Dominant Firms, Barriers to Entry Capital and Antitrust Policy

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Abstract

1 Introduction

In the last decade the theory of competition has moved away from the static theory, based on the perfect-imperfect of competition paradigm, to a dynamic theory. Competition in the traditional sense is price competition and the deviation from perfectly competitive prices is shown to result in welfare losses. Accordingly antitrust and competition laws in the U.S. and Europe had adhered to the static blueprint of the perfect competition paradigm.

The recent research direction moves away from the structure-conduct-performance paradigm, a long time framework for industrial organization studies and regulatory policy, and stresses that the dynamics of competition, does not necessarily depend on market structure. The new direction gives more relevance to the competitive behavior (for example rivalry in oligopolistic setting). It views competition as price competition as well as competition for product and process innovation. Accordingly, industrial organization and antitrust literature have attempted to integrate more dynamic and evolutionary view points into the studies. The major change of the paradigm came from both, first, the view that entry dynamic is always an important

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source of potential competition and, second, the view that strategic behavior of incumbents may result in prices below monopoly prices (limiting pricing) and in a drive for new product and process innovation to prevent entry or to preempt the rivals' strategies. As the overall usefulness of perfect competition framework has become more questionable as a guideline for antitrust regulation and competition policy it is still controversial what the features of a new antitrust rule and competition policy should be and how they should be designed for the new paradigm of competition dynamics.

Along the line of the new paradigm the present paper presumes that, as firms are exposed to the dynamics of competition, they attempt to restrict or inhibit competition through competition restricting investments. Our paper is based on earlier work by Brock (1983) and Dechert and Brock (...) who had studied barriers to entry capital to restrict competition. Yet, nowadays we know that firms not only build up entry preventing capital to reduce market competition (through engaging in increasing returns activity, advertisement, political lobbying, protection of innovations through patents, creating excess capacity and so on) but also can restrict competition through investments that inhibit competitive behavior (for example, investment in coalition formation, lobbying and presuring for anti-competitive regulatory measures, etc). These are all examples of investment that restrict the dynamics of competition (price competition as well as competition in welfare improving product and process innovation). This paper is concerned with such type of investments.

The main idea of our paper comes from earlier industrial organization literature that has shown that the threat of entry limits the price setting power of dominant firms and stimulates the incumbents to undertake innovations —both leading to welfare improvements. In that literature it already has been shown that the dominant firms, as incumbents, strive to build up entry preventing capital. In such an environment of heterogeneous firms, incumbents and entering firms, the dynamics of competition has been studied. The above mentioned paper by Brock (1983), had argued that when dominant firms face a threat of competitive fringe firms in the industry they will have an incentive to prevent it. Investing into barriers to entry capital through engaging in production activities with increasing returns and high adjustment cost of investment as well as through advertising, lobbying and excess capacity and patent protection, the dominant firm can create thresholds above which fringe firms cannot induce price competition and stimulate innovations. Brock (1983) has shown that if the dominant firms builds up entry-deterring capital, this might produce thresholds beyond which the incumbents can reduce or eliminate the dynamics of competition. Commencing with Brock's (1983) specific study on barriers to entry capital we propose to consider quite a general type of competition restricting investment, as above discussed, that incumbents can undertake to inhibit competition. We then also can show that depending on how the other firms and the regulatory institutions respond to this type of investment, complex dynamics, multiple steady states and thresholds, separating different domains of attraction, may emerge. Since the effectiveness of competition restricting investment indeed depend in part on regulatory rules set and enforced by antitrust institutions, we show how an antitrust and competition policy can be designed that may prevent the build up of such a competition restricting capital, strengthening incentives for price and innovation competition.

In this context the antitrust and competition policy should be to stimulate, encourage, and if necessary, restore the dynamics of markets by prohibiting the restrictions of competition. Yet, one can view the dominant firms as playing a game against the regulatory agencies, but the regulatory agency set adverse conditions, as for example has been discussed in the robust control literature (see Zhang and Semmler (2005). Yet as our results show the regulatory agency does not persistently have to intervene. Below some threshold there are forces that revive competition, yet above that threshold not. Competition policy should, through some regulatory instruments, increase the domains of attraction where competition takes place. Yet, we also show in our paper that it is quite intricate to detect the superior or inferior domains of attraction. We use dynamic programming to compute those domains of attraction.

