	0.722

How good are the tools for detecting different type of measurement invariance

1. Posterior Predictive P-values (PPP),

2. Bayesian Information Criterion (BIC),

3. Deviance Information Criterion (DIC).

How good are the tools for detecting different type of measurement invariance

MG-BSEM: Priors for differences in MG-BSEM	Fit measure	Approximate measurement (non)invariance 0.000	0.001	0.005	0.010	0.025	0.050
0.000	PPP	**0.176**	0.202	0.010	0.000	0.000	0.000
	BIC	0.000	0.000	0.000	0.000	0.000	0.000
	DIC	**0.753**	0.224	0.005	0.000	0.000	0.000
0.001	PPP	0.131	**0.229**	0.080	0.015	0.000	0.000
	BIC	0.000	0.000	0.000	0.000	0.000	0.000
	DIC	0.234	**0.665**	0.088	0.003	0.000	0.000
0.005	PPP	0.136	0.207	**0.331**	0.292	0.061	0.005
	BIC	0.000	0.000	0.000	0.000	0.000	0.000
	DIC	0.013	0.111	**0.707**	0.227	0.000	0.000
0.010	PPP	0.086	0.081	0.228	0.224	0.281	0.133
	BIC	0.000	0.000	0.000	0.000	0.000	0.000
	DIC	0.000	0.000	0.175	**0.499**	0.064	0.000
0.025	PPP	0.103	0.060	0.143	**0.327**	0.302	0.342
	BIC	0.071	0.020	0.008	0.000	0.000	0.000
	DIC	0.000	0.000	0.005	0.083	0.164	0.023
0.050	PPP	0.368	0.222	0.208	0.141	**0.355**	**0.520**
	BIC	**0.929**	**0.980**	**0.992**	**1.000**	**1.000**	**1.000**
	DIC	0.000	0.000	0.020	0.189	**0.772**	**0.977**

