

Currency Crises and Monetary Policy Rules

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March 23, 2005

Abstract

This paper studies optimal monetary policy rules by the central bank confronted by foreign investors' state-dependent reactions and self-fulfilling expectations. We extend Taylor type monetary policy rules by allowing the central bank to give some weight to the level of precautionary foreign reserve balances as one of its targets. We show that a currency crisis scenario can easily be created when the weight is zero, and it can be avoided when the weight is positive. The impacts of the central bank's monetary control on the output level, the inflation rate, the exchange rate, and the foreign reserve level are investigated as well. In solving our model variants we apply both the Hamiltonian as well as the Hamilton-Jacobi-Bellman (HJB) equation, the latter leading to a dynamic programming formulation of the problem. The flexible use of both the Hamiltonian as well as dynamic programming allows us to explore safe domains of attractions in a variety of complicated model variants. Given the uncertainties the central banks face, we also show of how central banks can enlarge safe domains of attraction.

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1 Introduction

Many central banks around the world have reacted to the Asian financial crisis by building up large precautionary reserve balances. By incorporating a *currency crisis* scenario into a linear-quadratic monetary control problem, we study Taylor type monetary policy rules that allow for foreign reserve targets. We show that an anticipation of a depreciation of the domestic currency causes a decrease in the central bank's foreign reserves. Additionally, when foreign investors observe a shortage of foreign reserves, this may trigger a panic caused by the fear of a termination of the currency conversion and increases a risk premium of the currency. A sharp depreciation of the currency gives incentives to foreign investors to pull their funds out of the country as quickly as possible. The prediction therefore becomes self-fulfilling.¹ This can cause a scenario known as a *currency crisis*. On the other hand, this particular type of currency run may not become so extreme when the central bank has enough reserves.² This paper studies optimal monetary policy rules by the central bank confronted by such foreign investors' state-dependent reactions and self-fulfilling expectations. We presume that the monetary policy rules may also depend on the weight that the central bank gives to the level of precautionary foreign reserve balances as one of its targets. The model can exhibit multiple steady states. The impacts of the central bank's monetary control on the output level, the inflation rate, the exchange rate, and the foreign reserve level are investigated as well. In solving our model variants we apply both the Hamiltonian as well as the Hamilton-Jacobi-Bellman (HJB) equation leading to a dynamic programming algorithm. The flexible use of both the Hamiltonian as well as dynamic programming allows us to explore a variety of model variants.

The remainder of the paper is organized as follows. Section 2 provides some empirical backgrounds of our paper. Section 3 introduces the monetary policy model. In Section 4 we explore the stabilization effects of monetary policy when the central bank disregards the reserves in the welfare function. Sections 5 and 6 include the control of the foreign reserves as one of its targets.

¹For more specifications of this mechanism, see Krugman (1979, 2000) and Aghion et al. (2000).

²Of course, credit lines from the IMF will also reduce currency runs.

2 Currency Crises and the Aftermath

Financial liberalization as well the introduction of floating exchange rates has actively been pursued by many governments since the 1980s. Yet, at the same time in the last twenty years, many countries have experienced major episodes of currency crisis and financial instability, some times with devastating effects on economic activity. The major examples are the Mexican (1994) and the Asian (1997-1998) currency and financial crises, where the liberalization of financial markets has led to a currency crisis, sudden reversal of capital flows followed by financial bankruptcies with consequently declining economic activity and large output losses.³

The stylized facts, illustrating the causes and consequences of currency and financial crises, are usually considered as the following ones.⁴

1. Before the currency crisis there is a deterioration of balance sheets of the economic units (households, firms, banks, the government and the country). The current account deficit to GDP ratio rises and the external debt to reserve ratio rises.
2. The crisis is triggered by a sudden reversal of capital flows and unexpected depreciation of the currency. Domestic interest rates usually jump up (partly initiated by central bank policy). Subsequently stock prices fall and a banking crisis occurs with large loan losses by banks and subsequent contraction of credit (sometimes moderated by a bail out of failing banks by the government).
3. The subsequent financial crisis frequently entails a large output loss due to large scale bankruptcies of firms and financial institutions and the decrease of investment and consumption spending.

Yet, after the experience of those shocks countries have become more cautious. In particular, those countries that were heavily exposed to currency and financial crises, have attempted to build up large precautionary currency reserves.

³For details, see Krugman (1979, 2000), Flaschel and Semmler (2003) and Stiglitz (2002).

⁴For details, see Kaminsky and Reinhart (1999).

Table 1 illustrates this tendency for some countries and regions.⁵ After the financial crisis, emerging markets in particular in the Far East including China, Indonesia, South Korea, Malaysia, Philippine, and Thailand, have built up large stockpiles of international reserves. Aizenman and Marion (2002) conducted a detailed statistical analysis using panel data of 122 developing countries and concluded that East Asian markets have changed their behavior in terms of holding foreign exchange reserves after the currency and financial crises occurring in East Asia.

	industrial	developing	Africa	Asia	Europe (developing)	Middle East	Western
1994	50.5	49.5	1.9	30.5	3.6	5.3	8.3
2002	39.6	60.4	2.9	38.5	7.1	5.7	6.7

Table 1: Change of the share of world reserves 1994-2002, in percentage (reserves minus gold)

As table 1 demonstrates the increase in reserves has particularly occurred in developing countries and East Asia. Table 2 shows the increase of reserve holdings since 1997 for particularly East Asian countries.⁶

	China	Taiwan	HongKong	South Korea	Singapore
1997	16	32	52	3	72
2001/2	16	44	67	25	88

Table 2: Increase of international reserve holdings since 1997 as percentage of GDP (reserves minus Gold)

Both tables 1 and 2 clearly illustrate the attempt of the affected countries to build up precautionary international reserves in order to withstand currency runs and currency crises.

Moreover, as the work by Aizenman and Turnovsky (2002) shows sufficient international reserves of a country that is an extensive international borrower can serve as a collateral for the creditor who want to secure international loans. The increase of reserves in heavily borrowing countries reduces

⁵The data are taken from Aizenman and Marion (2002, appendix).

⁶The data are taken from Aizenman and Marion (2002, appendix).

default risk and may raise the welfare of both the high income lending countries as well as the emerging market economies.⁷

In the subsequent sections of our paper we will study the dynamics of foreign currency reserves for macro economies of emerging markets when the monetary authority has become aware of those intricate issues as above discussed and attempts to control the reserves in the context of a monetary policy control model where monetary policy rules of Taylor type are pursued by the monetary authority.

3 A Monetary Policy Model

Next, we present a model for a representative country that pursues a monetary policy in an open economy. First, we presume that the central bank has two target variables: the inflation rate and the output level. We assume that those can be indirectly affected by the central bank through controlling the short-term nominal interest rate. The central bank's objective is to minimize a social welfare loss an economy deviating from those target levels. The social welfare loss is computed as:

$$L = \int_0^{\infty} e^{-rt} \left[\frac{\lambda_1}{2} (\pi - \pi^*)^2 + \frac{\lambda_2}{2} (y - y^*)^2 \right] dt \quad (1)$$

where π is the inflation rate, y is the output level, π^* and y^* are those target levels respectively. The weights λ_1 and λ_2 measure the relative importance of those target variables.