The remainder of the paper is organized as follows. Section 2 introduces the preliminary model, taking first, prices as constant. We present a number of example to illustrate different outcomes in different variants. Section 3 introduces price reaction by employing a downward demand function. Here we also compute the welfare loss due to restricted competition established through competition restricting investment. Section 4 studies antitrust and competition policy as resulting from our theoretical and numerical study. Section 5 concludes the paper. The appendix gives a brief summary of the dynamic programming method used to solve some of our model variants.

2 Model

2.1 Industry Environment

We presume a dominant firm in an industry. We can also interpret the dominant firm as a group of firms whose activities are highly coordinated. Yet for short we will use the term dominant firm. We presume that the dominant firm and the competitive fringe compete for a given market demand d. The dominant firm may have an incentive to restrict the other firms' behavior through investing in competition restricting capital. We here study the traditional case of a dominant firm that builds up entry-deterring capital.

The dominant firm's problem is to maximize the discounted future net cash flows:

$$\max_{x} \int_{0}^{\infty} e^{-rt} \left[q - C(q) - x - \varphi(x)\right] dt \tag{1}$$

where q is output of the dominant firm, $C(q)$ is the cost of production, and $C' > 0$. Let's assume a linear cost function for simplicity, $C' = c > 0$ where $1-c > 0$. This implies the dominant firm may enjoy on increasing returns in terms of production technology. x is entry-deterring gross investment, and φ is adjustment costs with properties $\varphi'(x) \gtreqless 0$ for $x \gtreqless 0$ and $\varphi'' > 0$. We assume that the price of a unit of investment good is 1.

Entry-deterring capital accumulation is:

$$\dot{E} = x - \delta_E E \tag{2}$$

where δ_E is the depreciation rate.

Output of the dominant firm is residual demand:

$$q = s(E; \rho, \chi)d \tag{3}$$

where $0 < s(E) < 1$ is a market share of the leading firm with properties; $s(0) = 0$, $s(+\infty) = 1$, $s'(E) \geq 0$, $s'(0) = s'(+\infty) = 0$. ρ is a parameter which measures the efficiency of the entry-preventing capital to enlarge the dominant firm's enlarging its market share, $\partial s/\partial \rho > 0$. χ is a parameter which represents how an antitrust and competition policy can be designed that may prevent the build up of entry-deterring capital, $\partial s/\partial \chi < 0$.

At the end, obviously, entry-deterring capital cannot be negative $-E \leq 0$. From the non-negativity condition, we can find a new constraint[1]

$$h = -E \ \leq 0 \Rightarrow \ \dot{h} = -\dot{E} = -[x - \delta_E E] \leq 0 \text{ whenever } h = 0. \tag{4}$$

Let the Lagrangian be written as

$$\mathcal{L} = s(E)d - C(q) - x - \varphi(x) + \lambda(x - \delta_E E) - \theta\dot{h} \tag{5}$$

The maximum principle gives the following set of first-order conditions:

$$\mathcal{L}_x = -1 - \varphi'(x) + \lambda + \theta = 0 \tag{6}$$

$$\mathcal{L}_\theta = -\dot{h} = x - \delta_E E \geq 0 \quad \theta \geq 0 \qquad \theta\mathcal{L}_\theta = 0 \tag{7}$$

$$-E \leq 0 \qquad \theta E = 0 \tag{8}$$

(8) is the complementary-slackness condition appended to (7) which ensures that (7) is valid only when the constraint is binding ($E = 0$).

At points where $\dot{\theta}$ is differentiable,

$$\dot{\theta} \leq 0 \quad (= 0 \text{ when } -E < 0). \tag{9}$$

$$\dot{E} = x - \delta_E E \tag{10}$$

$$\dot{\lambda} = (r + \delta_E)\lambda - (1 - C'(q))s'(E)d + \theta\delta_E \tag{11}$$

plus transversality conditions.