How good are the tools for detecting different type of measurement invariance

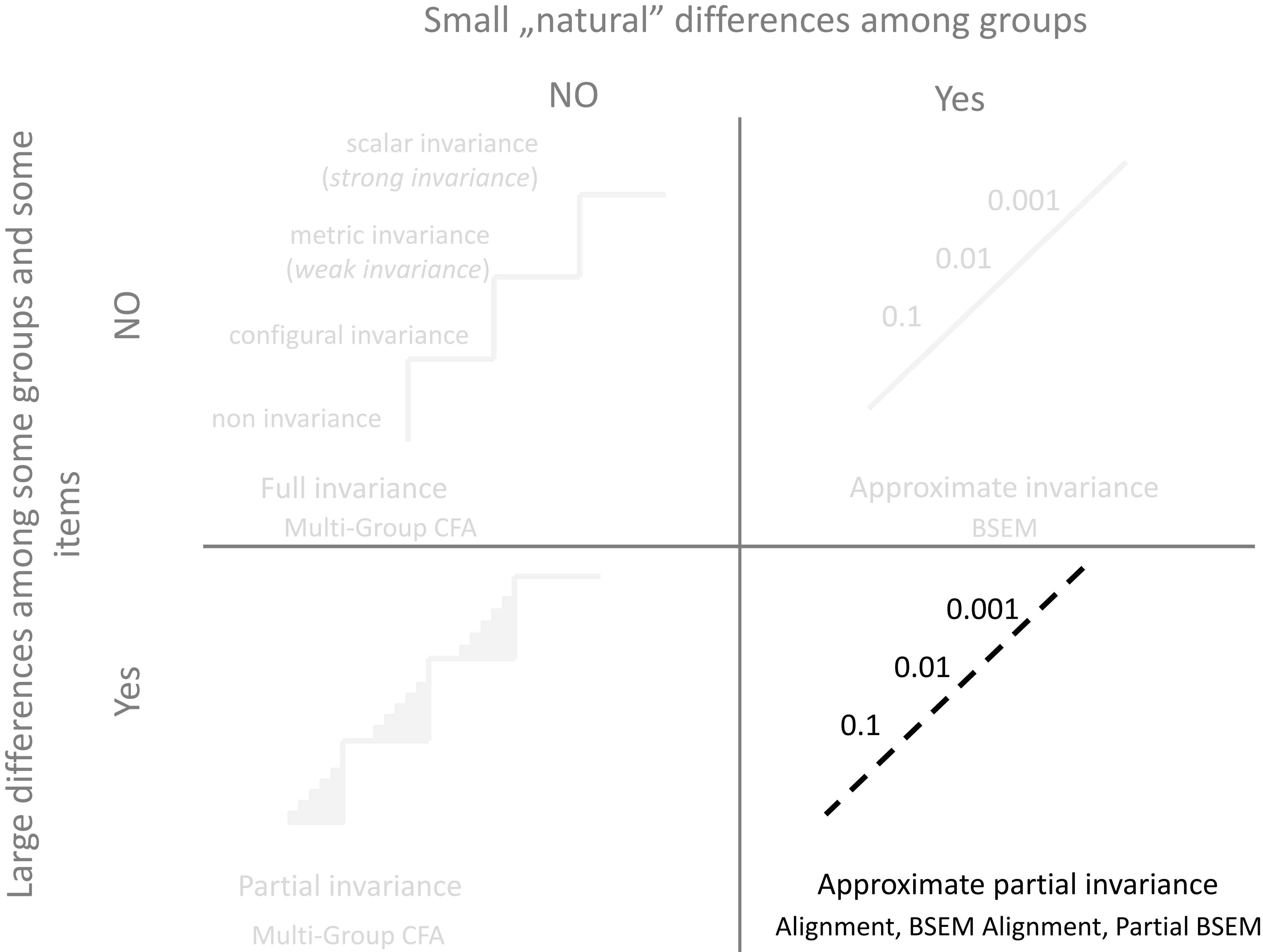
MG-BSEM		Fit measure	Approximate measurement (non)invariance					
			0.000	0.001	0.005	0.010	0.025	0.050
Priors for differences in MG-BSEM	0.000	PPP	**0.176**	0.202	0.010	0.000	0.000	0.000
		BIC	0.000	0.000	0.000	0.000	0.000	0.000
		DIC	**0.753**	0.224	0.005	0.000	0.000	0.000
	0.001	PPP	0.131	**0.229**	0.080	0.015	0.000	0.000
		BIC	0.000	0.000	0.000	0.000	0.000	0.000
		DIC	0.234	**0.665**	0.088	0.003	0.000	0.000
	0.005	PPP	0.136	0.207	**0.331**	0.292	0.061	0.005
		BIC	0.000	0.000	0.000	0.000	0.000	0.000
		DIC	0.013	0.111	**0.707**	0.227	0.000	0.000
	0.010	PPP	0.086	0.081	0.228	0.224	0.281	0.133
		BIC	0.000	0.000	0.000	0.000	0.000	0.000
		DIC	0.000	0.000	0.175	**0.499**	0.064	0.000
	0.025	PPP	0.103	0.060	0.143	**0.327**	0.302	0.342
		BIC	0.071	0.020	0.008	0.000	0.000	0.000
		DIC	0.000	0.000	0.005	0.083	0.164	0.023
	0.050	PPP	0.368	0.222	0.208	0.141	**0.355**	**0.520**
		BIC	**0.929**	**0.980**	**0.992**	**1.000**	**1.000**	**1.000**
		DIC	0.000	0.000	0.020	0.189	**0.772**	**0.977**

How good are the tools for detecting different type of measurement invariance

MG-BSEM		Fit measure	Approximate measurement (non)invariance					
			0.000	0.001	0.005	0.010	0.025	0.050
Priors for differences in MG-BSEM	0.000	PPP	**0.176**	0.202	0.010	0.000	0.000	0.000
		BIC	0.000	0.000	0.000	0.000	0.000	0.000
		DIC	**0.753**	0.224	0.005	0.000	0.000	0.000
	0.001	PPP	0.131	**0.229**	0.080	0.015	0.000	0.000
		BIC	0.000	0.000	0.000	0.000	0.000	0.000
		DIC	0.234	**0.665**	0.088	0.003	0.000	0.000
	0.005	PPP	0.136	0.207	**0.331**	0.292	0.061	0.005
		BIC	0.000	0.000	0.000	0.000	0.000	0.000
		DIC	0.013	0.111	**0.707**	0.227	0.000	0.000
	0.010	PPP	0.086	0.081	0.228	0.224	0.281	0.133
		BIC	0.000	0.000	0.000	0.000	0.000	0.000
		DIC	0.000	0.000	0.175	**0.499**	0.064	0.000
	0.025	PPP	0.103	0.060	0.143	**0.327**	0.302	0.342
		BIC	0.071	0.020	0.008	0.000	0.000	0.000
		DIC	0.000	0.000	0.005	0.083	0.164	0.023
	0.050	PPP	0.368	0.222	0.208	0.141	**0.355**	**0.520**
		BIC	**0.929**	**0.980**	**0.992**	**1.000**	**1.000**	**1.000**
		DIC	0.000	0.000	0.020	0.189	**0.772**	**0.977**