The economy is assumed to be represented by the following macroeconomic relationships:

$$y = \alpha - \beta(i - \pi^e) + X(e^e); \quad X'(e^e) > 0 \quad (2)$$

$$\pi = \pi^e + \eta(y - y^*) \quad (3)$$

$$\dot{e}^e = i - i^f - \rho(i - i^f, R); \quad \rho_{i-i^f} > 0, \quad \rho_R < 0 \quad (4)$$

⁷Of course, recently monetary authorities of some of the East Asian countries have bought up foreign currencies in order to stabilize their currencies and to keep their export markets flourishing.

where e^e is the expected exchange rate of domestic for foreign currency. An increase in e^e means an anticipation of depreciation of the domestic currency. Equ. (2) represents an IS relationship. The real interest rate negatively affects the firm's investment decisions. α and β are positive constants. X is the net imports which positively reacts to the depreciation of the expected exchange rate. Equ. (3) is a Phillips curve. The domestic inflation rate π is positively correlated with the deviation from the natural output (or NAIRU) level y_n . In the current version we neglect the open economy component in the Phillips-curve.⁸ For simplicity, we assume that the policy maker sets $y^* = y_n$. In our most simplest variant we also presume that π^e is a constant.⁹

The interest rate parity condition (4) is assumed to consist of two factors: the uncovered interest rate parity where i is the domestic riskless rate and i^f is the foreign riskless rate, and ρ is a risk premium which compensates the agents for the uncertain risk.¹⁰ It is commonly recognized that there are two types of risk factors; exchange rate risk and country risk. The volatility of the exchange rate causes exchange rate risk. High volatility brings about high uncertainty, therefore a high risk premium. Therefore, the first factor in ρ is $i - i^f$ that captures the volatility of the exchange rate. Country risk is the possibility of losses related to foreign financial instruments. We think that the central bank's foreign reserve accumulation is one of the key factors that affects country risk.

As above shown the currency runs in the concerned countries created for the central banks a serious shortage of foreign currency reserves. When foreign investors observe a shortage of foreign reserves, there is a high risk of staying in the domestic currency. Investors have to account for a possible rapid depreciation of the currency in the future. The second factor in ρ is thus R that captures country risk.

The currency reserve dynamics in our model can be written as

$$\dot{R} = X(e^e) + F(e^e, R); F_{e^e} < 0, F_R > 0 \quad (5)$$

where

⁸For including the affect of the exchange rate in the Phillips-curve, see Ball (1999) and Flaschel, Gong and Semmler (2005).

⁹This assumption will be relaxed in the numerical exploration of our model, using dynamic programming.

¹⁰For details of such risk premium, see Evans and Lyons (2005).

$$F(e^e, R) = n(e^e) - z(e^e, R); \quad j'(e^e) < 0, \quad z_{e^e} > 0, \quad z_R < 0 \quad (6)$$

The first term tells us that an increase in net imports will bring more foreign currency reserves. The term $F(e^e, R)$ captures the net inflow of financial funds that also causes an increase in currency reserves. Equ. (6) simply shows that the net inflow is equal to the inflow n minus the outflow z .¹¹ While n is a function of e^e only, z is a function of two factors: e^e and R . The reason why investors give weight to foreign reserves in their decision of pulling their funds out is somewhat related to investors' psychology. Since they understand that a serious shortage of foreign reserves may bring about a panic caused by the fear of a rapid depreciation of the currency, foreign investors will show a more sensitive reaction, in case of lower reserves, to an anticipated depreciation of the domestic currency. In short, equ. (6) tells us that investors' reaction is state-dependent.

Using the IS and Phillips relationships, the welfare loss can be rewritten in terms of two state variables e^e and R , and a control variable i . Therefore, the problem for the policy maker will be to minimize the following loss function:

$$\begin{aligned} \underset{i}{Min} L = & \int_0^{\infty} e^{-rt} \left[\frac{\lambda_1}{2} (\pi^e + \eta(\alpha - \beta(i - \pi^e) + X(e^e) - y^*) - \pi^*)^2 \right. \\ & \left. + \frac{\lambda_2}{2} (\alpha - \beta(i - \pi^e) + X(e^e) - y^*)^2 \right] dt \end{aligned} \quad (7)$$

s.t. to equ. (4), equ. (5) and the

$$\text{boundary conditions for } e^e \text{ and } R. \quad (8)$$

The current value Hamiltonian of this problem is

¹¹In the short-run, the outflow and the inflow of the foreign currency due to financial investments depends on the difference of the domestic and the rest of the world's interest rates and the expected exchange rate. Here we neglect the difference in the interest rates and assume the inflow to be constant. Those simple assumptions allow us to highlight the mechanism of the currency crises resulting from the investors speculative behavior.

$$\begin{aligned}
H &= \frac{\lambda_1}{2} \{ \pi^e + \eta(\alpha - \beta(i - \pi^e) + X(e^e) - y^*) - \pi^* \}^2 \\
&\quad + \frac{\lambda_2}{2} (\alpha - \beta(i - \pi^e) + X(e^e) - y^*)^2 \\
&\quad + q_1(i - i^f - \rho(i - i^f, R)) \\
&\quad + q_2(X(e^e) + F(e^e, R)).
\end{aligned} \tag{9}$$

The FOCs are

$$\begin{aligned}
\frac{\partial H}{\partial i} &= -\lambda_1 \beta \eta (\pi^e - \pi^*) \\
&\quad - \beta (\lambda_1 \eta^2 + \lambda_2) (\alpha - \beta(i - \pi^e) + X(e^e) - y^*) \\
&\quad + q_1 (1 - \rho_{i-i^f}(i - i^f, R)) \\
&= 0
\end{aligned} \tag{10}$$

$$\begin{aligned}
\dot{q}_1 &= r q_1 - (X'(e^e) - F_{e^e}(e^e, R)) q_2 \\
&\quad - \lambda_1 \eta X'(e^e) (\pi^e - \pi^*) \\
&\quad - (\lambda_1 \eta + \lambda_2) (\alpha - \beta(i - \pi^e) + X(e^e) - y^*)
\end{aligned} \tag{11}$$

$$\dot{q}_2 = (r - F_R(e^e, R)) q_2 + \rho_R(i - i^f, R) q_1 \tag{12}$$

$$\dot{e} = i - i^f - \rho(i - i^f, R) \tag{13}$$

$$\dot{R} = X(e^e) + F(e^e, R). \tag{14}$$

For the sake of deriving more specific results, we use the following specific functions.

$$\rho(R) = \frac{\sigma_1}{R} + \frac{\sigma_2}{2} (i - i^f)^2 \tag{15}$$

where the first term is country risk associated with a currency run and the second term is exchange rate risk driven by the volatility of the exchange rate. The terms σ_1 and σ_2 are constant coefficients.

$$X(e^e) = \varepsilon e^e - \bar{m} \quad (16)$$

where ε shows the elasticity of im(ex)ports to a change in exchange rate, and \bar{m} is the autonomous import level, and

$$\begin{aligned} F(R, e^e) &\equiv n(e^e) - z(e^e, R) \\ &= \frac{\nu}{e^e} - \frac{\mu}{R} e^{e^2} \end{aligned} \quad (17)$$

where ν and μ are constant coefficients.

