University of Amsterdam

Research Institute of Child Development and Education

Using cross-level invariance constraints when testing multilevel mediation using SEM

Suzanne Jak S.Jak@uva.nl

Overview

- Multilevel data
- Multilevel mediation and factor analysis
- Problems
  - Interpretation
  - Estimation
- Small simulation study

Multilevel data

Data in clusters

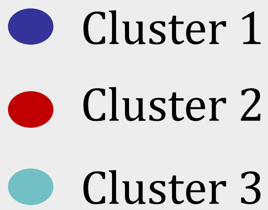
Multilevel hypotheses

- Typology of variables:
  - Level 1 variables: all variables on which individuals in the same cluster can have different scores
  - Level 2 variables: all variables on which individuals in the same cluster can not have different scores
- Most Level 1 variables have variance at level 2 as well! E.g. the average job performance differs across companies, the average math ability may differ across school classes
- Hypotheses may involve variables at different levels
  - E.g. Math self-efficacy mediates the influence of classroom climate on math achievement

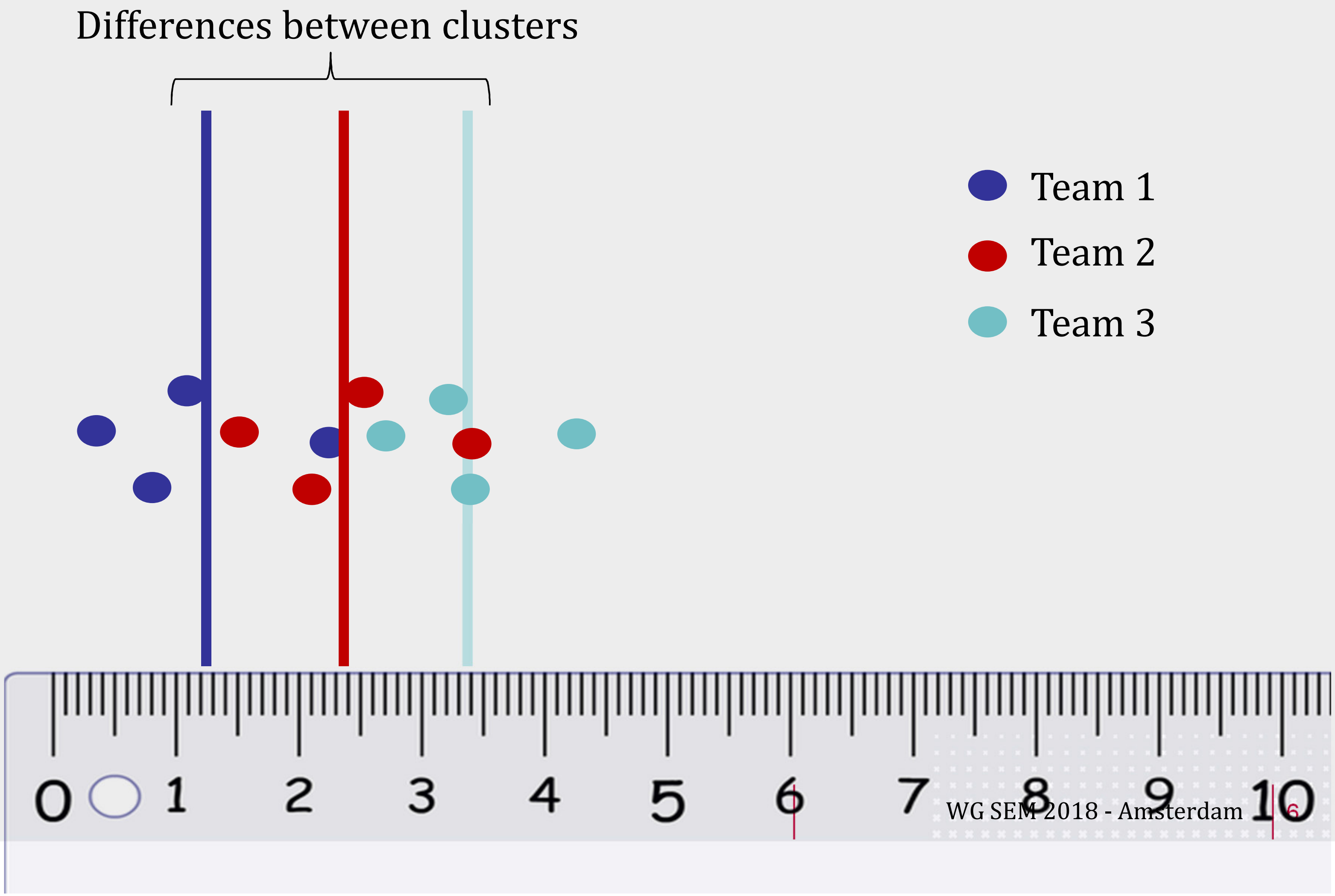
Multilevel variable decomposition

$$y_{ij} = \gamma + u_j + \eta_{ij}$$

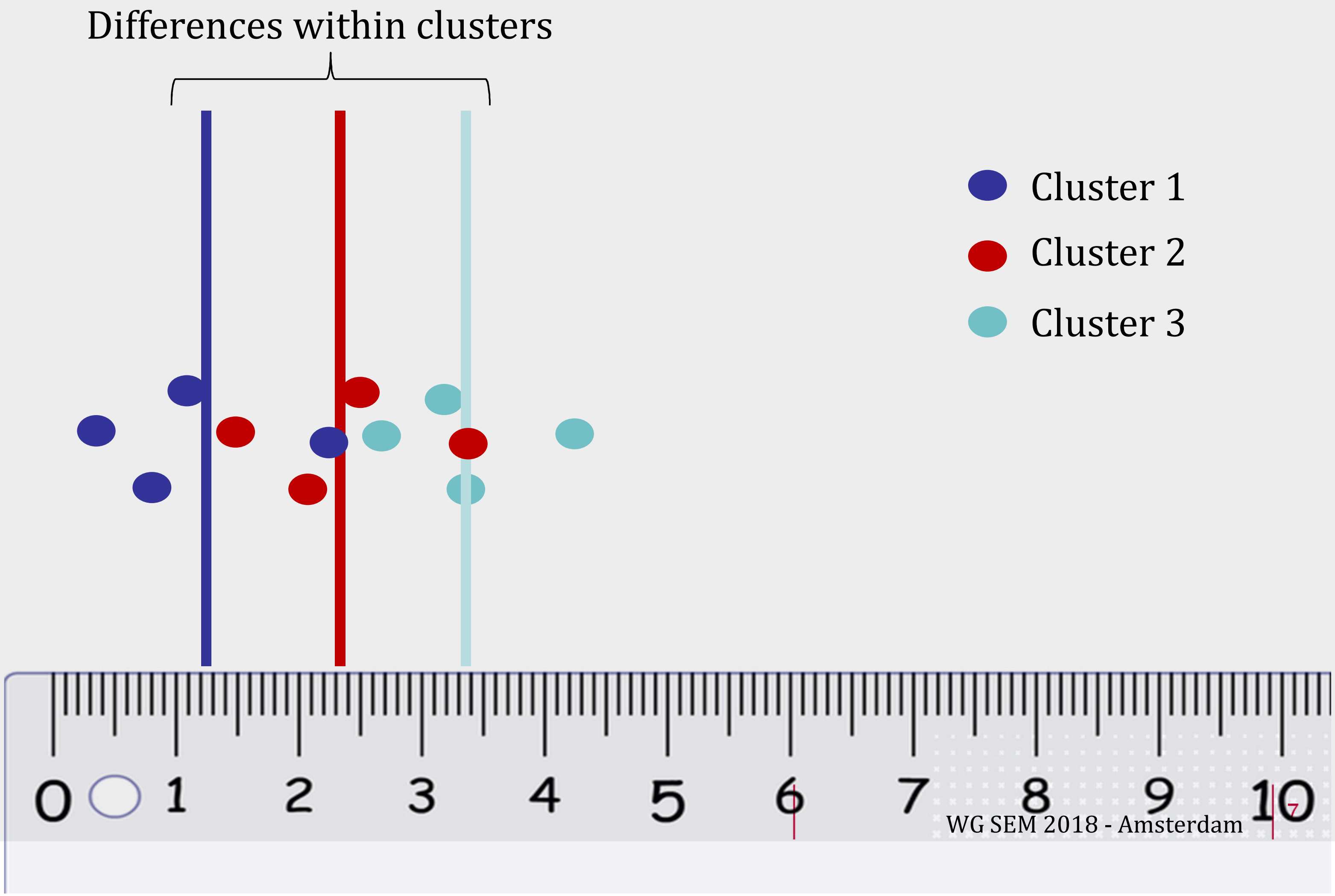
Observed score of person i in cluster j	Grand mean	Cluster deviation from grand mean	Individual deviation from cluster mean



Multilevel variable decomposition



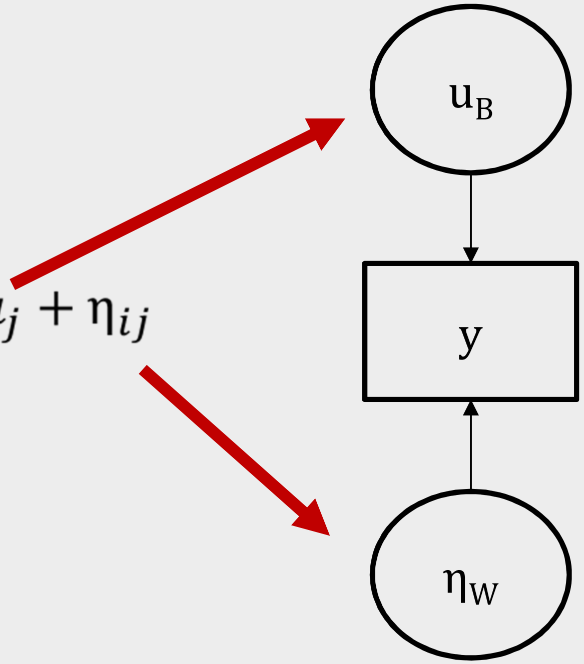
Multilevel variable decomposition



Within / between formulation

- Observed variable is decomposed into a within- and a between-component

$$y_{ij} = \gamma + u_j + \eta_{ij}$$



Between level / Level 2

Within level / Level 1

University of Amsterdam

Multilevel mediation

Psychological Methods
2010, Vol. 15, No. 3, 209–233

1082-989X/10/$12.00 DOI: 10.1037/a0020141

A General Multilevel SEM Framework for Assessing Multilevel Mediation

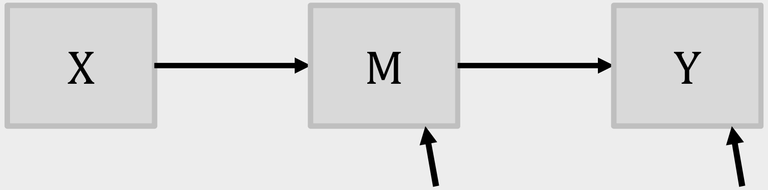
Kristopher J. Preacher
University of Kansas