How good are the tools for detecting different type of measurement invariance

MG-BSEM		Fit measure	Approximate measurement (non)invariance					
			0.000	0.001	0.005	0.010	0.025	0.050
Priors for differences in MG-BSEM	0.000	PPP	**0.176**	0.202	0.010	0.000	0.000	0.000
		BIC	0.000	0.000	0.000	0.000	0.000	0.000
		DIC	**0.753**	0.224	0.005	0.000	0.000	0.000
	0.001	PPP	0.131	**0.229**	0.080	0.015	0.000	0.000
		BIC	0.000	0.000	0.000	0.000	0.000	0.000
		DIC	0.234	**0.665**	0.088	0.003	0.000	0.000
	0.005	PPP	0.136	0.207	**0.331**	0.292	0.061	0.005
		BIC	0.000	0.000	0.000	0.000	0.000	0.000
		DIC	0.013	0.111	**0.707**	0.227	0.000	0.000
	0.010	PPP	0.086	0.081	0.228	0.224	0.281	0.133
		BIC	0.000	0.000	0.000	0.000	0.000	0.000
		DIC	0.000	0.000	0.175	**0.499**	0.064	0.000
	0.025	PPP	0.103	0.060	0.143	**0.327**	0.302	0.342
		BIC	0.071	0.020	0.008	0.000	0.000	0.000
		DIC	0.000	0.000	0.005	0.083	0.164	0.023
	0.050	PPP	0.368	0.222	0.208	0.141	**0.355**	**0.520**
		BIC	**0.929**	**0.980**	**0.992**	**1.000**	**1.000**	**1.000**
		DIC	0.000	0.000	0.020	0.189	**0.772**	**0.977**


Small „natural" differences among groups
NO
Yes
Large differences among some groups and some items
NO
Yes
scalar invariance
(*strong invariance*)
metric invariance
(*weak invariance*)
configural invariance
non invariance
Full invariance
Multi-Group CFA
0.001
0.01
0.1
Approximate invariance
BSEM
Partial invariance
Multi-Group CFA
0.001
0.01
0.1
Approximate partial invariance
Alignment, BSEM Alignment, Partial BSEM