Then, the FOCs can be rewritten as:

$$\begin{aligned} \frac{\partial H}{\partial i} &= -\lambda_1 \beta \eta (\pi^e - \pi^*) \\ &\quad -\beta (\lambda_1 \eta^2 + \lambda_2) (\alpha - \beta (i - \pi^e) + \varepsilon e^e - \bar{m} - y^*) \\ &\quad + q_1 (1 - \sigma_2 (i - i^f)) \\ &= 0 \end{aligned} \quad (18)$$

$$\begin{aligned} \dot{q}_1 &= r q_1 - \left(\epsilon + \frac{\nu}{e^{e^2}} + 2\mu \frac{e^e}{R} \right) q_2 \\ &\quad - \lambda_1 \eta \epsilon (\pi^e - \pi^*) \\ &\quad - (\lambda_1 \eta + \lambda_2) (\alpha - \beta (i - \pi^e) + \varepsilon e^e - \bar{m} - y^*) \end{aligned} \quad (19)$$

$$\dot{q}_2 = \left\{ r - \mu \left(\frac{e^e}{R} \right)^2 \right\} q_2 - \frac{\sigma_1}{R^2} q_1 \quad (20)$$

$$\dot{e}^e = i - i^f - \frac{\sigma_1}{R} - \frac{\sigma_2}{2} (i - i^f)^2 \quad (21)$$

$$\dot{R} = \varepsilon e^e - \bar{m} + \frac{\nu}{e^e} - \frac{\mu}{R} e^{e^2} \quad (22)$$

From (18),

$$q_1 = \frac{\lambda_1 \beta \eta (\pi^e - \pi^*) + \beta (\lambda_1 \eta^2 + \lambda_2) (\alpha - \beta (i - \pi^e) + \varepsilon e^e - \bar{m} - y^*)}{1 - \sigma_2 (i - i^f)} \quad (23)$$

Incorporating equ. (23) into eqs. (19)-(22) and solving $\dot{q}_1 = \dot{q}_2 = \dot{e}^e = \dot{R} = 0$ for i , e^e , R , and q_2 gives us the steady states of the model.

4 Monetary Policy without Reserve Control: Numerical Examples

Next, we introduce numerical examples. Hereby we presume that the monetary authority does not attempt to control the foreign reserves but only aims at controlling the inflation rate and output through steering the interest rate.

For the case of a monetary policy without explicit control of foreign reserves as target of the monetary authority we specify for the model, as developed in section 2, the parameters as represented in table 1. Note that in our example 4.1 of table 1 the function $\rho = 0$ means that there is no risk premium for the asset holding in the domestic currency. Yet, we have introduced with $\nu > 0$, $\mu > 0$, an effect on foreign exchange reserves triggered by the level of exchange rates and the level of reserves. Note that we here also presume the most simplest variant where, as in the output determination of equ. (2) and the Phillips-curve of equ. (3) the expected inflation rate, π^e , is a constant. We for convenience also take $\bar{m} = 0$.

$\lambda_1 = 1$	$\alpha = 5$
$\lambda_2 = 1$	$\beta = 1$
$\pi^* = .03$	$\eta = .01$
$\pi^e = .03$	$\sigma_1 = .00$
$y^* = 10$	$\sigma_2 = .00$
$i^f = .05$	$\varepsilon = 1$
$r = .02$	$\bar{m} = .00$
	$\nu = 100$
	$\mu = 10$

Table 3: Parameters for example 4.1, $\rho = 0$

The parameters of table 3 deliver us a unique steady state. The steady state values, for the reserves R , interest rate, i , the exchange rate, e , output y , and the inflation rate, π , are shown in table 4. We have computed the steady state for the above parameter values, using eqs. (19)-(23), employing the software package Mathematica. We obtain the following steady state (SS) as reported in table 4.¹² As we can observe there is a region for R between

¹²The eigenvalues obtained from the 4D system in q_1, q_2, e, R at this unique steady state

zero and 10.1 that may attract the currency reserves away from zero. This is indeed confirmed by the study of the dynamics of the steady state.

SS	R	i	e	y	π
1	10.1043	0.05	5.02	10	0.03

Table 4: Steady state values for example 4.1

The dynamics about the steady state of the state variables R and e are computed using a dynamic programming algorithm as applied to dynamic economic problems in Grüne and Semmler (2004). In the appendix a sketch of the algorithm is provided. Figure 1 shows the value function about the unique steady state.

are $\{2.46829, -2.44829, 1.0, -0.989899\}$. The steady state is thus a saddle point in terms of state and co-state variables. It is, however, a general mathematical result, that a saddle point in the state-costate space corresponds to a stable point in the state space dynamics of a HJB-equation.

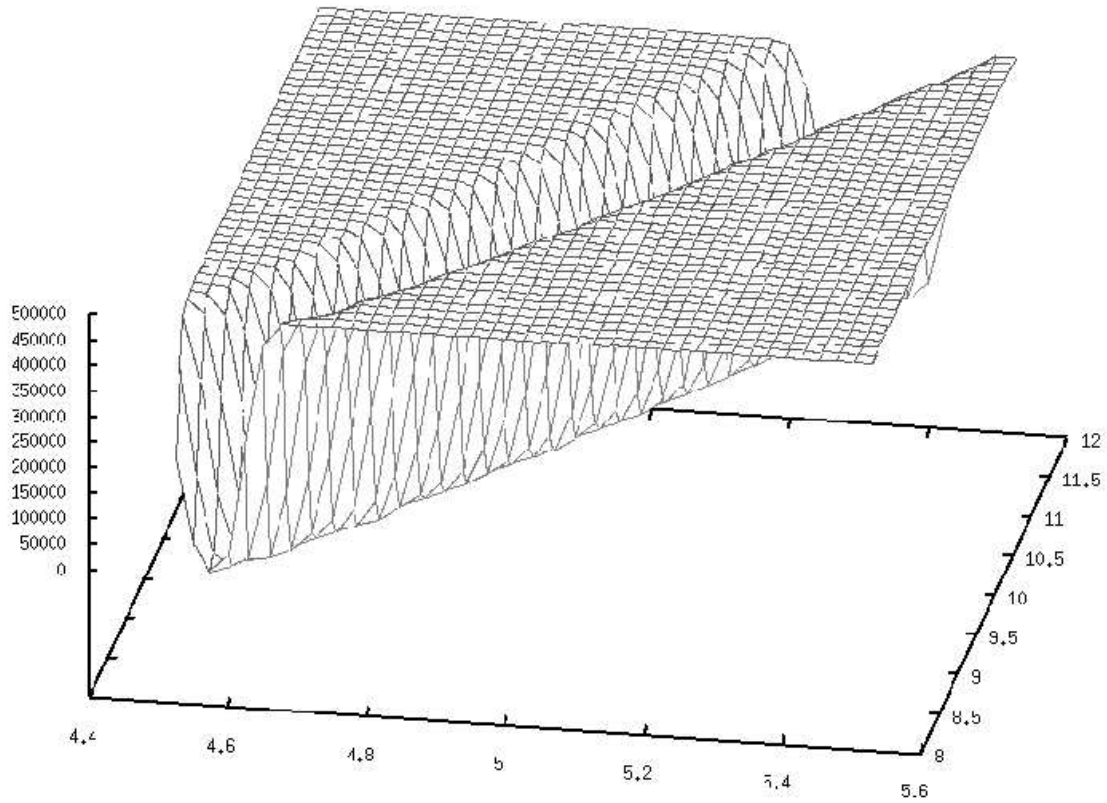


Figure 1: Value function about the unique steady state, example 4.1

The figure 1 shows that there is a deep valley of the value function about the steady state demonstrating that the value function indeed achieves a minimum about the steady state values in the vicinity of the steady state $R^* = 10.104$ and $e^* = 5.02$.

