

Demand Driven Goodwin Growth Cycles with a Three-Regimes Wage Phillips Curve

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Abstract

We introduce the results of a non-parametric estimate of the wage-Phillips Curve into a simplified version of the model by Flaschel and Krolzig (2006). The resulting non-linearity in the wage inflation-employment relation translates into a non-linearity in the reduced form of the model, namely the wage-price spiral. A dynamical analysis is then provided, both in wage-led and profit-led effective demand regimes. In a profit-led economy, shown to be the empirically relevant case in the case of the US economy, there are 2 stable equilibria of Goodwin (1967) growth cycle type, identified as a stable depression and a stable boom. Both steady states are surrounded by trajectories that cycle around their basins of attraction if the economy remains viable in the economic part of the phase space. The obtained type of growth fluctuations can be verified by a long phase cycle estimation for the US economy using a method developed by Kauermann, Teuber and Flaschel (2008).

Keywords: profit-led demand, profit squeeze, long phase cycles, multiple steady states, booms and depressions.

JEL CLASSIFICATION SYSTEM: E24, E31, E32.

1 Introduction

Standard Applied Macro literature expresses labor market and goods market dynamics by a single Phillips curve, in which the cost pressure on the two markets is working on a single inflation rate. This reduced form requires simple assumptions on markup pricing and Okun's law: prices have to be a constant mark-up on wages and the rate of utilization of both the workforce and the capital stock are always strictly positively correlated. Structuralist macro models, on the other hand, often consider two separate Phillips curves, one for the labor market and one for the goods market, in order to analyze the interacting dynamics of the adjustment processes on both markets, namely the wage-price spiral.

In this paper, we introduce the results of a non-parametric estimate of the wage-Phillips Curve, showing a non linear relation between wage inflation and employment rate, into a simplified version of the model by Flaschel and Krolzig (2006).¹ This non-linearity translates into a non-linearity the wage-price spiral, a dynamic equation in which the dependent variable is the wage share growth rate, and the predetermined variables are (the levels of) the employment rate and the capital utilization rate. Assuming an Okun's type law to hold, we can derive a relation between the growth rate of the wage share and the capital utilization rate. Equations of this kind are used in the related literature (e.g. Taylor, 2004) to derive a *distributive curve* for the economy. The building block of the paper is to study how the estimated non-linearity in the wage-Phillips curve affects the distributive schedule.

In structuralist fashion, we consider also the relation between the growth rate of capital utilization and the wage share. Although in the related literature this so-called demand regime of an economy is a functional relation in which what matters are *levels*, we can characterize, without changing the moral of the story, a negative dependence on the wage share of the capital utilization growth rate as profit-led demand regime, and a positive impact on the wage share on capital utilization growth rate as wage-led demand regime.

Combining the non-linear distributive schedule with the effective demand schedule, we analyze in depth the dynamic properties of the economy, both in wage-led and profit-led effective demand regimes.

The paper is organized as follows. In section 2, we review the baseline model we are considering, assuming no productivity growth. Then, we introduce the non-linearity in the demand pressure term into the wage Phillips curve, and insert the demand regime of the economy into the reduced form of the baseline model, to derive in Section 4 a dynamic equation whose form will change accordingly to whether the demand is profit-led or wage-led. After the characterization of the properties of this single dynamic equation, in Section 5 we will build on it adding a second dimension, namely the cross-relation between the capital utilization growth and the wage share level. This added dimension will lead to a 2D system in which the cross-dual interaction between wage share and capital utilization

¹See also Flaschel, Kauermann and Semmler (2007) for a nonparametric estimation of the wage – price spiral of the PcGets estimate of this spiral in Flaschel and Krolzig (2006).

is investigated. In this case, however, the characterization of the different equilibria cannot be conclusive, since two of them are still of the structurally unstable type of the Goodwin (1967) growth cycle model. Finally, in Section 6, we will consider both dual and cross-dual dynamics. Focusing on the profit-led case, which is empirically relevant for the US economy, we will show that this basically complete system will exhibit regions where convergence to good or bad (depressed) steady states occurs, and regions where the economy either cycles around these two basins of attraction or escapes from this situation by means of accelerating inflation or deflation. An explanation for the estimation methodology is provided in the Appendix.

2 Cross-over Wage-Price Dynamics

The starting point is a simplified version of the model of the wage-price spiral estimated by Flaschel and Krolzig (2006), whose structural form is given by:²

$$\begin{aligned}\hat{w} &= \beta_w (\bar{U}^l - U^l) + \kappa_w (\hat{p} + n_x) + (1 - \kappa_w)(\pi + n_x), & \hat{w} &= \dot{w}/w \quad \text{wage inflation} \\ \hat{p} &= \beta_p (\bar{U}^c - U^c) + \kappa_p (\hat{w} - n_x) + (1 - \kappa_p)\pi, & \hat{p} &= \dot{p}/p \quad \text{price inflation}\end{aligned}\quad (1)$$

where: U^l is the rate of unemployment of labor, U^c the rate of unemployment of capital, the bars indicate NAIRU values of the two variables, n_x stands for the rate of Harrod-Neutral technical change, π is the inflationary climate. In what follows we assume that there is no labor productivity growth: $\hat{x} = n_x = 0$.³ We define the real wage as $\omega \equiv w/p$, so that the wage share $\psi \equiv w/p/x = \omega/x$. We define furthermore the labor employment rate as $e \equiv 1 - U^l$, $\bar{e} \equiv 1 - \bar{U}^l$, and similarly the capital employment rate as $u \equiv 1 - U^c$, $\bar{u} \equiv 1 - \bar{U}^c$. The two equations in (1) are then modified as follows:

$$\begin{aligned}\hat{w} &= \beta_w ((1 - \bar{e}) - (1 - e)) + (1 - \kappa_w)\pi + \kappa_w \hat{p} = \beta_w (e - \bar{e}) + (1 - \kappa_w)\pi + \kappa_w \hat{p} \\ \hat{p} &= \beta_p ((1 - \bar{u}) - (1 - u)) + \kappa_p \hat{w} + (1 - \kappa_p)\pi = \beta_p (u - \bar{u}) + (1 - \kappa_p)\pi + \kappa_p \hat{w}\end{aligned}$$