Recovery of group means for partial + AMI situation

Groups affect.	AMI	Non-invariant	MG-CFA			PMG-CFA			MG-BSEM			PMG-BSEM			AMG-CFA		
			Cor	RMSE	95C	Cor	RMSE	95C	Cor	RMSE	95C	Cor	RMSE	95C	Cor	RMSE	95C
25%	0.005	1	**0.984**	0.062	0.786	**0.987**	**0.056**	0.828	**0.985**	0.078	0.851	**0.988**	0.061	**0.928**	**0.986**	0.061	0.773
		2	0.979	0.073	0.723	**0.987**	0.061	0.774	**0.981**	0.094	0.796	**0.987**	0.066	**0.905**	**0.981**	0.073	0.696
		3	0.972	0.078	0.716	**0.984**	0.062	0.781	0.976	0.107	0.751	**0.985**	0.068	**0.925**	0.968	0.088	0.659
		4	0.970	0.077	0.745	**0.980**	0.065	0.803	0.974	0.111	0.755	**0.981**	0.073	**0.936**	0.954	0.096	0.668
	0.010	1	0.979	0.074	0.690	**0.983**	0.070	0.716	**0.981**	0.095	0.887	**0.984**	0.083	**0.919**	**0.980**	0.072	0.694
		2	0.976	0.075	0.708	**0.982**	0.067	0.740	0.977	0.099	0.857	**0.983**	0.077	**0.940**	0.975	0.079	0.655
		3	0.966	0.088	0.642	0.973	0.079	0.699	0.970	0.123	0.791	0.974	0.095	**0.900**	0.960	0.097	0.577
		4	0.965	0.093	0.633	0.966	0.089	0.670	0.970	0.137	0.763	0.968	0.105	**0.915**	0.951	0.110	0.538
50%	0.005	1	**0.980**	0.070	0.722	**0.987**	**0.059**	0.793	**0.981**	0.093	0.810	**0.988**	0.064	**0.920**	**0.983**	0.067	0.733
		2	0.969	0.078	0.679	**0.985**	**0.058**	0.806	0.971	0.116	0.728	**0.986**	0.062	**0.936**	0.955	0.084	0.648
		3	0.961	0.093	0.642	**0.983**	0.066	0.774	0.965	0.152	0.624	**0.983**	0.072	**0.924**	0.954	0.107	0.532
		4	0.946	0.102	0.622	0.971	0.078	0.744	0.955	0.178	0.564	0.972	0.088	**0.923**	0.924	0.127	0.476
	0.010	1	0.973	0.083	0.639	**0.980**	0.075	0.689	0.974	0.106	0.850	**0.981**	0.084	**0.918**	0.974	0.080	0.629
		2	0.963	0.094	0.595	0.975	0.082	0.651	0.967	0.139	0.763	0.977	0.097	0.885	0.961	0.099	0.536
		3	0.961	0.097	0.603	0.973	0.083	0.653	0.965	0.153	0.742	0.974	0.099	**0.907**	0.951	0.109	0.513
		4	0.945	0.109	0.559	0.952	0.101	0.611	0.953	0.186	0.652	0.953	0.115	0.899	0.922	0.133	0.437
75%	0.005	1	0.978	0.072	0.699	**0.987**	**0.059**	0.802	0.979	0.103	0.743	**0.988**	0.064	**0.924**	**0.982**	0.071	0.698
		2	0.949	0.096	0.598	0.969	0.069	0.768	0.952	0.164	0.572	**0.980**	0.077	**0.906**	0.951	0.102	0.543
		3	0.943	0.105	0.545	0.977	0.071	0.741	0.948	0.191	0.541	0.978	0.078	**0.920**	0.937	0.116	0.471
		4	0.935	0.119	0.498	0.962	0.091	0.664	0.944	0.229	0.481	0.962	0.103	0.894	0.919	0.138	0.392
	0.010	1	0.970	0.085	0.622	**0.980**	0.074	0.687	0.972	0.117	0.823	**0.981**	0.085	**0.914**	0.973	0.084	0.609
		2	0.961	0.091	0.606	0.975	0.076	0.693	0.963	0.147	0.749	0.977	0.089	**0.927**	0.958	0.099	0.530
		3	0.947	0.111	0.511	0.965	0.091	0.626	0.953	0.190	0.660	0.966	0.109	**0.900**	0.939	0.125	0.417
		4	0.930	0.125	0.482	0.936	0.115	0.571	0.938	0.215	0.620	0.937	0.120	**0.929**	0.912	0.141	0.382
100%	0.005	1	0.974	0.094	0.529	**0.986**	**0.060**	0.793	0.975	0.123	0.667	**0.987**	0.068	**0.924**	0.979	0.084	0.606
		2	0.953	0.125	0.443	**0.982**	0.072	0.727	0.957	0.193	0.522	**0.983**	0.082	**0.908**	0.954	0.120	0.459
		3	0.939	0.147	0.381	0.975	0.089	0.659	0.944	0.259	0.455	0.975	0.103	0.873	0.933	0.150	0.364
	0.010	1	0.967	0.107	0.506	0.978	0.081	0.662	0.968	0.142	0.735	0.979	0.099	0.885	0.968	0.103	0.512
		2	0.954	0.128	0.410	0.974	0.093	0.601	0.958	0.183	0.655	0.975	0.110	0.884	0.952	0.122	0.407
		3	0.941	0.142	0.407	0.961	0.114	0.535	0.948	0.236	0.586	0.961	0.142	0.863	0.937	0.141	0.366