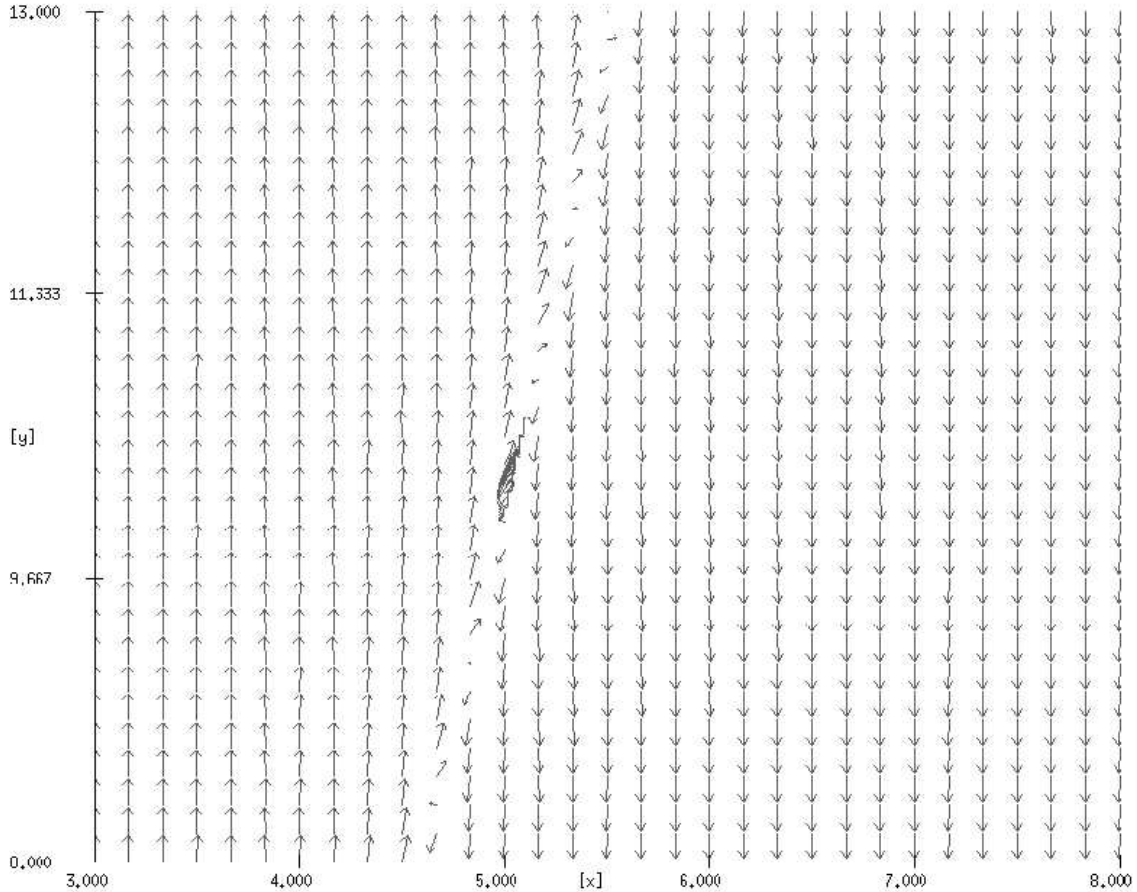


Figure 2: Value function about the unique steady state, example 4.1

Figure 2, which depicts the vector field for the solution of the dynamic programming problem (19)-(23), represented by the state equations (21)-(22), demonstrates the dynamics about the steady state. As can be observed from the trajectories all trajectories move toward the unique steady state \bar{R}, \bar{e} in the vicinity of the steady state. Thus, all currency reserves moved by a shock to the region between zero and the steady state \bar{R} , will safely move back to the steady state \bar{R} .

In a next variant, example 4.2 we pursue a dynamic programming solution of our basic variant of section 3, but we take in the IS equation (2) the actual inflation rate π instead of $\pi^e = \text{constant}$. In equ. (3) in the Phillips-curve we then presume, in order to avoid lags that the price dynamics is determined solely by the output gap. Also, in order to observe a stronger price reaction

we presume an $\eta=0.15$. The remainder of the parameters are the same as in table 3 (example 4.1).

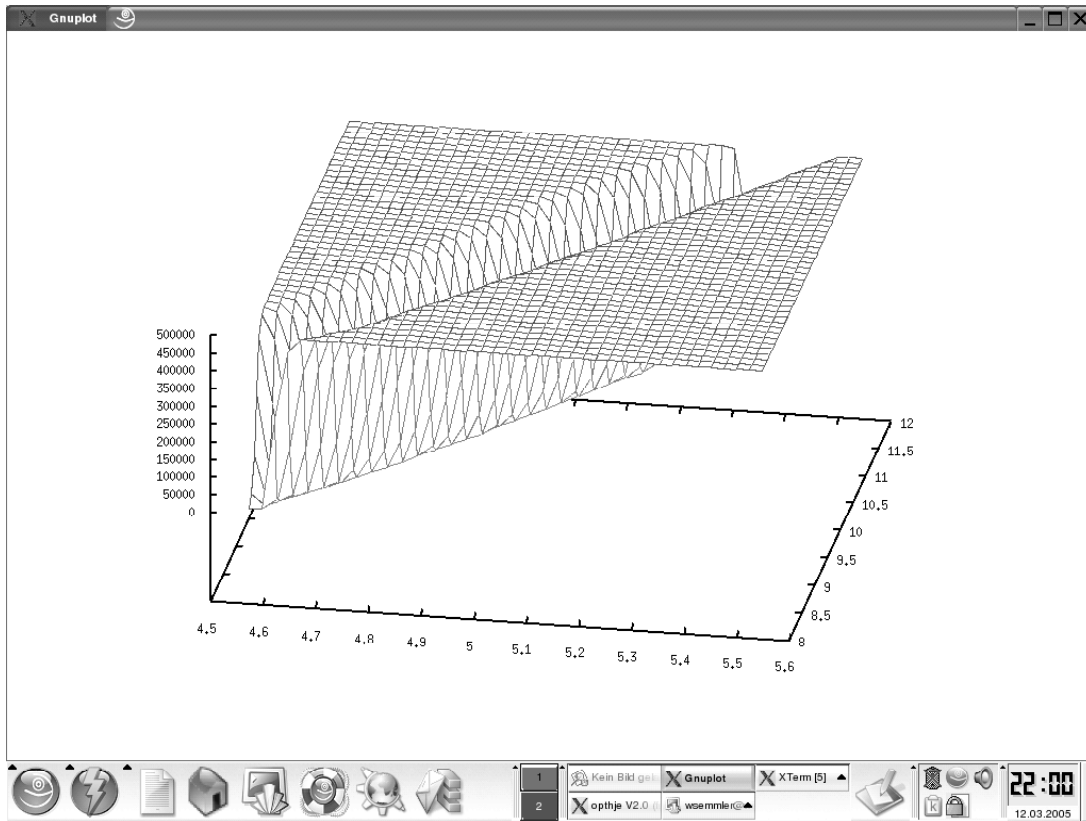


Figure 3: Value function about the steady state (actual inflation rate in the IS-curve), example 4.2.

As figure 3 for example 4.2 shows the value function changes only slightly, and as the computation of the vector field for this case has shown, the steady state equilibrium also only changes slightly. Yet the local dynamics does not change so that the positive steady state of R is the sole attractor between zero and the steady state of R . Since neither the shape of the value function significantly nor the dynamics change, we subsequently work with the simpler variant as proposed in equ. (1)-(4).