Subtracting π on both sides in both of the last equations, we obtain a two equation system in the variables $\hat{w} - \pi$, $\hat{p} - \pi$, which can be rewritten in matrix form as follows:

$$\begin{pmatrix} 1 & -\kappa_w \\ -\kappa_p & 1 \end{pmatrix} \begin{pmatrix} \hat{w} - \pi \\ \hat{p} - \pi \end{pmatrix} = \begin{pmatrix} \beta_w (e - \bar{e}) \\ \beta_p (u - \bar{u}) \end{pmatrix}$$

²In their model, the authors consider, in both equations, error-corrections for the deviation of the wage share from a certain level ψ_0 . For reasons of expositional simplicity, we do not analyze in this paper the consequences of this augmentation in both the money wage and the price Phillips curve.

³This assumption can seem unnecessarily strong. However, the ultimate task of the paper is to study the dynamics in the (u, ψ) phase space, and the rate of productivity growth, as it is easily verified, affects only the intercept of the distributive curve in that plane unless an equation for \hat{n}_x as a function of either $(U^l - \bar{U}^l)$, $(U^c - \bar{U}^c)$ or real wage growth is added to the model. In fact, equation (2) becomes: $\hat{\psi} = \kappa [(1 - \kappa_p)\beta_w (e - \bar{e}) - (1 - \kappa_w)\beta_p (u - \bar{u})] + [\kappa - (1 - \kappa_p)] n_x$. Specifying an equation for n_x and then add a third dimension to the system (2) requires additional hypotheses on the dependence of productivity growth on the relevant variables in (1), hypotheses we decided not to make in the present framework.

Defining $\kappa = (1 - \kappa_p \kappa_w)^{-1}$, the solution of (2) yields:

$$\begin{aligned}\hat{w} - \pi &= \kappa [\beta_w(e - \bar{e}) + \kappa_w \beta_p(u - \bar{u})] \\ \hat{p} - \pi &= \kappa [\kappa_p \beta_w(e - \bar{e}) + \beta_p(u - \bar{u})]\end{aligned}$$

Finally, using the definition of $\hat{\psi} \equiv \hat{w} - \hat{p}$ implied by the assumed absence of labor productivity growth, in subtracting the second equation from the first one, we derive the following reduced form:

$$\hat{\psi} = \kappa [(1 - \kappa_p)\beta_w(e - \bar{e}) - (1 - \kappa_w)\beta_p(u - \bar{u})] \quad (2)$$

The last equation shows that the wage share responds to both utilization rates in labor input and capital. Assuming, as done in the benchmark model, linear coefficients and a simple Okun's law⁴ $e = u$, $\bar{e} = \bar{u}$, and given $\kappa > 0$, we will then have that the real wage adjustment responds positively (negatively) to economic activity iff

$$\alpha \equiv (1 - \kappa_p)\beta_w - (1 - \kappa_w)\beta_p > 0 \quad (\text{respectively } < 0).$$

Following Proaño et al. (2007), we denote the case of a positive response of $\hat{\psi}$ on economic activity ($\alpha > 0$) as *labor market-led* wage-adjustment process, whereas we will say that the wage-adjustment is *goods market-led* when $\alpha < 0$ holds.

On the other hand, it is common in structuralist macroeconomic modeling to consider also the way in which changes in wage share affect the capital utilization rate, to determine the so-called *Effective Demand Regime* of the economy. If the capital utilization rate reacts positively to the wage share, the demand for goods is said to be wage-led, while if variations in the wage share cause changes in capital utilization rate of opposite sign the demand regime is said to be profit-led.

As Flaschel and Krolzig (2006) pointed out, the sign of the parameter α , combined with the characteristics of the demand regime of the economy, determines whether real wage adjustments have stabilizing or destabilizing effects. Wage adjustments will have stabilizing effects if the negative response of investment to changes in real wages outweighs the positive response of consumption, and if wages are more flexible to labor demand pressures than prices to goods market pressures (or both v.v.). Conversely, if investment reacts less than consumption to a change in real wages and wages remain more flexible than prices (in terms each of their own demand pressures), or both v.v., then real wage adjustments will show destabilizing effects. The four possible scenarios are presented in Table 1, taken from Proaño et al. (2007):

It is not within the scopes of the present paper to analyze in depth such type of effects, already broadly discussed in the quoted work.

⁴See Okun (1970) for the original formulation which gives the equation we shall work with in this paper if Okun's elasticity parameter is set equal to 1 (in place of 1/3).

	wage-led goods demand	profit-led goods demand
labor market-led real wage adjustment	adverse (divergent)	normal (convergent)
goods market-led real wage adjustment	normal (convergent)	adverse (divergent)

Table 1: Four Baseline Real Wage Adjustment Scenarios

3 Non-Linearities in the Wage Demand-Pressure Term

Let us now consider the first term in brackets in (2), describing how the wage share reacts to the employment rate level. Flaschel and Krolzig (2006) estimated a VAR for the US economy, assuming a linear relation between ψ and e . In what follows, we will consider instead a p-spline estimation of the money wage Phillips curve, where all parameters are now considered to be an unknown, assumed to be smooth, function of their subsequent variable (see the appendix to this paper).⁵ This estimation gives rise to the non-linear relationship between wage inflation and demand pressure on the labor market shown in Figure 1: the curve is increasing up to an employment rate of slightly more than 92%, then has an almost flat or at most slightly decreasing region, and eventually becomes again increasing for values of the unemployment rate smaller than 6%.

By looking at the plot of the first derivative of the function, we see that its curvature displays several changes. The first increasing portion is virtually linear; right after an employment rate of about 91%, the curve becomes concave, until an inflexion point around a 6.5 % unemployment rate, after which the curve becomes increasing and convex again. Eventually, there is another inflexion point around an employment rate of 95.5% or so, and the final portion of the curve is increasing but concave. As seen in the bottom graph, the unconditional mean of the first derivative of β_w is around 0.6.