Recovery of group means for partial + AMI

Groups affect.	AMI	Non-invariant	MG-CFA			PMG-CFA			MG-BSEM			PMG-BSEM			AMG-CFA		
			mean	RMSE	95C	mean	RMSE	95C	mean	RMSE	95C	mean	RMSE	95C	mean	RMSE	95C
25%	0.005	1	0.304	0.010	0.884	0.304	0.010	0.893	0.297	0.010	0.936	0.307	0.011	0.945	0.308	0.013	0.902
		2	0.304	0.011	0.871	0.304	0.010	0.884	0.289	0.013	0.907	0.309	0.012	0.938	0.307	0.012	0.888
		3	0.303	0.011	0.861	0.303	0.010	0.888	0.282	0.019	0.865	0.311	0.014	0.940	0.306	0.013	0.870
		4	0.303	0.011	0.847	0.304	0.011	0.876	0.275	0.025	0.810	0.313	0.015	0.936	0.306	0.013	0.835
	0.010	1	0.305	0.014	0.858	0.306	0.013	0.867	0.306	0.012	0.944	0.315	0.017	0.936	0.308	0.015	0.873
		2	0.305	0.014	0.847	0.305	0.014	0.860	0.300	0.011	0.944	0.317	0.019	0.937	0.308	0.015	0.862
		3	0.304	0.014	0.835	0.305	0.014	0.855	0.292	0.013	0.919	0.319	0.020	0.929	0.307	0.015	0.842
		4	0.304	0.015	0.818	0.305	0.015	0.838	0.287	0.016	0.888	0.323	0.024	0.919	0.307	0.017	0.804
50%	0.005	1	0.304	0.011	0.883	0.304	0.010	0.891	0.289	0.014	0.908	0.308	0.012	0.941	0.306	0.012	0.899
		2	0.302	0.011	0.859	0.302	0.011	0.887	0.275	0.025	0.809	0.311	0.014	0.940	0.304	0.012	0.878
		3	0.303	0.011	0.834	0.303	0.010	0.885	0.263	0.037	0.681	0.317	0.017	0.936	0.304	0.012	0.847
		4	0.303	0.011	0.812	0.304	0.011	0.866	0.250	0.050	0.499	0.323	0.023	0.917	0.304	0.013	0.780
	0.010	1	0.303	0.013	0.849	0.303	0.013	0.861	0.298	0.011	0.938	0.316	0.017	0.936	0.306	0.015	0.866
		2	0.303	0.013	0.841	0.304	0.013	0.866	0.286	0.016	0.896	0.321	0.022	0.928	0.305	0.014	0.852
		3	0.304	0.014	0.814	0.304	0.013	0.851	0.276	0.025	0.828	0.327	0.028	0.914	0.306	0.015	0.826
		4	0.304	0.014	0.793	0.306	0.015	0.824	0.266	0.034	0.732	0.335	0.035	0.896	0.306	0.016	0.769
75%	0.005	1	0.303	0.010	0.868	0.304	0.010	0.891	0.282	0.019	0.864	0.311	0.013	0.938	0.306	0.012	0.891
		2	0.302	0.010	0.837	0.302	0.010	0.880	0.262	0.038	0.666	0.316	0.017	0.931	0.303	0.012	0.856
		3	0.302	0.010	0.806	0.302	0.010	0.880	0.245	0.055	0.442	0.322	0.022	0.914	0.302	0.011	0.813
		4	0.304	0.013	0.767	0.306	0.012	0.852	0.232	0.068	0.279	0.333	0.033	0.890	0.305	0.015	0.739
	0.010	1	0.305	0.014	0.848	0.305	0.014	0.859	0.294	0.012	0.930	0.320	0.021	0.929	0.308	0.015	0.865
		2	0.303	0.013	0.819	0.304	0.013	0.855	0.276	0.025	0.827	0.326	0.027	0.912	0.304	0.014	0.831
		3	0.304	0.015	0.780	0.306	0.014	0.833	0.262	0.038	0.676	0.336	0.036	0.875	0.305	0.015	0.786
		4	0.303	0.015	0.744	0.306	0.015	0.796	0.248	0.052	0.499	0.347	0.047	0.860	0.303	0.017	0.729
100%	0.005	1	0.306	0.018	0.823	0.303	0.011	0.883	0.275	0.026	0.791	0.313	0.016	0.934	0.307	0.016	0.868
		2	0.309	0.022	0.762	0.303	0.012	0.879	0.250	0.050	0.501	0.327	0.028	0.892	0.307	0.021	0.786
		3	0.314	0.032	0.686	0.305	0.015	0.858	0.232	0.068	0.303	0.355	0.055	0.736	0.310	0.029	0.696
	0.010	1	0.306	0.019	0.803	0.303	0.014	0.856	0.287	0.016	0.896	0.324	0.025	0.910	0.308	0.018	0.833
		2	0.311	0.026	0.746	0.303	0.016	0.837	0.267	0.034	0.720	0.342	0.042	0.842	0.309	0.024	0.761
		3	0.313	0.030	0.693	0.303	0.019	0.793	0.249	0.051	0.506	0.382	0.082	0.580	0.310	0.029	0.703

Conclusions

- Don't use short scales (3-, 4-items)
- PI situation is not a problem for PMG (if non-invariance items are known)
- PI situation is not a problem for AMG models (if there is little non-invariance item
- Approximate MI (AMI) is problematic
  - AMI>0.001 recovery of means is very difficult
  - SEM Path coefficients are reasonably robust (even MG-CFA will do)
- AMI with partial non-invariance is even more problematic

Limitations and further work

- Add additional conditions which are useful and realistic for applied researchers
- Problem of detection of biased items in partial invariance situations
- Treat Partial Invariance (PI) items as unknown in similar simulations
  - Assess effects of misspecification of PI items
- Impact of MI on more complicated SEM models