In a next variant, example 4.3, we permit the function $\rho > 0$ due to a risk premium arising from low currency reserves. We again refer to the simple variant eqs. (1)-(4). We set $\sigma_1 = 0.3$. The other parameter R remain the

same as in table 3. We still keep $\sigma_2 = 0$. Due to the introduction of a risk premium $\sigma_1 > 0$, multiple steady state equilibria may arise.¹³ This is shown in table 5.

SS	R	i	e	y	π
1	10.2578	0.079	5.04925	10.00	0.03
2	0.5197	0.627	1.74967	6.15	-0.008
3	-0.0515	-5.773	-0.803366	10.00	0.03
4	-0.5197	-0.527	-1.74967	3.80	-0.031

Table 5: Multiple steady states for example 4.3, $\sigma_1 = 0.3$, $\sigma_2 = 0$, $\rho > 0$

Note that the highest steady state is almost the same as in table 4, for example 4.1, with $\sigma_1 = 0$, $\sigma_2 = 0$, yet there are 3 more steady states arising, with a negative R indicating that the country may have to obtain a credit line on external reserves for example from the IMF. Overall, due to country risk ($\sigma_1 > 0$) the domain of attraction of the steady state $\bar{R} = 10.25778$ has decreased and shocks may produce a more vulnerable situation for the country's currency reserves.¹⁴

The next example, example 4.4, allows also for the function $\rho > 0$, and thus a risk premium, but a reaction of the foreign currency reserves to the exchange rate and the level of foreign reserves so we have both $\sigma_1 > 0$, $\sigma_2 > 0$. We here again employ the simple variant of equs. (1)-(4) but now with $\rho > 0$ due to $\sigma_1 > 0$, $\sigma_2 > 0$. We employ the parameters as shown in table 6:

¹³The eigenvalues obtained from the 4D system in q_1, q_2, e, R at each steady state (from SS1 to SS4) are: $\{-2.43215, 2.3969, 1.02606, -0.960707\}$, $\{-114.343, 112.387, 0.992782 + 2.75418 i, 0.992782 - 2.75418 i\}$, $\{-114.246, 112.286, 4.84355, -2.85307\}$, $\{-2453.37, 2409.97, 22.8586, 20.574\}$. Note that SS(1) is stable, SS(2) is unstable, SS(3) is stable and SS(4) again unstable.

¹⁴Note that a large negative ρ , due to a negative R , an external credit line of a country, might not be very reasonable, so we might disregard the steady with a negative R or constrain the ρ by a lower bound.

$\lambda_1 = 1$	$\alpha = 5$
$\lambda_2 = 1$	$\beta = 1$
$\pi^* = .03$	$\eta = .01$
$\pi^e = .03$	$\sigma_1 = 2.0$
$y^* = 10$	$\sigma_2 = 2.0$
$i^f = .05$	$\varepsilon = 1$
$r = .02$	$\bar{m} = .00$
	$\nu = 100$
	$\mu = 10$

Table 6: Parameters for example 4.4, $\rho > 0$ ($\sigma_1 > 0$, $\sigma_2 > 0$)

Also example 4.4 gives rise to multiple steady states. We have left aside the detailed exploration of the number of steady state equilibria but explore instead, using dynamic programming, in what direction the largest steady state moves.

Note that the difference of this example and the previous one is that we have taken both $\sigma_1 > 0$, $\sigma_2 > 0$ ($\sigma_1 = 2$, $\sigma_2 = 2$), indicating that the home countries risk premium rises due to two effects.

Both figures 4 and 5 show that with the country's risk premium becoming positive, $\rho > 0$ due to $\sigma_1 > 0$, $\sigma_2 > 0$ the required equilibrium reserves move up and also e increases slightly.

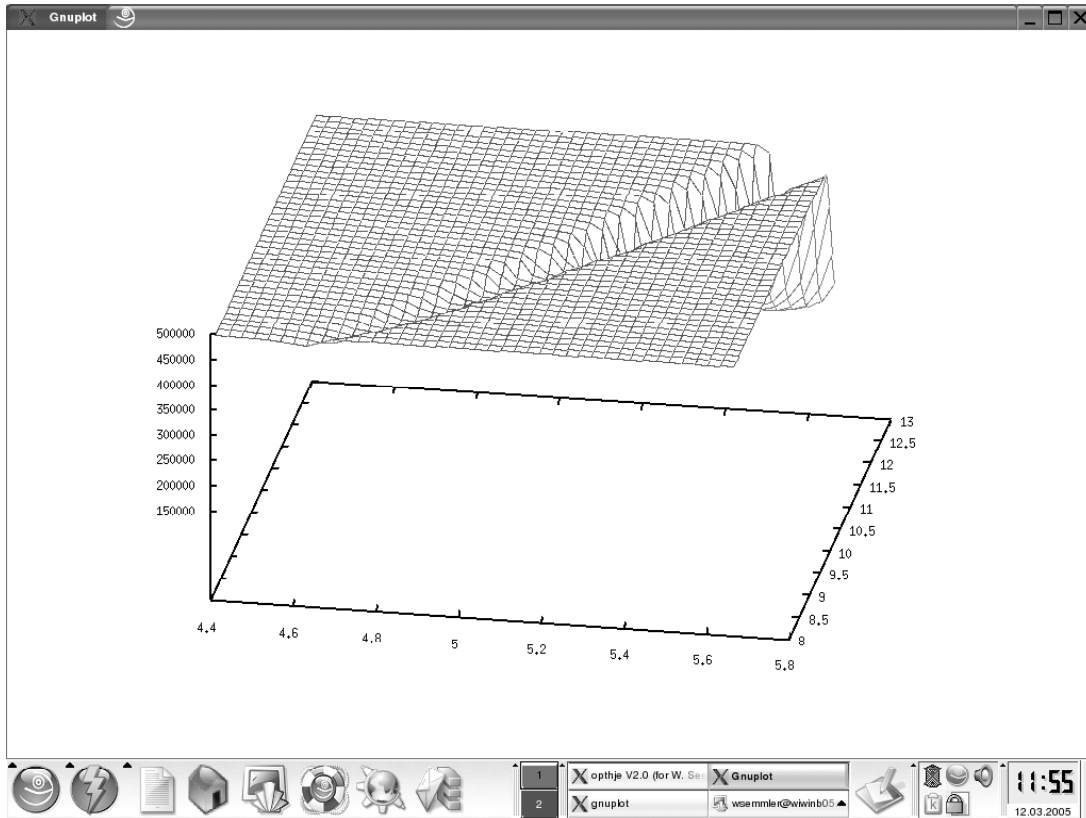


Figure 4: Minimum of the value function moves up (equilibrium R is rising), example 4.4