Although the US Labor market is not worldwide known for the strength of its labor unions, a standard economic intuition behind the behavior of the curve could lay on a bargaining power argument relative to labor supply. For high levels of unemployment, the workers' bargaining power is small: they (or the labor union representing them) will be satisfied with only small increases or even decreases in the nominal wage in order to increase the employment rate. Corresponding to the center of the curve, there is a flat region where labor is resisting wage inflation decreases at the given expected price inflation, a situation widely familiar through Keynes' discussion of it. Finally, as soon as the economic activity is above a NAIRU-type full employment rate⁶, workers will exercise their increased bargaining power in requiring significantly more than proportional increases in wage inflation (as compared

⁵The other estimated p-spline functions are basically linear ones – also for the price Phillips curve – with the exception of the inflation climate which however does not matter for the law of motion of real wages.

⁶However, we will see later on that the NAIRU concept itself is made ambiguous by implications of this functional form.

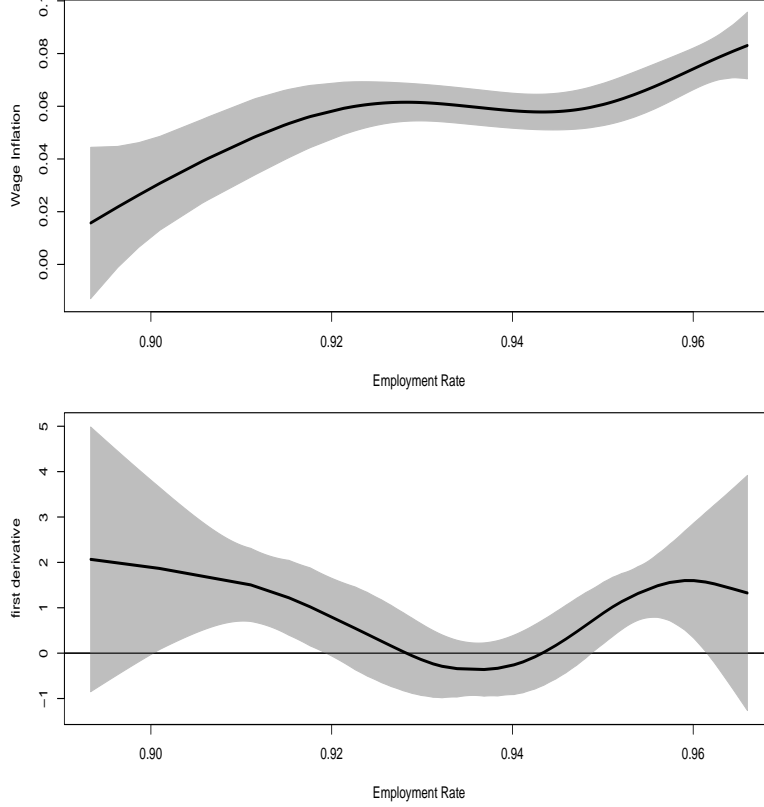


Figure 1: P-spline estimation of the wage-inflation/employment-rate schedule and its first derivative (with confidence intervals shown as grey areas, see the appendix for a brief discussion of penalized spline estimation)

to price inflation). On the basis of the nonlinearity shown in Figure 1, assuming that price flexibility is higher than wage flexibility in the middle range of the money wage Phillips curve, we get for the dynamics of the real wage the situation shown in Figure 2. The opposite situation will hold in the cases where money wage inflation reacts strongly to the demand pressure in the labor market.

In view of such arguments, let us consider the term after the minus sign in (2) in more detail. To keep things simple, we still assume that the price flexibility parameter β_p is a constant coefficient, so that the second term in square brackets in (2) is just a linear function. Assume also, as done in the benchmark model, the simple Okun's Law $e = u$ to hold, to modify (2) as follows:

$$\hat{\psi}(u) = \kappa [(1 - \kappa_p)\beta_w(u - \bar{u}) - (1 - \kappa_w)\beta_p(u - \bar{u})] \quad (3)$$

Hence, $\hat{\psi}(u) > 0$ when $\beta'_w > \frac{1-\kappa_w}{1-\kappa_p}\beta_p$, $\hat{\psi}(u) < 0$ otherwise. The local maxima and minima of this composite function will lay where $\beta'_w = \frac{1-\kappa_w}{1-\kappa_p}\beta_p$; as for the other values of u in the domain, we will have that the growth rate of the wage share will increase when $\beta'_w > \frac{1-\kappa_w}{1-\kappa_p}\beta_p$, and decrease when the inequality is reversed. Loosely speaking, if the coefficient $\frac{1-\kappa_w}{1-\kappa_p}\beta_p$ is

positive but “not too large” (less than, say, 1.5, according to Figure 1), plotting $\hat{\psi}$ against the capital utilization rate will lead to the picture in Figure 2.⁷