Michael J. Zyphur
University of Melbourne

Zhen Zhang
Arizona State University

Several methods for testing mediation hypotheses with 2-level nested data have been proposed by researchers using a multilevel modeling (MLM) paradigm. However, these MLM approaches do not accommodate mediation pathways with Level-2 outcomes and may produce conflated estimates of between- and within-level components of indirect effects. Moreover, these methods have each appeared in isolation, so a unified framework that integrates the existing methods, as well as new multilevel mediation models, is lacking. Here we show that a multilevel structural equation modeling (MSEM) paradigm can overcome these 2 limitations of mediation analysis with MLM. We present an integrative 2-level MSEM mathematical framework that subsumes new and existing multilevel mediation approaches as special cases. We use several applied examples and accompanying software code to illustrate the flexibility of this framework and to show that different substantive conclusions can be drawn using MSEM versus MLM.

Multilevel mediation



Most common models (McNeish, 2017)

	X	M	Y
Level	1	1	1
	1	2	1
	1	2	2
	2	1	1
	2	2	1
	2	2	2
	1	1	2
	2	1	2

Each of the three variables can be on the within-level or on the between-level (Preacher, Zyphur & Zhang, 2010)

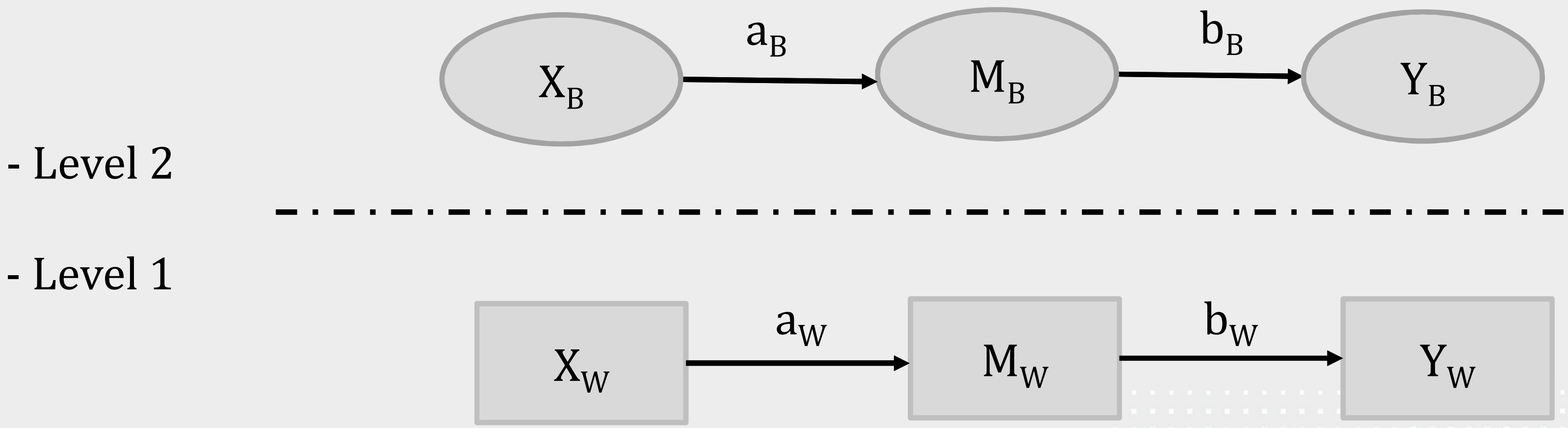
UNIVERSITY OF AMSTERDAM

Example 1-1-1 mediation

Students nested in classes

Student-level indirect effect: $a_W * b_W$
Class-level indirect effect: $a_B * b_B$

- X: Student self-esteem
- M: Student effort
- Y: Student math performance



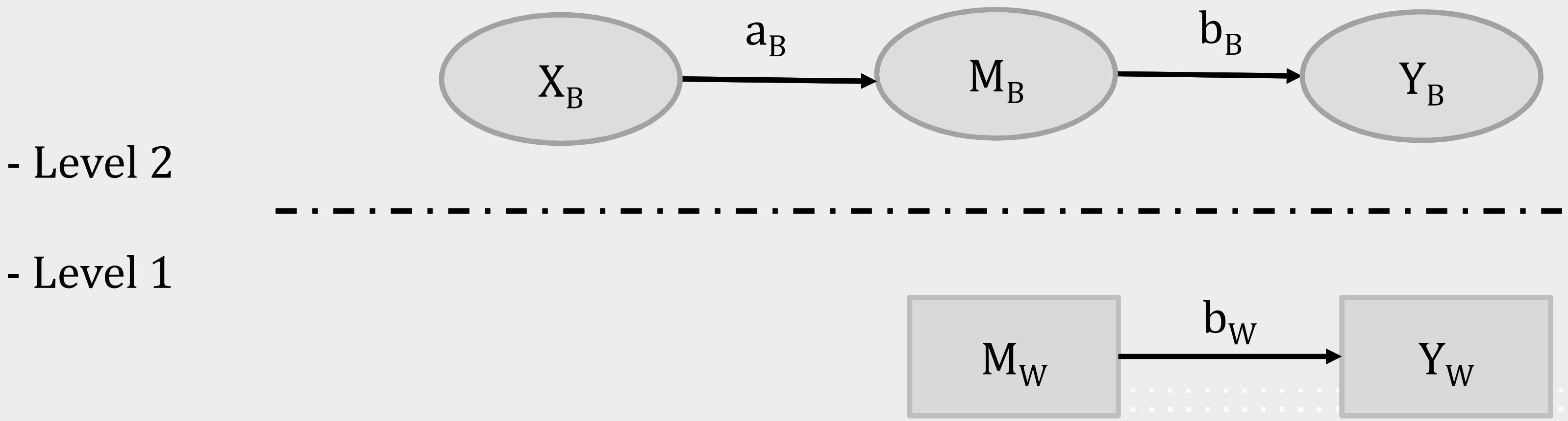
Example 2-1-1 mediation

If any of the three variables is a between-level variable, mediation occurs at the between-level only