[1]Since h is not allowed to exceed 0, then whenever $h = 0$, we must forbid h to increase. Thus, the problem has a state-space constraint.

2.2 Optimal Entry-Deterring Investment Rules

Our primary interest is the optimal entry-deterring investment. Whenever entry-deterring capital is positive, $E > 0$ (constraint not biding), from (8) and (9), we know that $\theta = \dot{\theta} = 0$. Therefore, from (6), we can have the optimal entry-deterring investment rule:

$$\begin{cases} x > 0 & \lambda > 1 \\ x = 0 \quad \text{for} & \lambda = 1 \\ x < 0 & \lambda < 1 \end{cases} \qquad \text{when } E > 0. \tag{12}$$

Since λ is the discounted value of the sum of marginal future net cash flows by increasing a unit of entry-deterring capital,[2] (12) suggests that if it is greater than 1 (which comes from the assumption of the price of a unit of investment goods set 1), the firm invests more until λ decreases to 1, and vice versa. Note that λ is affected by the parameters such as δ_E, ρ, and χ. High depreciation of the entry preventing efforts makes the dominant firm discourage Low efficiency of entry-deterring investment and strong regulation enforced by antitrust institutions will discourage the dominant firm's entry-preventing efforts.

On the other hand, when the constraint is binding for some time period, it follows that $E = \dot{E} = 0$. Thus, from (10), the optimal entry-deterring investment rule is

$$x = 0 \qquad \text{when } E = 0. \tag{13}$$

This case arises when the market share of the dominant firm is negligibly small. In the static theory of competition this has been interpreted as a perfectly competitive market environment. The firm switches between the rules (12) and (13) as the state of its entry-deterring capital changes.

2.3 Dynamic System

To make the economic implication clearer, we make a 2D system in terms of x and E. From (6) and (11), we derive an equation of motion for x:

[2] From (11), $\dot{\lambda} - (r + \delta_E)\lambda = -s'(E; \rho, \chi)d$. By solving this first order differential equation, we obtain:
$\lambda_t = d \int_t^\infty s'(E; \rho, \chi) e^{-(r+\delta_E)\tau} d\tau$.

$$\dot{x} = \frac{1}{\varphi''(x)}[(r + \delta_E)(1 + \varphi'(x)) - (1 - C'(q))s'(E)d - \theta r + \dot{\theta}] \ . \tag{14}$$

(14) together with (2) describes our system. Figure ## depicts the phase diagram of the system. In this economy, we possibly have two attractors; one attractor in the positive region, another one at zero and a repellor is somewhere in the middle. This case is recognized as a typical state-dependency and threshold problem, i.e. if the industry tends toward high concentration equilibrium or ends up with a competitive environment depends on how much entry-deterring capital the dominant firm has accumulated. An industry tends toward a higher concentration when the dominant firm accumulates entry-deterring capital beyond a certain level that is called a "threshold". If this is not the case, with a different parameter set, we can have a sole attractor at zero which suggests that the industry will be settled in a competitive environment regardless of the stock of entry-deterring capital by the dominant firm. This is likely to happen when the depreciation of the entry-deterring capital is high or/and when the regulatory agency imposes a strong regulation. Both cases discourage the dominant firm to accumulate and hold entry-deterring capital.

Yet, overall we want to remark here that the local analysis of computing the number of equilibria does not necessarily imply that those are actually reached. Using dynamic programming we will show that more a complex behavior can arise.

We next check the stability of the system around each positive steady state. Note that $\theta = \dot{\theta} = 0$ for any $x^*, E^* > 0$. The associated Jacobian matrix J:

$$J = \begin{bmatrix} \frac{\partial \dot{E}}{\partial E} & \frac{\partial \dot{E}}{\partial x} \\ \frac{\partial \dot{x}}{\partial E} & \frac{\partial \dot{x}}{\partial x} \end{bmatrix}_{x^*, E^* > 0} = \begin{bmatrix} -\delta_E & 1 \\ \Theta & r + \delta_E \end{bmatrix}. \tag{15}$$

where

$$\Theta \equiv \frac{1}{\varphi''(x)}[C''(q)(s'(E)d)^2 - (1 - C'(q))s''(E)d]. \tag{16}$$

From the assumption of a linear production cost, $C'' = 0$ and $1 - c > 0$ in (16). Therefore, the sign of Θ depends on only s''. Since the market share s has a S-shape, $s'' > 0$ above the reflection point and $s'' < 0$ below the reflection point.