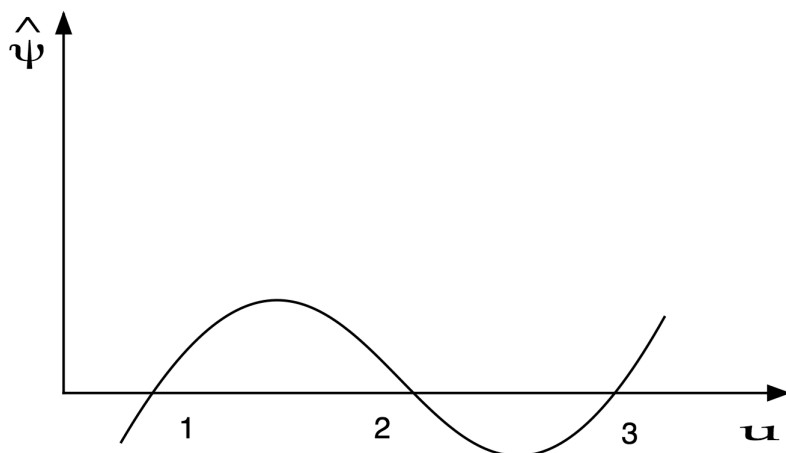


Figure 2: The reduced-form wage-price spiral under the Okun’s Law $e = u$

The combination of data and the assumption on β_p suggests a non-monotonic relation between wage share adjustment and the capital utilization rate. In structuralist macro textbooks the (long-run) relation between levels $\psi(u)$ is known as *Distributive Curve*. The sign of the first derivative of the function matters, in what $\frac{\partial \psi}{\partial u} > 0$ is interpreted saying that the economy is ‘Marxist’ or it exhibits ‘profit-squeeze’, given that a raise in capacity utilization will result in a rising wage share and thus into a falling profit share. Conversely, $\frac{\partial \psi}{\partial u} < 0$ means that the economy exhibits ‘forced saving’ along Kaldorian lines (Taylor, 2004).

Note, however, that so far we have been dealing with log-derivatives and not levels on the y -axis. The adjustment process we are considering has already been described by saying the economy is *labor-market led* in correspondence to the steady states 1 and 3, whereas it is *goods-market led* in proximity of equilibrium 2. Since a rising wage share will determine the profit share to fall, we can assimilate the labor-market led case with (maybe cyclical) profit-squeeze and the goods-market led scenario with (maybe cyclical) forced saving, but one should keep in mind the long-run *vs.* short-run distinction. Hence, some effort needs to be done in order to better characterize these dynamics, and reconcile the two points of view. Such an exercise is outside the scope of this paper. For what is compelling here, it’s enough to say that, given the estimated non-linearity of the wage Phillips curve with respect to the demand pressure term, and the assumption on the cost term in the reduced form (2), Figure 2 shows that a labor-market led distributive curve corresponds to the two unstable equilibria, while in the stable equilibrium the distributive curve is goods-market led.

The literature we are referring to considers also the long-run relation between levels $u(\psi)$, traditionally called *Effective Demand Regime*. Again, the focus is on the sign of the first

⁷In the terminology we used above, this curve shows that the economy is labor market led around 1 and 3, but goods market led around 2. In the case of the 2D dynamics we will consider later on, the curve will be an isocline (in rescaled form).

derivative of the function to determine whether the demand regime of the economy is wage-led $\frac{\partial u}{\partial \psi} > 0$ or profit-led $\frac{\partial u}{\partial \psi} < 0$. Combining the distributive curve with the demand regime of the economy, a 2D dynamical system of equations is then generally studied. The behavior of the system depends on the slope of the two curves, which in Taylor (2004) are derived as isoclines in the phase space (u, ψ) .

Traditionally, such systems are considered to be composed of linear equations. Given the results in Figure 2 as a prior, and the implied reduced form distributive curve, the dynamics of the model is now more involved than, for example, in Barbosa-Filho and Taylor (2006). In order to isolate the basic implications of the hypothesis made so far, we will start plugging the relation between the capital utilization rate and the wage share into the curve $\hat{\psi}(u)$ derived from our empirical results. This substitution will allow us to analyze the relation between wage share growth and its level.

This one-dimensional simplification has the advantage of being very simple to analyze, but nevertheless already able to generate interesting dynamics, varying according to different demand regime scenarios.

4 The One-Dimensional Baseline Dynamics

For the sake of simplicity, assume $u(\psi)$ to be a linear function, as done, among others, in Barbosa-Filho and Taylor (2006). Plugging this simple equation for the demand regime into the distributive curve, we can derive a dynamic relation between the wage share adjustment process and its own level; the shape of this function will depend on the characteristics of the demand regime of the economy. The two possible cases, corresponding to a profit-led and a wage-led scenario respectively, are represented in Figures 3 and 4.

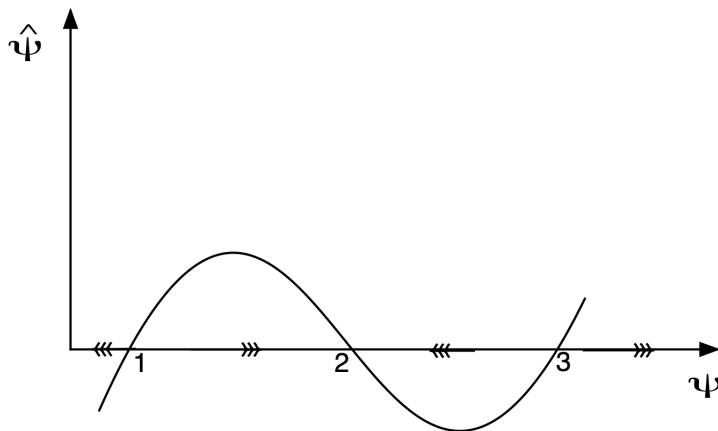


Figure 3: Real Wage Dynamics in a Wage-Led Demand Regime ($u_\psi > 0$): Corridor stability

In these two simple one-dimensional plots, the analysis of the transitional dynamics is straightforward. The estimated non-linearity in the demand-pressure term, combined with

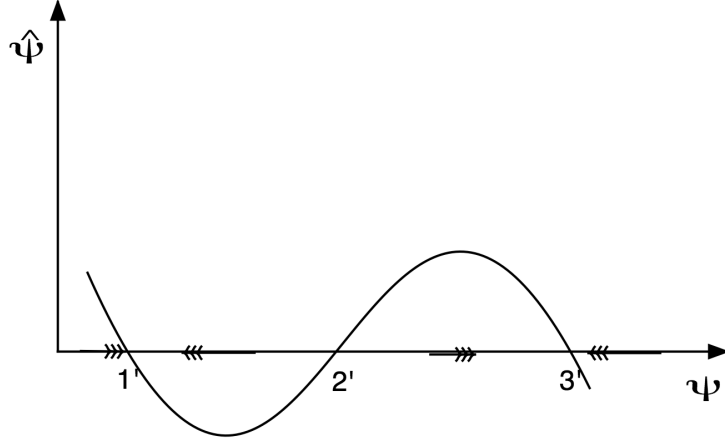


Figure 4: Real Wage Dynamics in a Profit-Led Demand Regime ($u_\psi < 0$) : Stable depressions or stable booms

the assumed linear cost-pressure and demand regime, leads to three different values for ψ ensuring a constant wage share. It is clear, however, that the slope of the demand regime will play a crucial role in determining the stability of the zeros of the equation we are studying.