Employees nested in teams

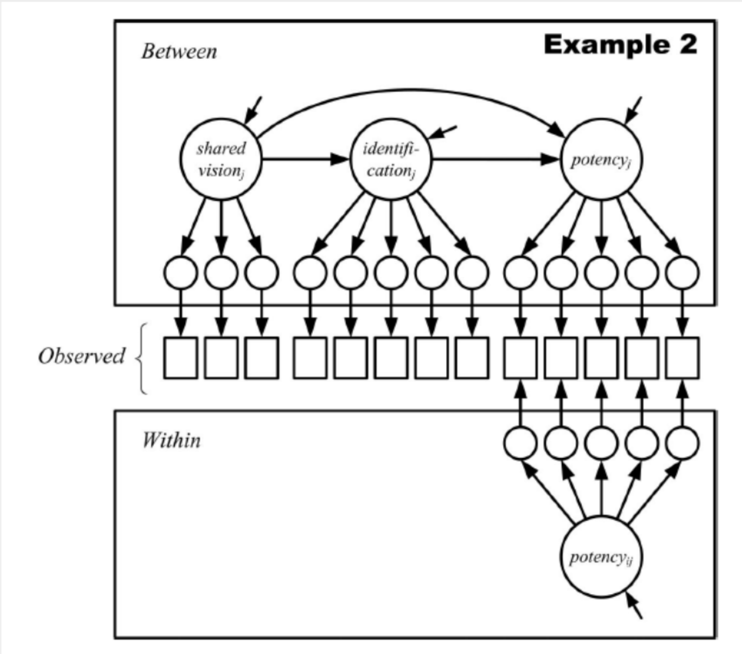
Indirect effect: a_B*b_B

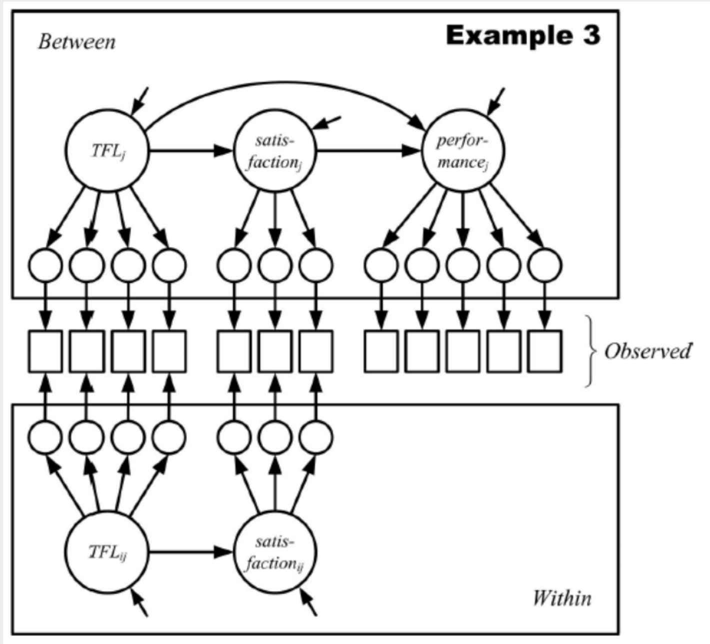
- X: Treatment variable 'Training on the job'
- M: Job-related skills
- Y: Job performance



Multilevel mediation with latent variables

Examples from Preacher et al. (2010)