- SS2 (the middle steady state) occurs below the reflection point. Therefore $s'' > 0$ and $\Theta < 0$. The phase diagram also indicates $\frac{\partial \dot{x}}{\partial E} > 0$ in the vicinity of the SS2. Thus,

$$J = \begin{bmatrix} - & + \\ - & + \end{bmatrix}. \tag{17}$$

Det $J = -\delta_E(r+\delta_E) + \frac{1}{\varphi''}(1-c)s''d$, Tr $J = r > 0$, and the discriminant $\Delta = (\text{Tr } J)^2 - 4\text{Det } J$. Det $J > 0$ holds for relatively small r, δ_E, c and large d which ensure multiple steady states. [PROOF COMES HERE.] Those facts tell that the dynamics in the vicinity of the SS2 is a source (the SS2 is a repellor). It can be a spiral source for $\Delta < 0$ or a node source for $\Delta < 0$.

- SS3 (the upper steady state) occurs above the reflection point. $s'' < 0$ and $\Theta > 0$ holds. Thus,

$$J = \begin{bmatrix} - & + \\ + & + \end{bmatrix}. \tag{18}$$

Det $J < 0$. Therefore, the SS3 is a saddle.

2.4 Numerical Examples

Let us use specific functions for production costs, adjustment costs of entry-deterring investment, and market share determination.

$$C(q) = cq \tag{19}$$

$$\varphi(x) = \alpha x^2 \tag{20}$$

$$s(E) = \frac{E^\rho}{\chi^\rho + E^\rho} < 1 \tag{21}$$

We assume a constant marginal cost c for production, $\rho > 1$ represents the efficiency of the entry-preventing effort and χ captures the regulatory state of the industry. For convenience, we set up a default parameter set as:

Example: (Default A) $r = .02$, $\delta_E = .15$, $\rho = 5$, $\chi = 10$, $d = 10$, $c = .001$, $\alpha = .5$

	SS1 (attractor)	SS2 (repellor)	SS3 (attractor)
Entry-deterring capital E	0	5.02049	13.6949
Investment level x	0	0.753074	2.05423
Market share s	0	0.0309098	0.828093

2.4.1 E_0: Initial Entry-Deterring Capital

A natural monopoly has naturally high entry barriers due to expensive initial costs. Therefore the initial E is likely to be above the threshold in some industries (for example in utilities, like Gas, Electricity, etc.).

2.4.2 χ: Regulatory Environment Change

We presume that χ can be influenced by a policy maker in other words it is a policy parameter.

Example A-1: (Very Weak Regulation) $r = .02$, $\delta_E = .15$, $\rho = 5$, $\chi = 1$, $d = 10$, $c = .001$, $\alpha = .5$

	SS1 (attractor)	SS2 (repellor)	SS3 (attractor)
Entry-deterring capital E	0	0.243818	2.43846
Investment level x	0	0.0365727	0.365769
Market share s	0	0.0008609	0.988534

Example A-2: (Strong Regulation) $r = .02$, $\delta_E = .15$, $\rho = 5$, $\chi = 20$, $d = 10$, $c = .001$, $\alpha = .5$

	SS1 (attractor)	SS2 (repellor)	SS3 (attractor)
Entry-deterring capital E	0	15.9242	18.7755
Investment level x	0	2.38862	2.81632
Market share s	0	0.242417	0.421675

Example A-3: (Very Strong Regulation) $r = .02$, $\delta_E = .15$, $\rho = 5$, $\chi = 30$, $d = 10$, $c = .001$, $\alpha = .5$

	SS1 (unique attractor)
Entry-deterring capital E	0
Investment level x	0
Market share s	0