In a wage-led scenario, the only stable equilibrium is 2: as long as the wage-share lays within the open interval $(1,3)$, we have a stable movement towards the steady state. Conversely, when the demand regime is profit-led, equilibrium 2' is unstable and 1', 3' are stable. Hence, while a wage-led demand determines a corridor stability around an intermediate steady state, in the profit-led scenario we find two extreme equilibria, which we could call a stable depression (1') and stable boom (3').

It is worth to notice, going back to Table 1, that the stable steady state in the wage-led scenario occurs for the goods market-led portion of the wage-adjustment curve, whereas the two stable equilibria in the profit-led demand regime correspond to the regions of the function $\hat{\psi}(u)$ where movements in wage inflation are labor market-led.

5 The Goodwin Cross-Dual Extension of the Model

So far, we used the relation between wage share adjustment process and capital utilization rate to derive a sketch of the basic dynamic properties of the wage- price spiral considered, under the non-linear relation between the money wage adjustment process and the employment rate. A more complete analysis requires the consideration of how the adjustment process of capital utilization rate is affected by variations in the wage share, or equivalently the effective demand regime adjustment process. To study this interaction, we need to add a dimension to the single equation system given by (2) under the assumed Okun's law. The

simplest way to do so is to consider a linear relation:

$$\hat{u} = \beta_u(\psi - \bar{\psi}) \quad (4)$$

where, as before, $\bar{\psi}$ indicates a steady level for the wage share. It is clear from the analysis above that a profit-led economy corresponds to the case in which $\beta_u < 0$, and then (4) is an error-correction equation; a wage-led economy requires instead $\beta_u > 0$. A steady state for the non-linear relation (4) implies a horizontal isocline in the phase plane (u, ψ) corresponding to $\psi = \bar{\psi}$. Considering Figure 2, we have now however 3 vertical isoclines for a zero wage share rate of growth, corresponding to capital utilization rates of 1, 2, 3 respectively.⁸

Let us now analyze qualitatively the dynamical properties of this system. In order to study the behavior of the wage share relative to the capital utilization rate, we have to consider all the several cases arising from the nonlinear functional form we are using. When $0 \leq u \leq u_1$ the wage share will decrease, given the positive slope of the function in that part of its domain. When $u_1 \leq u \leq u_2$, the wage share will rise: before the curve reaches its local maximum, we have that $u \geq u_1$, and the positive slope of $\beta_w(\cdot)$ will produce an increase in ψ ; when the curve starts to decrease, we have that $u < u_2$ and the negative slope of the function in that region will let the wage share to keep rising. The same lines of reasoning apply, *mutatis mutandis*, for $u_2 \leq u \leq u_3$: before the local minimum is achieved, the wage share will decrease because the capital utilization rate is above the steady state level u_2 ; from that on, the new steady state level u_3 will lead the dynamics and, given the positive slope of the curve in that region, the wage share will keep decreasing. Finally, when $u_3 < u \leq 1$, the positive slope of the curve implies an increase in the wage share.

As for the dynamics of the capital utilization rate, if the demand regime is wage-led, then $\beta_u > 0$, and above the $\dot{u} = 0$ isocline the capital utilization rate will increase; below the line u will fall. In the profit-led demand regime instead, given $\beta_u < 0$, u will decrease above the $\dot{u} = 0$ isocline, whereas it will rise below the $\dot{u} = 0$ isocline.

The combinations of the two different demand regime scenarios with the dynamics of the wage share are presented in Figures 5 and 6. The dashed lines indicate possible escape regions. Note that, because of the model's specification, in principle it is mathematically possible that at least the wage share goes above one. This is surely a weakness: in order to avoid it we should respecify the setup with the inclusion of additional non-linearities able to ensure that the wage share is not allowed to exit the $[0,1]$ interval. We will consider this issue further below.

The Jacobian matrix, evaluated at one of the steady states (denoted by a o), is:

$$J_o = \begin{pmatrix} 0 & \beta_u u_o \\ \kappa[(1 - \kappa_p)\beta'_w(\cdot) - (1 - \kappa_w)\beta_p]\psi_o & 0 \end{pmatrix}$$

⁸This is different from Rose's (1967) original situation of a single steady state (despite the presence of our nonlinear money wage Phillips curve in this early model of employment dynamics) and the difference is due to the different type of wage-price spiral that is used in Rose's original contribution.