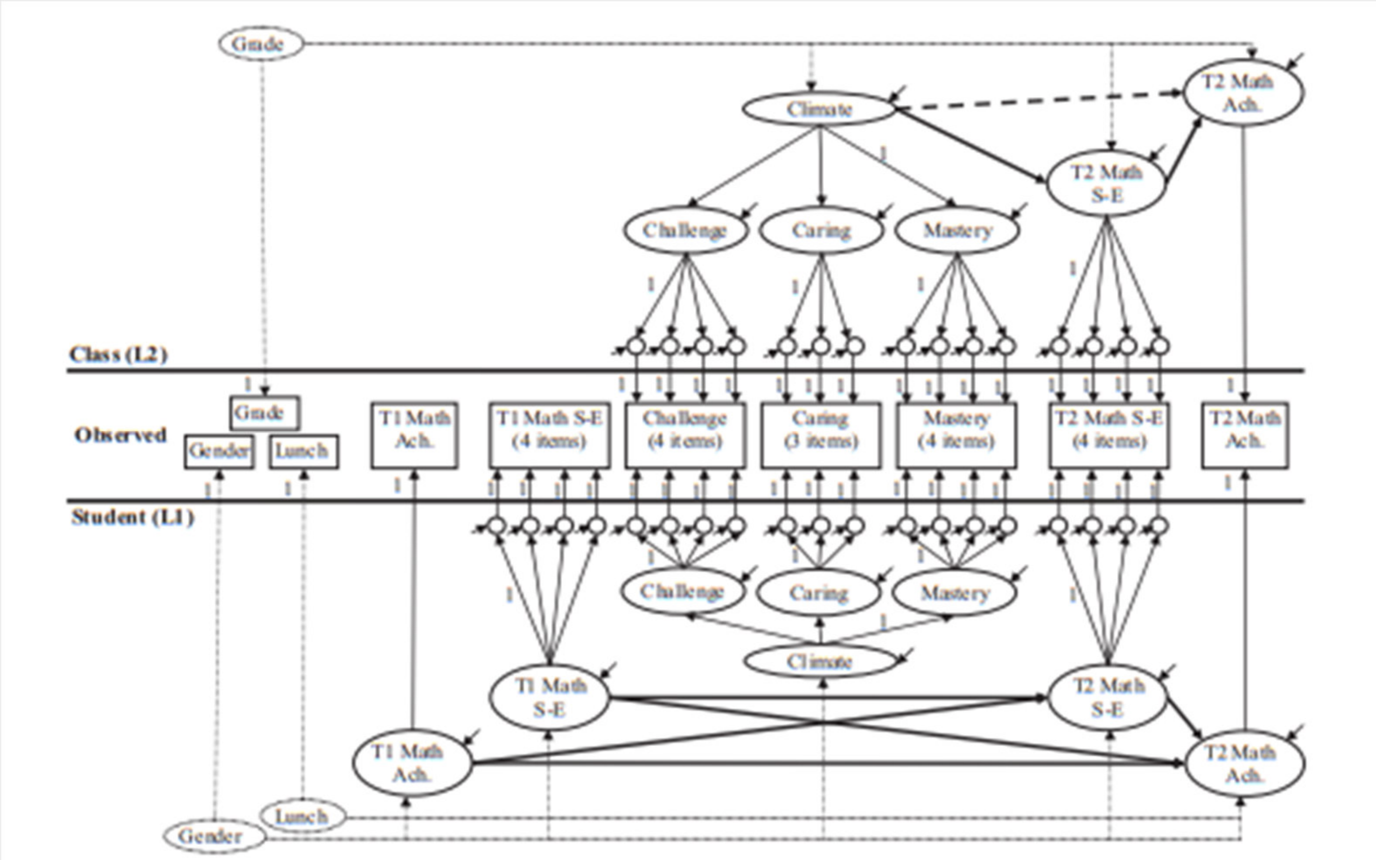




UNIVERSITY OF AMSTERDAM

Multilevel mediation with latent variables

Example from Morin et al. (2014)



Multilevel mediation with latent variables

- Li and Beretvas (2013)
- Comparing mediation models with composite scores vs. latent variables
  - Serious convergence issues with $N_{between} < 80$
  - Low power to detect indirect effect
- “Unfortunately, MLSEM cannot be recommended over the use of composite scores for the majority of conditions examined”

University of Amsterdam

Multilevel confirmatory factor analysis

Article

Journal of Educational and Behavioral Statistics
2016, Vol. 41, No. 5, pp. 481–520
DOI: 10.3102/1076998616646200

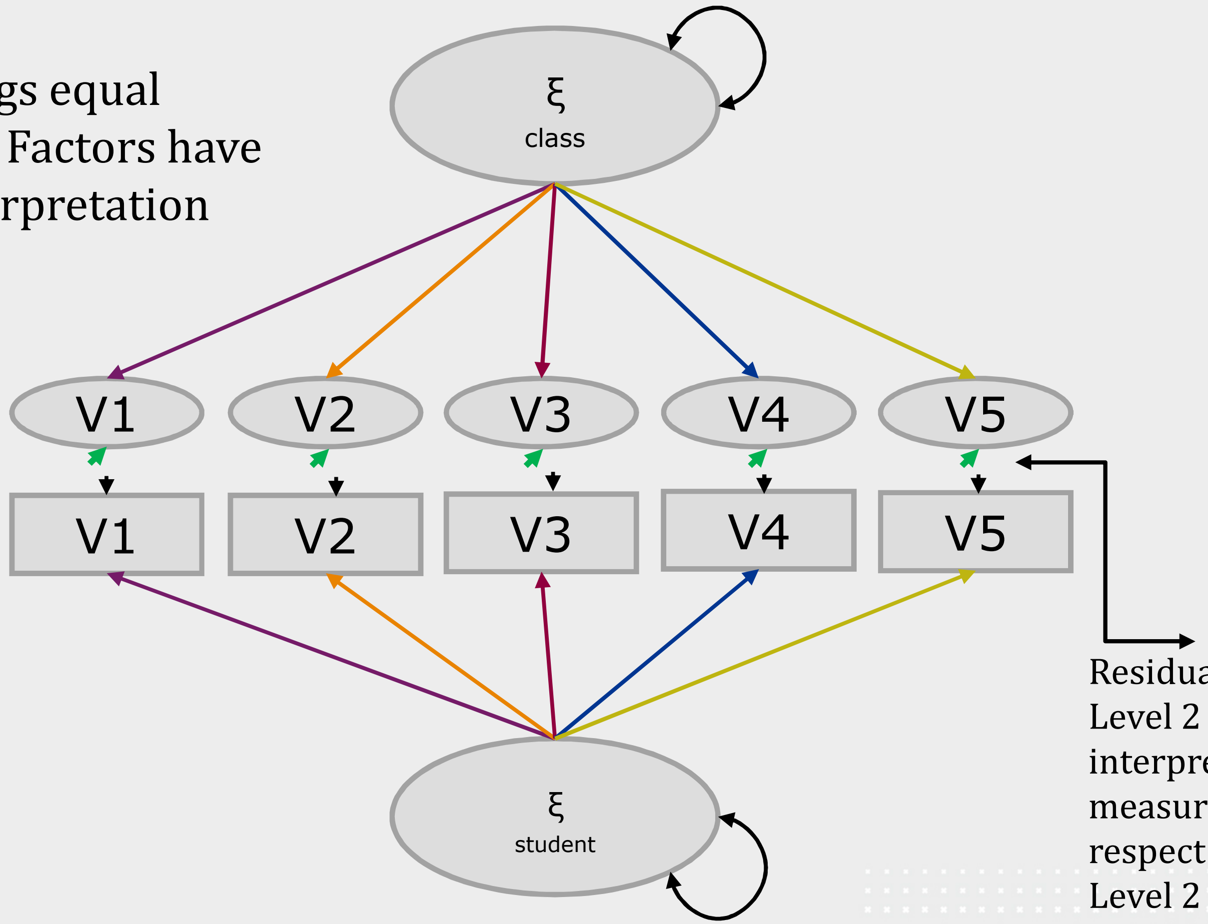
Construct Meaning in Multilevel Settings