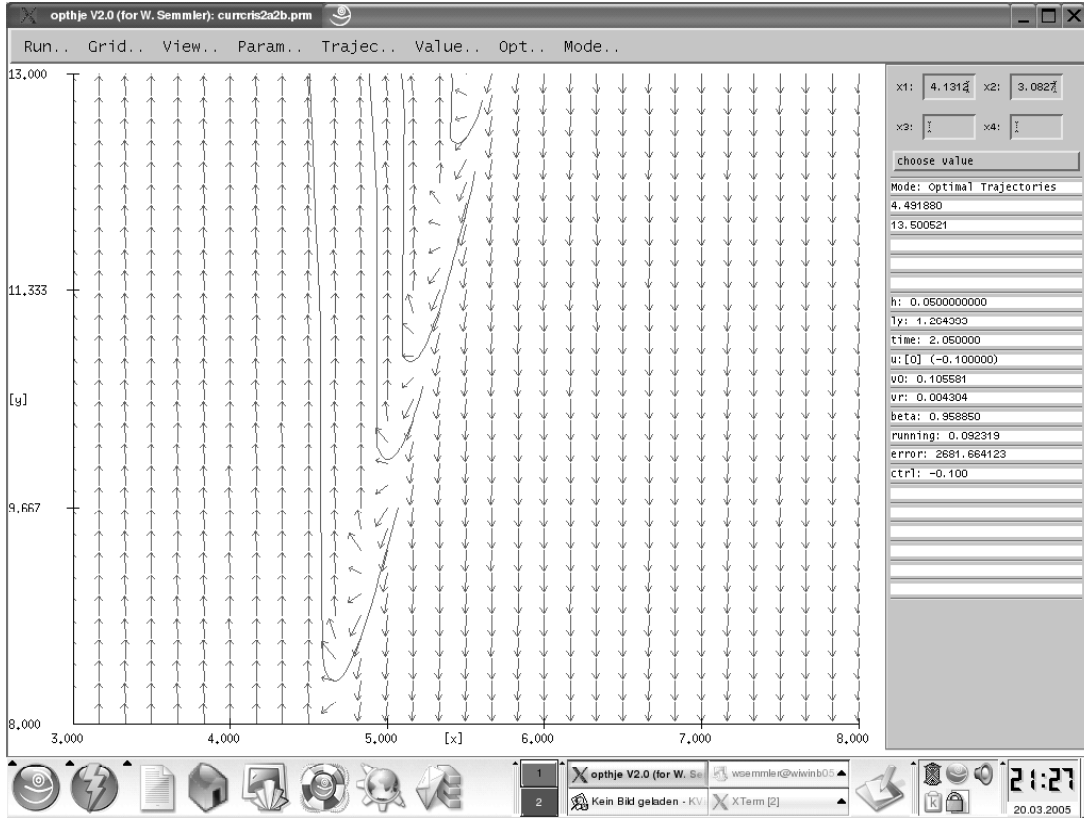


Figure 5: Vector field pointing upward, example 4.4

As figures 4 and 5 demonstrate the required equilibrium reserves are upward shifting when the home country's risk premium is rising due to $\sigma_1 > 0$, $\sigma_2 > 0$, but the domain of attraction of the upper steady state of R has been reduced.

5 Targeting the Foreign Reserves

As eluded to in section 2, after the experience of the currency financial crises, the currency reserve level has become one of the most important concerns for many central banks. By building up large of precautionary reserve balances, central banks keep the ability to defend their currency value and avoid a currency crisis scenario. In this section, we consider a central bank which has three target variables: the inflation rate, the output level and its foreign

currency reserves. The short-term nominal interest rate is again the only control variable for the central bank.

This social welfare loss is computed as:

$$L = \int_0^{\infty} e^{-rt} \left[\frac{\lambda_1}{2} (\pi - \pi^*)^2 + \frac{\lambda_2}{2} (y - y^*)^2 + \frac{\lambda_3}{2} (R - R^*)^2 \right] dt \quad (24)$$

where now R , the stock of foreign currency reserves, R^* , its target level and λ_3 , its relative importance, are additionally included in the central bank's objective function.

Suppose that the economy is as before described by the IS equation (2), the Phillips curve (3), the interest rate parity condition (4), and currency reserve dynamics (5). Then, the problem for the monetary policy authority can be rewritten as:

$$\begin{aligned} \underset{i}{Min} L = & \int_0^{\infty} e^{-rt} \left[\frac{\lambda_1}{2} (\pi^e + \eta(\alpha - \beta(i - \pi^e) + X(e^e) - y^*) - \pi^*)^2 \right. \\ & + \frac{\lambda_2}{2} (\alpha - \beta(i - \pi^e) + X(e^e) - y^*)^2 \\ & \left. + \frac{\lambda_3}{2} (R - R^*)^2 \right] dt \end{aligned} \quad (25)$$

s.t. eqs. (4), (5) and the boundary conditions for e^e and R
The current value Hamiltonian of this problem is

$$\begin{aligned} H = & \frac{\lambda_1}{2} \{ \pi^e + \eta(\alpha - \beta(i - \pi^e) + X(e^e) - y^*) - \pi^* \}^2 \\ & + \frac{\lambda_2}{2} (\alpha - \beta(i - \pi^e) + X(e^e) - y^*)^2 \\ & + \frac{\lambda_3}{2} (R - R^*)^2 \\ & + q_1 (i - i^f - \rho(i - i^f, R)) \\ & + q_2 (X(e^e) + F(e^e, R)). \end{aligned} \quad (26)$$

After an inclusion of the foreign reserve target, only (12) changes as follows:

$$\dot{q}_2 = (r - F_R(e^e, R)) q_2 - \lambda_3 (R - R^*) + \rho_R (i - i^f, R) q_1. \quad (27)$$

Therefore, by introducing the same specific functions (15)-(17), (27) can be rewritten as

$$\dot{q}_2 = \left\{ r - \mu \left(\frac{e^e}{R} \right)^2 \right\} q_2 - \lambda_3 (R - R^*) - \frac{\sigma_1}{R^2} q_1. \quad (28)$$

Incorporating (23) into (19), (21), (22), and (28) and solving $\dot{q}_1 = \dot{q}_2 = \dot{e}^e = \dot{R} = 0$ for i , e^e , R , and q_2 gives a new set of steady states.

6 Monetary Policy with Reserve Control: Numerical Examples

In the next examples we employ the assumption that the monetary authority also targets the exchange reserves. We fix a required foreign exchange reserve R^* which may reflect some monetary policy target resulting from experience and observations.

$\lambda_1 = 1$	$\alpha = 5$
$\lambda_2 = 1$	$\beta = 1$
$\lambda_3 = 10$	$\eta = .01$
$\pi^* = .03$	$\sigma_1 = .00$
$\pi^e = .03$	$\sigma_2 = .00$
$y^* = 10$	$\varepsilon = 1$
$R^* = 100$	$\bar{m} = 0$
$i^f = .05$	$\nu = 100$
$r = .02$	$\mu = 10$

Table 7: Parameters for example 6.1, $\rho = 0$

With the parameters as represented in table 7, which are the same as in table 3, but R^* and λ_3 are introduced, we obtain a multiplicity of steady states.¹⁵

¹⁵The eigenvalues obtained from the 4D system in q_1, q_2, e, R at each steady state (from SS1 to SS3) are: $\{3.27181, -3.25073, 0.00 + 3.14646 i, 0.00 - 3.14646 i\}$, $\{3.83562 \times 10^6, -3.83562 \times 10^6, 8.79169, -8.78159\}$, $\{3.46176 \times 10^6, -3.46176 \times 10^6, 0.00 + 8.7866 i, 0.00 - 8.7866 i\}$. Note that no reference on the stability properties of the equilibria can be made when the real parts of the eigenvalues are zero. Dynamic programming has to be used instead.

In fact, as the table 8 shows three steady states for our state variable R and e , and the corresponding interest rates, i , outputs, y and inflation rates, π , are obtained.