2.4.3 ρ: Efficiency of Entry-Deterring Effort

Example A-4: (High Efficiency) $r = .02$, $\delta_E = .15$, $\rho = 7$, $\chi = 10$, $d = 10$, $c = .001$, $\alpha = .5$

	SS1 (attractor)	SS2 (repellor)	SS3 (attractor)
Entry-deterring capital E	0	6.05303	13.4589
Investment level x	0	0.907954	2.01883
Market share s	0	0.0289113	0.888881

Example A-5: (Low Efficiency) $r = .02$, $\delta_E = .15$, $\rho = 2$, $\chi = 10$, $d = 10$, $c = .001$, $\alpha = .5$

	SS1 (attractor)	SS2 (repellor)	SS3 (attractor)
Entry-deterring capital E	0	0.997767	10.9931
Investment level x	0	0.149665	1.64897
Market share s	0	0.00985726	0.547202

2.4.4 δ_E: Depreciation of Entry-Deterring Capital

δ_E can be another policy parameter. For example, one can view this as representing the life time of a patent that the firm has obtained whereby δ_E is set by the regulatory agency.

Example A-6: (Low Depreciation) $r = .02$, $\delta_E = .01$, $\rho = 5$, $\chi = 10$, $d = 10$, $c = .001$, $\alpha = .5$

	SS1 (attractor)	SS2 (repellor)	SS3 (attractor)
Entry-deterring capital E	0	2.80561	22.5453
Investment level x	0	0.0280561	0.225453
Market share s	0	0.00173534	0.983122

Example A-7: (100% Depreciation) $r = .02$, $\delta_E = 1$, $\rho = 5$, $\chi = 10$, $d = 10$, $c = .001$, $\alpha = .5$

	SS1 (unique attractor)
Entry-deterring capital E	0
Investment level x	0
Market share s	0

2.4.5 r: Discount Rate

The future discount rate will be high when a product cycle is short and consumers' taste changes rapidly. High uncertainty of future market demand lets the dominant firm pursue a take profit and leave strategy.

Example A-8: (High Discount Rate) $r = .3$, $\delta_E = .15$, $\rho = 5$, $\chi = 10$, $d = 10$, $c = .001$, $\alpha = .5$

	SS1 (attractor)	SS2 (repellor)	SS3 (attractor)
Entry-deterring capital E	0	7.17392	10.5523
Investment level x	0	1.07609	1.58284
Market share s	0	0.159673	0.566792

Example A-9: (Very High Discount Rate) $r = .5$, $\delta_E = .15$, $\rho = 5$, $\chi = 10$, $d = 10$, $c = .001$, $\alpha = .5$

	SS1 (unique attractor)
Entry-deterring capital E	0
Investment level x	0
Market share s	0

3 Restricted Competition and Loss of Benefit

In the previous section, the dominant firm simply maximizes its market share for a given market demand. The maximization of the market-share, however, makes sense only with an inelastic market demand curve. In this section, we introduce a downward sloping market demand, $d(p)$. Therefore, the dominant firm faces a downward residual demand $sd(p)$. We assume that the market price is guided by the dominant firm. The objective of this section is to study the effects of the dominant firm's competition restricting activities on the market price and to explain the possible loss of economic benefits arising hereby.

3.1 Model

The dominant firm's objective is to maximize the discounted future net revenues

$$\max_{x} \int_0^\infty e^{-rt} \left[pq - C(q) - x - \varphi(x)\right] dt \tag{22}$$

subject to (2). The other assumptions are kept same. We conveniently assume that the price is a function of the market share of the dominant firm:

$$p = p(s) \quad \text{for } 0 \leq s \leq 1 \tag{23}$$

where $p'(s) > 0$, $p(0) = p^c$, $p(1) = p^m$. p^c $(= C'(q))$ and p^m are the competitive and monopolistic prices respectively. The dominant firm faces a downward market demand:

$$q = sd(p). \tag{24}$$

The dominant firm's revenue is $R(s) = p(s)sd(p)$. Most empirical studies in Industrial Organization have shown that there is some positive correlation of market share and rates of return.[3] Therefore, we will choose a set of parameters so that $R'(s) > 0$ for $0 \leq s \leq 1$.