Laura M. Stapleton
Ji Seung Yang
Gregory R. Hancock
University of Maryland

We present types of constructs, individual- and cluster-level, and their confirmatory factor analytic validation models when data are from individuals nested within clusters. When a construct is theoretically individual level, spurious construct-irrelevant dependency in the data may appear to signal cluster-level dependency; in such cases, however, and consistent with theory, a single-level analysis with a correction for dependency may be appropriate. Regarding cluster-level constructs, we discuss two types—shared and config-

Interpretation two-level factor model

Factor loadings equal across levels: Factors have the same interpretation across levels



Residual variance at Level 2 can be interpreted as measurement bias with respect to unmeasured Level 2 variables

Cross-level invariance

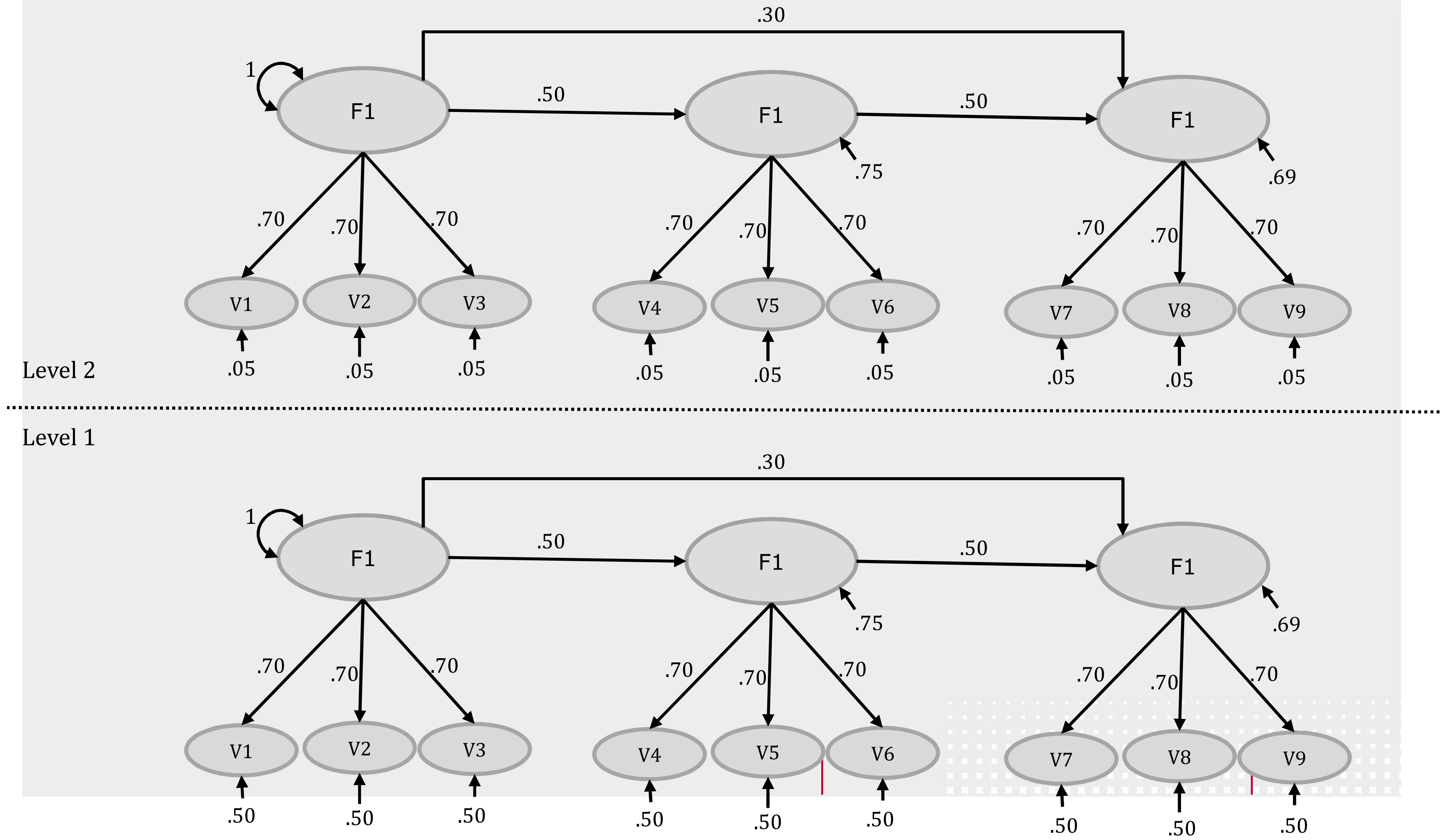
- Not mentioned by Preacher et al. (2010) or Li and Beretvas (2013)
- Li and Beretvas generated data with cross-level invariance, but did not constrain $\mathbf{\Lambda}$ when fitting the model
  - Interpretation problems
  - Estimation problems