SS	R	i	e	y	π
1	100.042	.05	14.6595	19.6395	0.126395
2	0.000205191	.05	0.12708	5.10708	-0.018929
3	-0.000221597	.05	-0.13038	4.84962	-0.021503

Table 8: Steady State Values, $\rho = 0$, example 6.1

Figure 6 depicts the value function about the high steady state $R = 100.042$ and $e = 14.6595$.

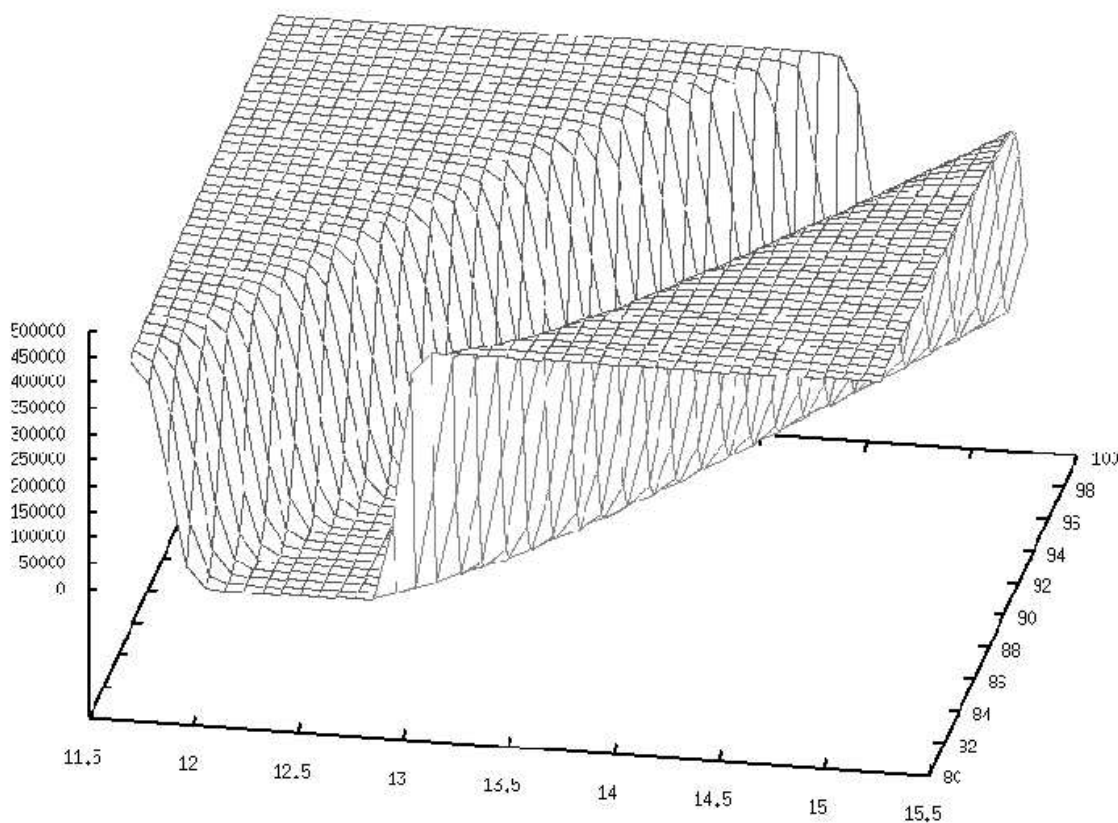


Figure 6: Value function about high steady state, example 6.1

As the value function shows the minimum again lies in a deep valley. Note that the required R^* is assumed to be very high and the introduced $\lambda = 10$ represents a strong reaction of the monetary authority to the deviation of foreign reserves to its target. This is purposely undertaken in order to obtain distinct and clearly separated steady states.

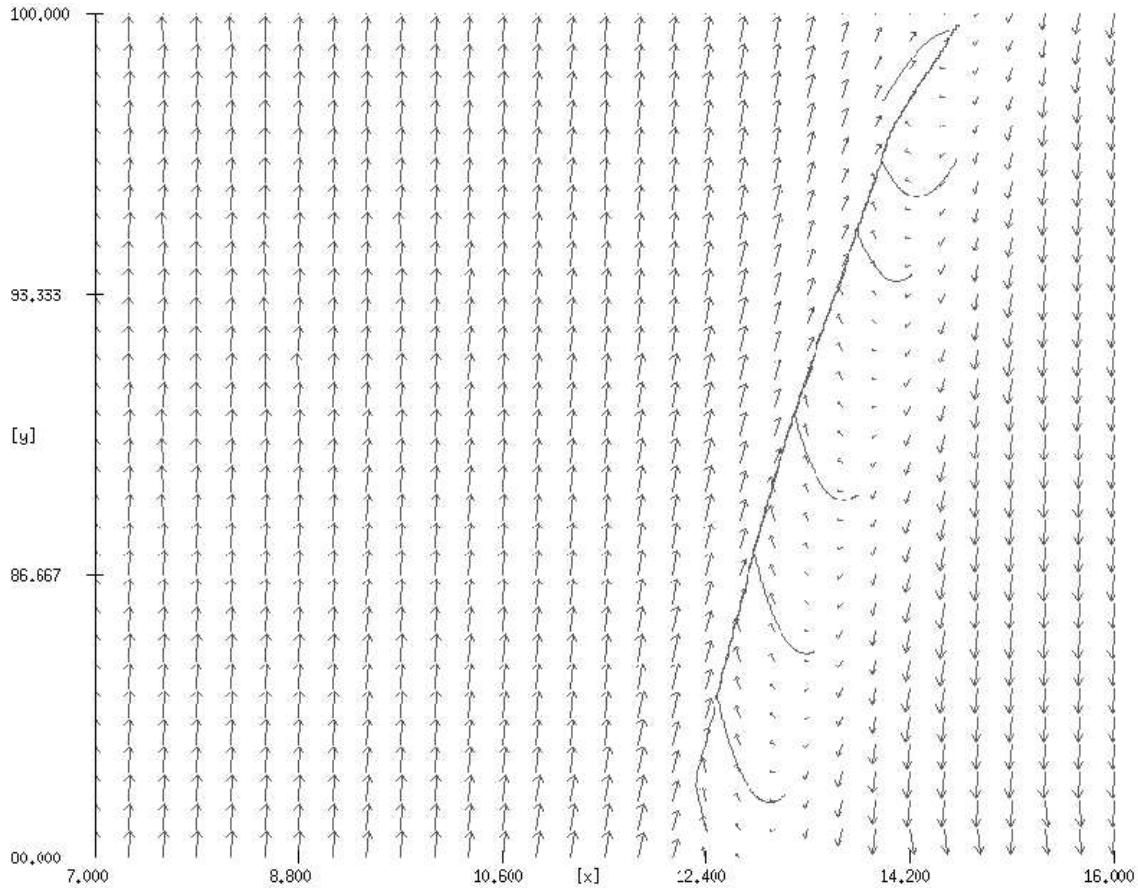


Figure 7: Vector field about the high steady state, example 6.1

As figure 7 shows there are converging dynamics toward the high steady state which is indicated by the arrows, thus the highest steady state $SS(1)$ is an attractor.

Next, we study an example, example 6.2 where, due to $\sigma_1 > 0$ and $\sigma_2 = 0$ In fact we take the same parameters as reported in table 7 for example 6.1, but employ $\sigma_1 = 0.3$, and $\sigma_2 = 0$. There is now a risk rate $\rho > 0$ responding

to low currency reserves.