The Lagrangian is written as

$$\mathcal{L} = p(s)s(E)d(p) - C(q) - x - \varphi(x) + \lambda(x - \delta_E E) - \theta\dot{h}. \tag{25}$$

[3] See for example Weis (1963). For an extensive survey of earlier literature, see Semmler (1984).

We share the first order conditions (6)-(10) from the previous section and only the equation of motion for λ is modified:

$$\begin{aligned}\dot{\lambda} &= (r+\delta_E)\lambda - p'(s)s'(E)s(E)d(p) \\ &\quad -\{p(s)-C'(q)\}\{s'(E)d(p)+s(E)d'(p)p'(s)s'(E)\}+\theta\delta_E.\end{aligned} \tag{26}$$

3.2 Dynamic System

The equ. (26) modifies the economic system as follows:

$$\begin{aligned}\dot{x} &= \frac{1}{\varphi''(x)}[(r+\delta_E)(1+\varphi'(x)) - p'(s)s'(E)s(E)d(p) \\ &\quad -\{p(s)-C'(q)\}\{s'(E)d(p)+s(E)d'(p)p'(s)s'(E)\}-\theta r+\dot{\theta}]\end{aligned} \tag{27}$$

and

$$\dot{E} = x - \delta_E E. \tag{28}$$

The system again has a state-dependent dynamic property with two attractors or a solo attractor.

3.3 Numerical Examples

Specific functions for the market price and the market demand should be defined. We choose linear functions for simplicity.

$$p(s) = p^c + (p^m - p^c)s \quad \text{for } 0 \le s \le 1 \tag{29}$$

$$d = b - ap \tag{30}$$

p^c, p^m, b and a are chosen so that $R(s) = p(s)sd(p)$ monotonically increases for $0 \le s \le 1$. This could happen for a relatively small difference $(p^m - p^c)$, large b and small a. We created a default parameter set as:

Example: (Default) $r = .02$, $\delta_E = .15$, $\rho = 5$, $\chi = 10$, $c = .001$, $\alpha = .5$, $p^m = 8$, $p^c = 2$, $b = 10$, $a = .5$.

	SS1 (attractor)	SS2 (repellor)	SS3 (attractor)
Entry-deterring capital E	0	4.14476	18.7069
Investment level x	0	0.621714	2.80603
Market share s	0	0.0120842	0.958175
Price Level	2.00	2.0725	7.74905
Market Demand	9.00	8.96375	6.12548

3.4 Loss of Benefit

Assuming that our economy is represented by the default example, what will be the economic consequence of the dominant firm's optimal entry-deterring activities? Under this economic system, the dominant firm will accumulate the entry-deterring capital to reach the high market share steady state if the firm holds, for some reason, the entry-deterring capital above the threshold level. For example, the dominant firm may hold critical patents, have excess capacity, have attracted a large customer stock through advertising and so on. The industry might also end up with a high market share of a few firms.

Using the basic microeconomic theory, we can compute the economic surplus for each steady state equilibrium that is an attractor. When the unrestricted competitive market is approached, the total economic surplus (ES) is the sum of producer's surplus and consumer's surplus:

$$ES_1 = (p^c - c)d(p^c) + \int_{p^c}^{\infty} d(p)dp = \int_{p^c}^{\infty} d(p)dp \tag{31}$$

where c is the constant marginal cost of production. Note that $p^c = c$ at the competitive equilibrium.

On the other hand, the high concentration equilibrium s^* is realized at

$$ES_2 = (p(s^*) - c)d(p(s^*)) + \int_{p(s^*)}^{\infty} d(p)dp. \tag{32}$$

Thus, the deadweight loss from the dominant firm's entry-deterring activities will be computed as:

$$ES_1 - ES_2 = \int_{c}^{p(s^*)} d(p)dp - (p(s^*) - c)d(p(s^*)) > 0. \tag{33}$$

The deadweight loss is always positive as long as the market demand is assumed to have a downward slope. For example, the default parameter set creates the following deadweight loss:

Example: (Default B)

	SS1 (attractor)	SS2 (repellor)	SS3 (attractor)
Deadweight Loss	0	1.07278	15.0081

Therefore, by leaving this industry as it is, positive benefit loss of the amount $ES_1 - ES_2$ will be created. This fact justifies some regulatory agency to intervene into the industry to prevent the loss of benefit.