Simulation study

- Effect of not-applying cross-level invariance constraints on convergence and power
- Generate 2000 datasets under model with cross-level invariance
- Fit model with and without across-level invariance with lavaan

Population model

ICC = .33



Results

- Non-convergence ICC = .17

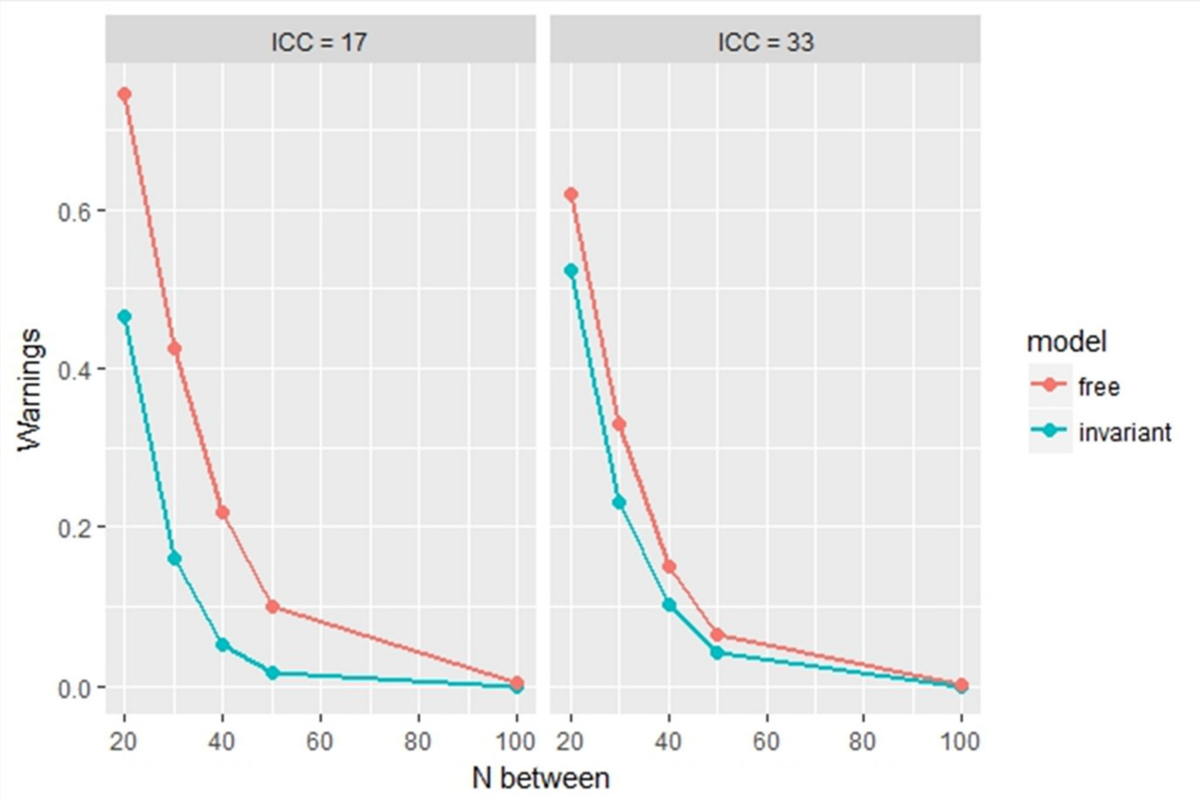
$N_{between}$	Invariant	Free
20	0	42
30	0	5
40	0	1
50	0	0
100	0	0