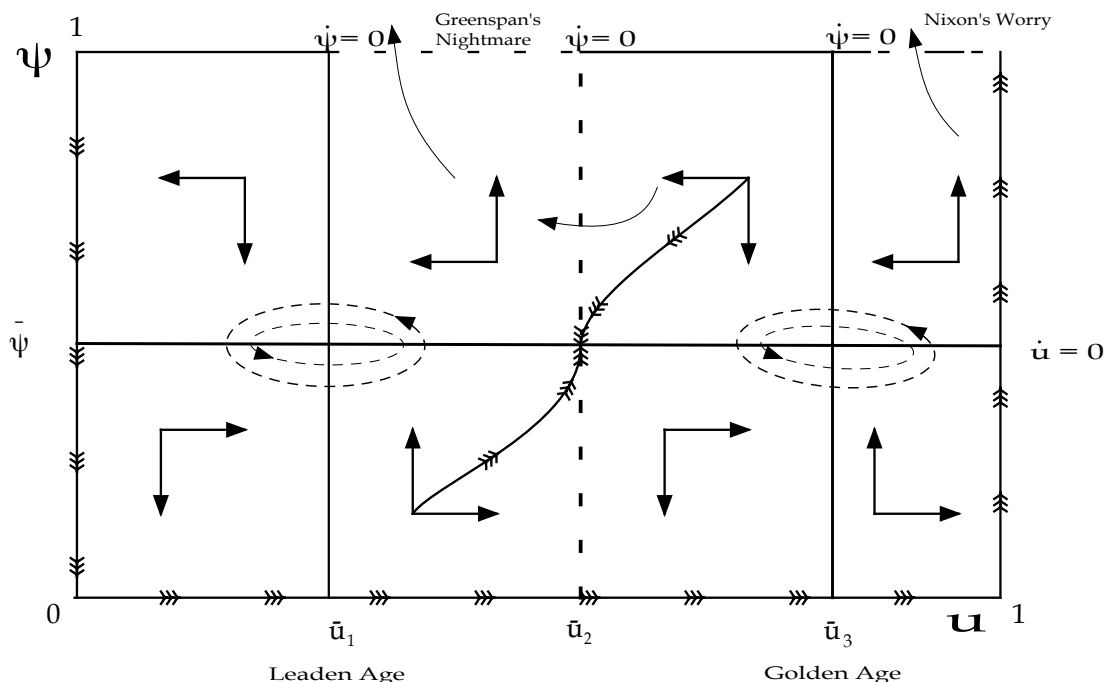


Figure 5: Phase Diagram for the Profit-Led Demand Regime: $\hat{u}_\psi < 0$, with escape regions in the boom and the depressed situation due to wage and price inflation dominance, respectively

The trace of this matrix is equal to zero, then its eigenvalues are purely imaginary (if the determinant is positive). On the other hand, the stability features of the different equilibria depend on whether the demand regime of the economy is wage-led or profit-led.

If the economy is profit-led, then $\beta_u < 0$. Looking at Figure 5, we see that for the steady-state values u_1 and u_3 , $(1 - \kappa_p)\beta'_w > (1 - \kappa_w)\beta_p$ holds, then the determinant of $J(\psi, u)$ (evaluated at the steady state) is positive and the two corresponding equilibria are stable, although not asymptotically stable. In fact – as shown in figure 5 – these steady state positions are of the center type dynamics of the Goodwin (1967) growth cycle model in the ranges where the curve in Figure 5 is downward sloping. We thus have two local Goodwin growth cycle situations (see Figure 5), and in between a saddlepoint equilibrium where the separatrices may either wind around the shown persistent cycles or escape in the boom region or the depression region.⁹

If the separatrices that depart from the steady state in the middle of Figure 5 wind around the Goodwin center type dynamics and return to the saddle (see Figure 7 for a representation of this possibility), they enclose the two stability basins of the steady states left and right and they may attract movements that are situated outside the separatrices. We then have that the paths generated by the dynamics in this area oscillate around depressions and booms in

⁹ It is easily seen that the steady state corresponding to the intersection between the horizontal isocline $\psi = \bar{\psi}$ and the vertical one $u = u_2$ is a saddle, because of the negative value of the determinant of the Jacobian evaluated at that point.

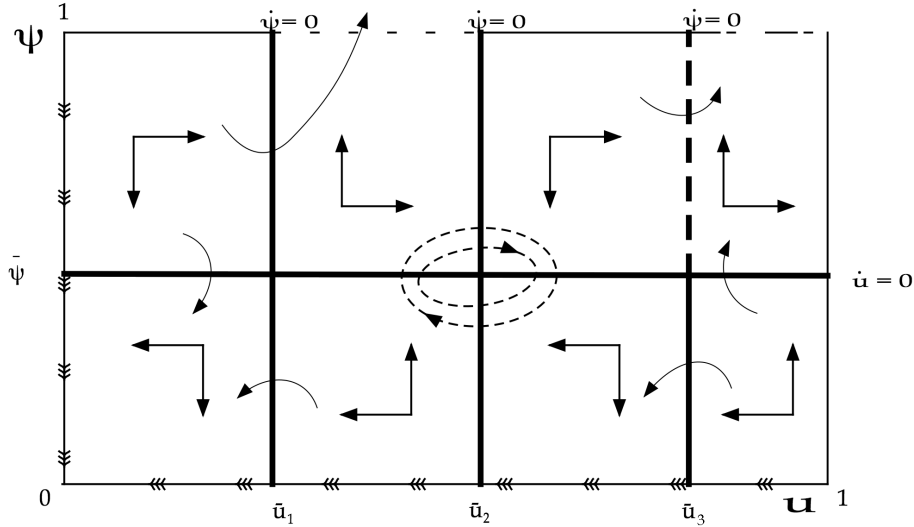


Figure 6: Phase Diagram for the Wage-Led Demand Regime: $\hat{u}_\psi > 0$

a sequential manner. We thus may be trapped in a depressed region or a boom region or may be thrown out of this situation and then oscillate between booms and depressions.

Suppose the initial condition to be, for example, in the top-right region in Figure 5: if the system does not escape out of the box, there is a movement toward north-west, until the isocline corresponding to u_3 is reached. Since we are above the isocline $\dot{u} = 0$, the system keeps moving to the left and then leaves the vertical isocline and starts going south-west. At that point, the dynamics might reach the saddle path, or move to the bottom part of the picture, or simply hit the isocline u_2 , thus keeping going west. At that point, (if the north-west movement associated with this region does not make the wage share to blow out), we will have again a movement to south-west. Not being able to determine at this stage of the analysis whether there is a basin of attraction somewhere in the top-left of the diagram, we can suppose the system to keep moving in this direction until the isocline u_1 is hit. Then, the wage share falls until it reaches the value $\bar{\psi}$. Then the system moves to the bottom, and again it might pass through all the bottom regions or reach the saddle path.