4 How Antitrust Policy Works

Based on the previous discussion, our question is whether any policy parameter can be used to reduce the possibility of the dominant firm's leading an industry to a high concentration equilibrium. We consider χ and δ_E as policy parameters. χ can be interpreted as general regulatory environment or climate set by laws, regulations, monitoring, finally imposing costs on the firm through penaltes, law suite costs and so on. Also when excessive advertisement, lobbying etc. is restricted, χ will be larger. δ_E represents the depreciation of the cumulative entry-deterring capital of the dominant firm. δ_E is larger when past advertisement or lobbying effort has become less effective due to the consumers' taste changes or any regulatory changes of the life time of the patent. Also the patent can become obsolete. δ_E can be a policy parameter if the regulatory agency has a control over the terms of the patent that the firm has obtained.

Using numerical examples, we can see how antitrust policy might effectively work.

Example B-1: (Weak Regulation) $r = .02$, $\delta_E = .15$, $\rho = 5$, $\chi = 30$, $c = .001$, $\alpha = .5$, $p^m = 8$, $p^c = 2$, $b = 10$, $a = .5$.

	SS1 (attractor)	SS2 (repellor)	SS3 (attractor)
Entry-deterring capital E	0	19.6056	39.8492
Investment level x	0	2.94084	5.97739
Market share s	0	0.106509	0.805265
Price Level p	2.00	2.63906	6.83159
Market Demand d	9.00	8.68047	6.58421
Deadweight Loss	0	1.73984	11.6642

Example B-2: (Strong Regulation) $r = .02$, $\delta_E = .15$, $\rho = 5$, $\chi = 40$, $c = .001$, $\alpha = .5$, $p^m = 8$, $p^c = 2$, $b = 10$, $a = .5$.

	SS1 (attractor)	SS2 (repellor)	SS3 (attractor)
Entry-deterring capital E	0	30.5258	46.5924
Investment level x	0	4.57887	6.98886
Market share s	0	0.20562	0.681959
Price Level p	2.00	3.23372	6.09175
Market Demand d	9.00	8.38314	6.95412
Deadweight Loss	0	2.61262	9.27432

Example B-3: (Very Strong Regulation) $r = .02$, $\delta_E = .15$, $\rho = 5$, $\chi = 50$, $c = .001$, $\alpha = .5$, $p^m = 8$, $p^c = 2$, $b = 10$, $a = .5$.

	SS1 (unique attractor)
Entry-deterring capital E	0
Investment level x	0
Market share s	0
Price Level p	2.00
Market Demand d	9.00
Deadweight Loss	0

Example B-4: (High Depreciation) $r = .02$, $\delta_E = .5$, $\rho = 5$, $\chi = 10$, $c = .001$, $\alpha = .5$, $p^m = 8$, $p^c = 2$, $b = 10$, $a = .5$.

	SS1 (attractor)	SS2 (repellor)	SS3 (attractor)
Entry-deterring capital E	0	6.718	12.9742
Investment level x	0	3.359	6.48711
Market share s	0	0.120365	0.786154
Price Level p	2.00	2.72219	6.71692
Market Demand d	9.00	8.6389	6.64154
Deadweight Loss	0	1.85122	11.2759

Example B-5: (100% Depreciation) $r = .02$, $\delta_E = 1$, $\rho = 5$, $\chi = 10$, $c = .001$, $\alpha = .5$, $p^m = 8$, $p^c = 2$, $b = 10$, $a = .5$.