2000 replications

UNIVERSITY OF AMSTERDAM

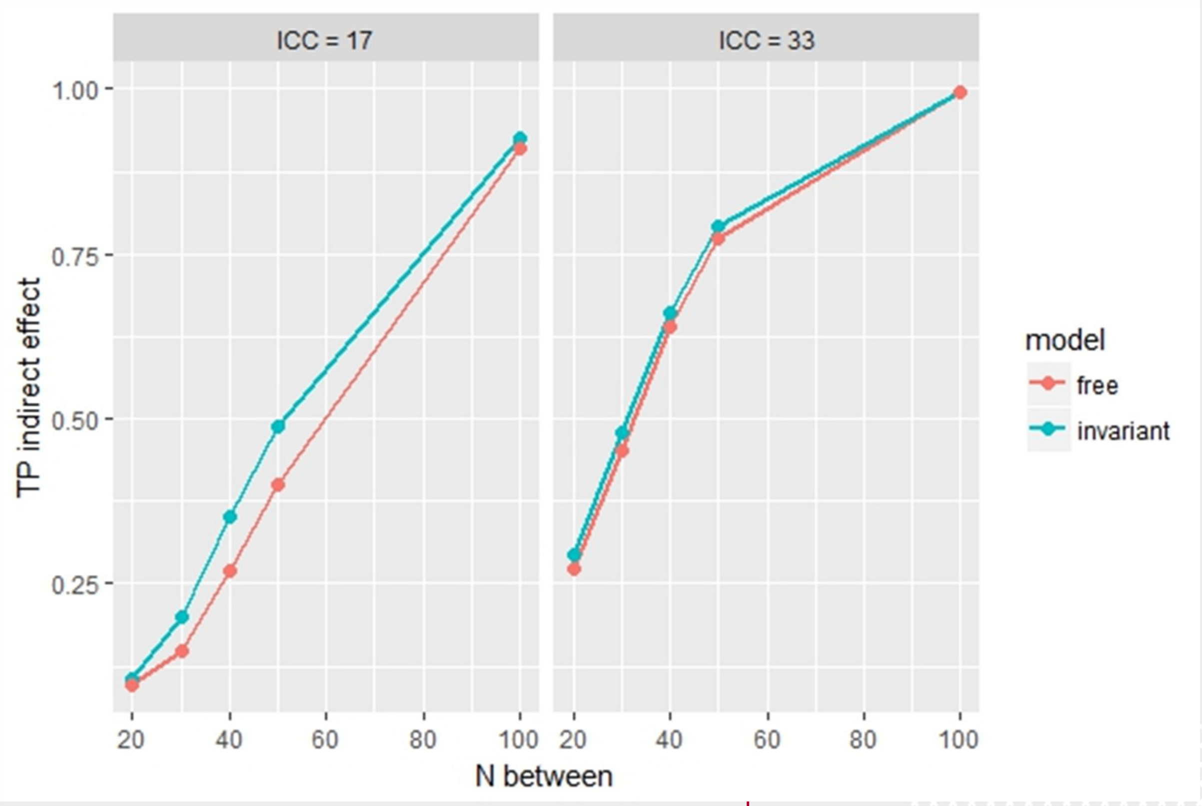
Results

- Warnings `("some estimated ov variances are negative")`



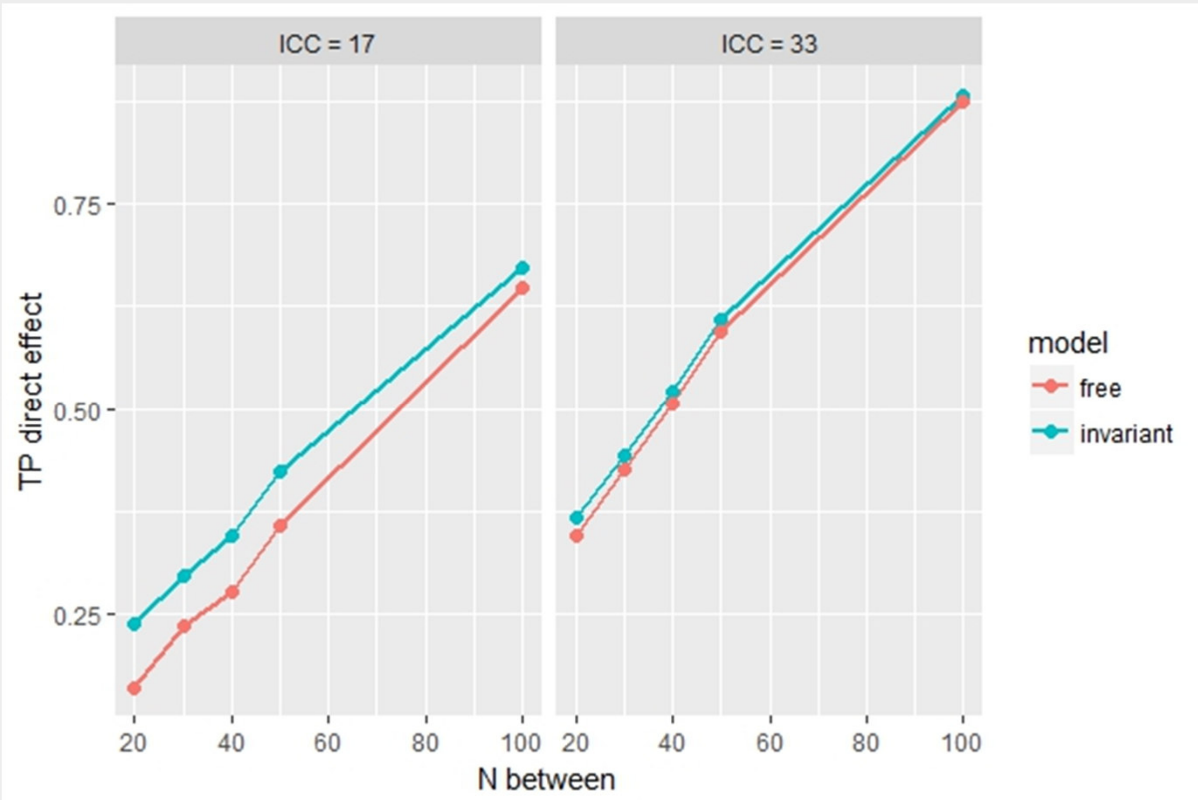
Results

- Significant indirect effect (based on delta-method)



Results

- Significant direct effect



Conclusion and discussion

- Cross-level invariance of lambda (if appropriate)
  - Facilitates interpretation
  - Enhances estimation and power
- If not appropriate
  - Biased mediational effects (Guenole, 2016)

Conclusion and discussion

- If strong factorial invariance across clusters holds: $\boldsymbol{\Lambda}_{\text{within}} = \boldsymbol{\Lambda}_{\text{between}}$ and $\boldsymbol{\theta}_{\text{between}} = 0$
  - Reduces number of parameters → less estimation problems?
- Need to extend simulation study
  - Vary N_{within}, vary ICC, bootstrap SEs

Thank you for listening!

- Questions?