SS	R	i	e	y	π
1	100.042	0.0530	14.6595	19.6365	0.126365
2	-0.0323294	-9.2294	-0.6874	13.5721	0.065720

Table 9: Steady State Values, $\sigma_1 = 0.3$, $\sigma_2 = 0$, $\rho > 0$, example 6.2

As shown in table 9 this time due to the introduction of the risk premium, arising from $\sigma_1 = 0.3$, only two steady states are detectable.¹⁶ Since the upper steady state (SS1) remains the same as in example 6.1 we do not expect, when employing the dynamic programming algorithm, different results in the local stability properties and the local behavior of the value function. We thus can by-pass a detailed dynamic programming study for this variant.

¹⁶The eigenvalues obtained from the 4D system in q_1, q_2, e, R at each steady state (from SS1 to SS2) are: $\{3.27182, -3.25075,$

$0.00 + 3.14645 i, 0.00 - 3.14645 i\}, \{-4561.26, 4480.39, 40.4486 + 35.3196 i, 40.4486 - 35.3196 i\}$ The latter case shows instability, but again no inference can be made for the case of zero real roots. Yet the dynamic programming result showed, as for the case of table 8 and depicted in figure 7 that the upper equilibrium is stable.

7 Conclusion

In this paper we have studied an extension of Taylor type monetary policy rules where the central bank is confronted by foreign investors' state dependent reactions and self-fulfilling expectations. In this context we have explored two different currency crises scenarios. We have focused on two factors; λ_3 , the weight that the central bank gives to the level of precautionary foreign reserve balances, and ρ , the risk premium factor in the interest rate parity condition. The factor ρ represents the foreign investors' state-dependent reactions which is likely to fuel currency runs. A summary of the outcomes can be stated as follows.

Scenario 1: Baseline monetary policy without reserve control ($\lambda_3 = 0$)

In our baseline monetary policy model, the central bank pursues only two targets, the output level and the inflation rate. When the risk premium factor is zero $\rho = 0$, there is a unique steady state in a positive state space, and the dynamic property of this steady state is represented by a saddle point when the Hamiltonian is used to study the local stability properties which corresponds to a stable steady state in the state space. When the risk premium factor $\rho > 0$ is introduced, multiple steady states arise and two domains of attraction appear in a positive state space. The upper steady state is close to the unique steady state obtained when $\rho = 0$ and it is a saddle point and the lower steady state occurs at a low reserve level and the dynamic property is an unstable focus. The latter indicates that the lower steady state is unstable in the state space. Therefore, the lower steady state divides the positive state space into two domains: an upper stable and a lower unstable domain, where the trajectories move downward. Overall, this reduces the stable domain of attraction and a currency crisis scenario may here develop, the reserve level continuously falls and, the central bank has to keep high interest rates which entails a recession and deflation. Since the lower steady state level of R moves up when the risk premium factor is strong, it suggests that higher ρ enlarges the domain of attraction of the lower steady state and a greater possibility of a currency crisis can be expected.

Scenario 2: Monetary policy with targeting the reserve ($\lambda_3 > 0$)

In this scenario the central bank targets the level of precautionary foreign reserve balances as well. When the risk premium factor is zero, multiple

steady states arise and thus two domains of attraction exist in a positive state space. Since the lower steady state is close to zero and the upper steady state of R is very high and close to the target reserve level, there is little possibility of a currency crisis. The lower steady state moves even closer to zero when the central bank gives higher weight to the level of precautionary foreign reserve balances, which means that the domain of attraction of the upper steady state enlarges as λ_3 increases. The central bank's actual reserve level tends to be as large as the central bank's desired target level. When the risk premium factor $\rho > 0$ is introduced, it turns out that we still can avoid the currency crisis scenario. There is a unique steady state in a positive state space, and the steady state of R remains close to the central bank's target level.

Overall, we may conclude that if the central bank takes the level of precautionary foreign reserve balances into account as one of its targets it is likely to avoid the economy flipping into a currency crisis scenario. Given the usual uncertainty that central banks face in terms of the data of the country, the underlying model and its parameters, the domains of bad outcomes and the size of shocks it appears as a very useful strategy of central banks to increase such domains of attraction by some strategy.

Appendix: The Numerical Solution of the Model

We here briefly describe the dynamic programming algorithm as applied in Grüne and Semmler (2004) that enables us to numerically solve the dynamic model as proposed in section 3. The feature of the dynamic programming algorithm is an adaptive discretization of the state space which leads to high numerical accuracy with moderate use of memory.

Such algorithm is applied to discounted infinite horizon optimal control problems of the type introduced in section 3. In our model variants we have to numerically compute $V(x)$ for

$$V(x) = \max_u \int_0^\infty e^{-r} f(x, u) dt$$

$$\text{s.t. } \dot{x} = g(x, u)$$

where u represents the control variable and x a vector of state variables.

In the first step, the continuous time optimal control problem has to be replaced by a first order discrete time approximation given by

$$V_h(x) = \max_j J_h(x, u), \quad J_h(x, u) = h \sum_{i=0}^{\infty} (1 - \theta h) U f(x_h(i), u_i) \quad (\text{A1})$$

where x_u is defined by the discrete dynamics

$$x_h(0) = x, \quad x_h(i+1) = x_h(i) + hg(x_i, u_i) \quad (\text{A2})$$

and $h > 0$ is the discretization time step. Note that $j = (j_i)_{i \in \mathbb{N}_0}$ here denotes a discrete control sequence.

The optimal value function is the unique solution of a discrete Hamilton-Jacobi-Bellman equation such as

$$V_h(x) = \max_j \{hf(x, u_o) + (1 + \theta h)V_h(x_h(1))\} \quad (\text{A3})$$

where $x_h(1)$ denotes the discrete solution corresponding to the control and initial value x after one time step h . Abbreviating

$$T_h(V_h)(x) = \max_j \{hf(x, u_o) + (1 - \theta h)V_h(x_h(1))\} \quad (\text{A4})$$

the second step of the algorithm now approximates the solution on grid Γ covering a compact subset of the state space, i.e. a compact interval $[0, K]$ in our setup. Denoting the nodes of Γ by $x^i, i = 1, \dots, P$, we are now looking for an approximation V_h^Γ satisfying

$$V_h^\Gamma(X^i) = T_h(V_h^\Gamma)(X^i) \tag{A5}$$

for each node x^i of the grid, where the value of V_h^Γ for points x which are not grid points (these are needed for the evaluation of T_h) is determined by linear interpolation. We refer to the paper cited above for the description of iterative methods for the solution of (A5). Note that an approximately optimal control law (in feedback form for the discrete dynamics) can be obtained from this approximation by taking the value $j^*(x) = j$ for j realizing the maximum in (A3), where V_h is replaced by V_h^Γ . This procedure in particular allows the numerical computation of approximately optimal trajectories.

In order to distribute the nodes of the grid efficiently, we make use of a posteriori error estimation. For each cell C_l of the grid Γ we compute

$$\eta_l := \max_{k \in C_l} | T_h(V_h^\Gamma)(k) - V_h^\Gamma(k) |$$

More precisely we approximate this value by evaluating the right hand side in a number of test points. It can be shown that the error estimators η_l give upper and lower bounds for the real error (i.e., the difference between V_j and V_h^Γ) and hence serve as an indicator for a possible local refinement of the grid Γ . It should be noted that this adaptive refinement of the grid is very effective for computing steep value functions and models with multiple equilibria, see Grüne and Semmler (2004).

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