This is the situation where the dynamics stays in the unit square of the phase space. However, as pointed above, there is the mathematical possibility that it escapes from it top right. If such a situation is approached, i.e., if the wage share is moving towards 1 on the right hand side (near the full capacity line $u = 1$), we have that wage inflation is outperforming price inflation: a situation that under the Nixon administration led to a wage and price stop in order to avoid accelerating wage price inflation rates. In this way the economy may be stopped to produce outcomes that become non-viable and be forced to stay in the unit-square domain of the phase space. This instability of the private sector in a golden age may then lead the economy into the depressed part of the phase space from where it may recover after a while or not.

Yet the economy may also produce ever increasing wage shares in the left hand part of the figure, in the form of price inflation that is lower than wage inflation and thus leads to real wage increases. The cause of this possibility must be found in the wage level or the wage inflation rate to be still somewhat rigid in the downward direction. We have called this situation Greenspan's nightmare and associate this term with prices that are falling faster than wages. Yet such a situation may already come about when price disinflation is larger than wage disinflation and may thereby already lead to an adverse adjustment in the wage share (in the real wage in fact).

But these are the only possibilities where the economy may be endangered in its viability: the zero axes cannot be touched in such a growth rate dynamics, and the full capacity ceiling limits the dynamics on the right hand side of the (economically meaningful part of) the phase space. Therefore, at this level of generality the possible scenarios of the dynamical model are either depressed or boom Goodwin cycles, or movements around these two situations or the two breakdown scenarios just discussed (as long as economic policy remains inactive in such a situation). Note that the Nixon problem is much easier to handle than the Greenspan problem, in particular if the latter is coupled with the occurrence of deflation and not just disinflation.

In Figure 6 we consider the same scenarios for the case of a wage-led economy. Given the positive value of β_u , the dynamics of the capital utilization rate are reversed. Thus, we find two saddle-point equilibria at 1 and 3, and an unstable one corresponding to 2. Notice that, albeit the two external saddle points are compatible with the 1D analysis above, it is hard to find a parallel between the single dimension stability and the possibility of collapse that may now exist, either via hyperinflation to the right or hyperdeflation to the left. However, considering the own-level effects on the growth rates of both capital utilization and wage share, will allow us to reconcile the 1D findings to the somewhat puzzling results of the present section.

We do not go into the details here however, since the clockwise orientation of the interior Goodwin cycles that this situation generates is counterfactual with empirical observations, at least for the US economy.¹⁰

Let us finally point out that numerical simulations are generally needed in order to assess the size of the basins of stability of the stable steady states. Nonetheless, our qualitative analysis shows that the dynamics displayed by the system are already rich and more complex than just of a limit cycle type.¹¹

¹⁰See figure 10 and its discussion below.

¹¹See the quotations from Hirsch and Smale below in this regard.

6 Adding Keynesian Dual Forces

In thinking about the 2D system studied above, an econometrician's point of view would be of heavy criticism about the assumed and unnecessary restrictions on the influence of the levels of the two variables considered onto their respective rate of growth. Such influences should, in principle, not be denied to exist, and their empirical validity should be assessed using the available data. What we will try to do in the remaining part of the paper is indeed to consider not only the cross-dual dynamics but to introduce dependence of $\hat{u}, \hat{\psi}$ on their respective *levels*. This attempt can be seen as an effort to combine the model by Flaschel and Krolzig (2006) with the one by Barbosa-Filho and Taylor (2006).

The first force to consider is the effect of the level of the wage share on its own rate of growth. In order to characterize this relation, affecting the reduced form of their model, Barbosa-Filho and Taylor (2006) use a combination of two arguments. The first one lays on an 'induced technical progress' relation between the level of the wage share and the rate of growth of labor productivity, known in the literature as *Kaldor-Verdoorn equation*. Generally, this effect is positive since higher wages to pay will induce firms in adopting more labor-savings techniques; however, the size of the parameter has to be estimated.

The second argument is that the bargaining power of the labor force increases with the wage share. Thus, the real wage rate of growth $\hat{\omega}$ should depend positively on the wage share. Assuming both relations to be linear, if the magnitude of the Verdoorn coefficient is higher than the constant value $\partial\hat{\omega}/\partial\psi$, or in other words if the induced technical change effect is higher than the bargaining power effect on the real wage, then the function $\hat{\psi}(\psi)$ should have a negative slope.

A different story with the same ending is told in Flaschel and Krolzig (2006). They assume that an increasing wage share will dampen the evolution of wage inflation, building on Blanchard and Katz (1999) to 'microfound' this negative relation with a bargaining argument.

What about the dependence of the growth rate of capacity utilization \hat{u} on its own level u ? Since the capacity utilization rate is defined as $u \equiv X/K$, where X is output and K is the installed capacity, its rate of growth is then given by $\hat{u} = \hat{X} - \hat{K}$. Barbosa-Filho and Taylor (2006) provide linear equations for the output growth rate and for capacity growth rate:

$$\begin{aligned}\hat{X} &= \alpha_0 + \alpha_u u + \alpha_\psi \psi \\ \hat{K} &= \beta_0 + \beta_u u + \beta_\psi \psi\end{aligned}$$

There is a general consensus on the basic Keynesian stability condition $\partial\dot{X}/\partial X < 0$ to hold. Translated into the present framework this means $\alpha_u < 0$. One has, however, to consider also the effect of an increase in the capital utilization rate on the productive capacity of the economy. Generally, capital formation responds positively to the level of economic activity (which can be interpreted as an acceleration principle). It follows immediately that the rate of growth of capacity is negatively affected by the level of capacity utilization.¹² Equation

¹²Note that we have assumed $\dot{x} = 0$ in our model, meaning that a profit-led scenario will occur if $\beta_\psi > 0$

(4) is then to be modified as follows:

$$\hat{u} = \beta_{u\psi}(\psi - \bar{\psi}) - \beta_{uu}(u - \bar{u}), \beta_{uu} > 0, \quad \beta_{u\psi} \begin{cases} > 0 & \text{if the economy is wage-led} \\ < 0 & \text{if the economy is profit-led} \end{cases} \quad (5)$$

and is now interacting with an ODE that is augmented by a negative impact of the real wage on its rate of growth, represented by $-\beta_{\psi\psi}(\psi - \bar{\psi})$, in the dynamical analysis to be conducted below.