	SS1 (unique attractor)
Entry-deterring capital E	0
Investment level x	0
Market share s	0
Price Level p	2.00
Market Demand d	9.00
Deadweight Loss	0

Both policies are successful to reduce the deadweight loss. It is also possible to make a competitive state as a sole attractor by rasing χ and δ_E. Regulatory agencies, however, have to be very careful about a difference between two policies on how deadweight loss is reduced. By rasing χ, the basin of attraction associated with the competitive state enlarges and the high market share equilibrium is pushed further up. High market shares will be achieved only with large entry-deterring capital accumulation. Thus, the dominant firm with a given entry-deterring capital is more likely to be absorbed in a competitive equilibrium. On the other hand, by rasing δ_E, the basin of attraction associated with the competitive state enlarges only slightly. Moreover, the high market share equilibrium is pushed down. This means that two attractors become closer. High market share is achieved even with small entry-deterring capital. Therefore, the absolute level of E cannot be a proxy of market share in this case. The possibility that the the dominant firm leads an industry to high concentration doesn't decrease much by raising δ_E. When δ_E is used as a policy parameter, it will be suggested that the regulatory agency sets an enough high δ_E to make a competitive equilibrium a solo attractor.

5 Conclusion

Appendix: The Numerical Solution of the Model

We here briefly describe the dynamic programming algorithm as applied in Grüne and Semmler (2004) that enables us to numerically solve the dynamic model as proposed in section 3. The feature of the dynamic programming algorithm is an adaptive discretization of the state space which leads to high numerical accuracy with moderate use of memory.

Such algorithm is applied to discounted infinite horizon optimal control problems of the type introduced in section 3. In our model variants we have to numerically compute $V(x)$ for

$$V(x) = \max_u \int_0^\infty e^{-r} f(x,u) dt$$

$$\text{s.t. } \dot{x} = g(x,u)$$

where u represents the control variable and x a vector of state variables.

In the first step, the continuous time optimal control problem has to be replaced by a first order discrete time approximation given by

$$V_h(x) = \max_j J_h(x,u), \quad J_h(x,u) = h \sum_{i=0}^{\infty} (1-\theta h) U f(x_h(i), u_i) \tag{A1}$$

where x_u is defined by the discrete dynamics

$$x_h(0) = x, \; x_h(i+1) = x_h(i) + hg(x_i, u_i) \tag{A2}$$

and $h > 0$ is the discretization time step. Note that $j = (j_i)_{i \in \mathbb{N}_0}$ here denotes a discrete control sequence.

The optimal value function is the unique solution of a discrete Hamilton-Jacobi-Bellman equation such as

$$V_h(x) = \max_j \{hf(x, u_o) + (1+\theta h) V_h(x_h(1))\} \tag{A3}$$

where $x_h(1)$ denotes the discrete solution corresponding to the control and initial value x after one time step h. Abbreviating

$$T_h(V_h)(x) = \max_j \{hf(x, u_o) + (1-\theta h) V_h(x_h(1))\} \tag{A4}$$

the second step of the algorithm now approximates the solution on grid Γ covering a compact subset of the state space, i.e. a compact interval $[0, K]$ in our setup. Denoting the nodes of Γ by $x^i, i = 1, ..., P$, we are now looking for an approximation V_h^Γ satisfying

$$V_h^\Gamma(X^i) = T_h(V_h^\Gamma)(X^i) \tag{A5}$$

for each node x^i of the grid, where the value of V_h^Γ for points x which are not grid points (these are needed for the evaluation of T_h) is determined by linear interpolation. We refer to the paper cited above for the description of iterative methods for the solution of (A5). Note that an approximately optimal control law (in feedback form for the discrete dynamics) can be obtained from this approximation by taking the value $j^*(x) = j$ for j realizing the maximum in (A3), where V_h is replaced by V_h^Γ. This procedure in particular allows the numerical computation of approximately optimal trajectories.

In order the distribute the nodes of the grid efficiently, we make use of a posteriori error estimation. For each cell C_l of the grid Γ we compute

$$\eta_l := \max_{k \in c_l} \mid T_h(V_h^\Gamma)(k) - V_h^\Gamma(k) \mid$$

More precisely we approximate this value by evaluating the right hand side in a number of test points. It can be shown that the error estimators η_l give upper and lower bounds for the real error (i.e., the difference between V_j and V_h^Γ) and hence serve as an indicator for a possible local refinement of the grid Γ. It should be noted that this adaptive refinement of the grid is very effective for computing steep value functions and models with multiple equilibria, see Grüne and Semmler (2004).

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