Defining $u_0 \equiv \beta_{uu}\bar{u} - \beta_{u\psi}\bar{\psi} > 0$, the isocline $\dot{u} = 0$ will be of the form:

$$\psi = -\frac{u_0}{\beta_{u\psi}} + \frac{\beta_{uu}\beta_{u\psi}}{u}$$

and it will have a positive intercept and negative slope in the plane (u, ψ) if the economy is profit-led, or a negative intercept and positive slope if the economy is wage-led. The other isocline is again given by:

$$\dot{\psi} = 0 : \quad \psi = \bar{\psi} + \frac{\kappa [(1 - \kappa_p)\beta_w(u - \bar{u}) - (1 - \kappa_w)\beta_p(u - \bar{u})]}{\beta_{\psi\psi}}$$

It is qualitatively seen identical to what is shown in figure 5, but quantitatively rescaled by the parameters $\beta_{\psi\psi}, \bar{\psi}$, as shown above.

A mathematical digression

Hirsch and Smale (1974) provide a large amount of insights into the limit sets of planar dynamical systems. A limit set of an initial point in phase space (and of the trajectory that starts from it) is the set of all limit points of sequences that lie on this trajectory, it is called an ω -limit-set if time runs to $+\infty$ and an α -limit-set if time goes to $-\infty$. In figure 7 we present an example (drawn from their book, see their p.240) where the ω -limit-set is given by the shown saddle points and its two stable arms when approached from the outside and by the left or right side of this limit set when approached from the inside (excluding the two – unstable – sources inside of them). They use this example as an example that violates the conditions of the Poincaré-Bendixson theorem, but interestingly enough the dynamics we are investigating in this section of the paper is very closely related to this example.

In a planar dynamical system – as the one we are investigating in this section – limits sets are fairly simple.

In fact, [Figure 7] is typical, in that one can show that a limit set other than a closed orbit or equilibrium is made up of equilibria and trajectories joining them. The Poincaré-Bendixson theorem says that if a compact limit set in the plane contains no equilibria it is a closed orbit. Hirsch and Smale (1974, p2.40).

and a wage-led scenario if $\beta_{\psi} < 0$.

We claim on the basis of this quotation that the dynamics to be considered below is of the type shown in figure 7 with the differences however that there is a further equilibrium at the origin of the phase plane and that the saddle arms do not enclose unstable equilibrium points, but stable ones.¹³ This assertion however only holds true if the situations we describe as Nixon's worry or Greenspan's nightmare do not happen, i.e., if there are upper turning points in income distribution before the value $\psi = 1$ is reached.¹⁴

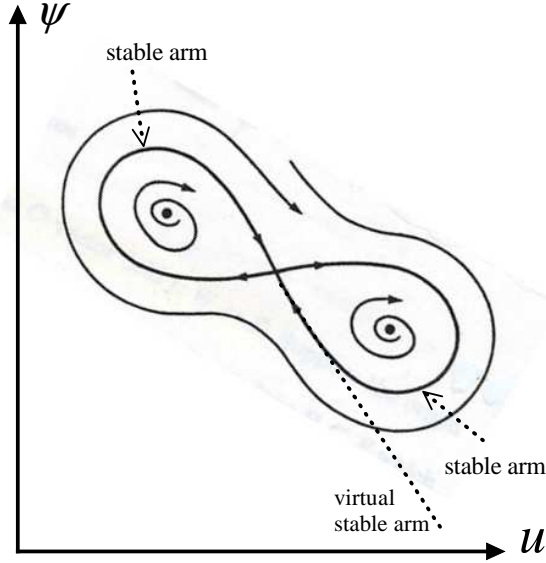


Figure 7: Limit sets in planar systems that are not closed orbits. Adapted from Hirsch and Smale (1974).

6.1 Stability Analysis of the Full 2D System

We are now able to fully characterize the dynamics of the economy according to the features of the demand regime. The Jacobian matrix at the steady states is now given by:

$$J = \begin{pmatrix} -\beta_{uu}u_o & \beta_{u\psi}u_o \\ \kappa[(1 - \kappa_p)\beta'_w(\cdot) - (1 - \kappa_w)\beta_p]\psi_o & -\beta_{\psi\psi}\psi_o \end{pmatrix}$$

In the profit-led case, $\beta_{u\psi} < 0$. Thus, when $(1 - \kappa_p)\beta'_w(\cdot) > (1 - \kappa_w)\beta_p$, that is when the distributive curve has a positive slope, the determinant is positive. Given the negative trace, both eigenvalues of the matrix are negative, and an equilibrium corresponding to this situation is asymptotically stable. Conversely, when the slope of the locus $\dot{\psi} = 0$ is negative,

¹³There is in addition the full capacity utilization barrier at $u = 1$ to the right where the dynamics undergoes a regime switching process.

¹⁴In linearized rational expectations models the virtual stable arm shown in figure 7 is used in place of the true one in order to determine the position where the economy is moving to, a fact that may lead to forward-looking reactions of economic agents that differ from those that would happen in the true nonlinear model, see Asada et al. (2003, p. 214) for details.

the corresponding equilibrium will be a saddlepoint, as before. Figure 8 displays the phase diagram corresponding to a profit-led economy when the slope of the locus $\dot{u} = 0$ is such that the two curves intersect three times. As shown in the figure, the dynamics around the steady states E_1 and E_3 feature the same counterclockwise behavior, although the negative trace of the Jacobian matrix ensures the convergence of these oscillations towards the steady states. As in Barbosa-Filho and Taylor (2006), this is due to the positive slope of the distributive curve in that regions, meaning, in their terminology, that there is a stabilizing profit-squeeze effect. On the other hand, since at the intermediate equilibrium E_2 the distributive curve has a negative slope and intersects the demand regime "from above", this steady state is a saddle point.¹⁵

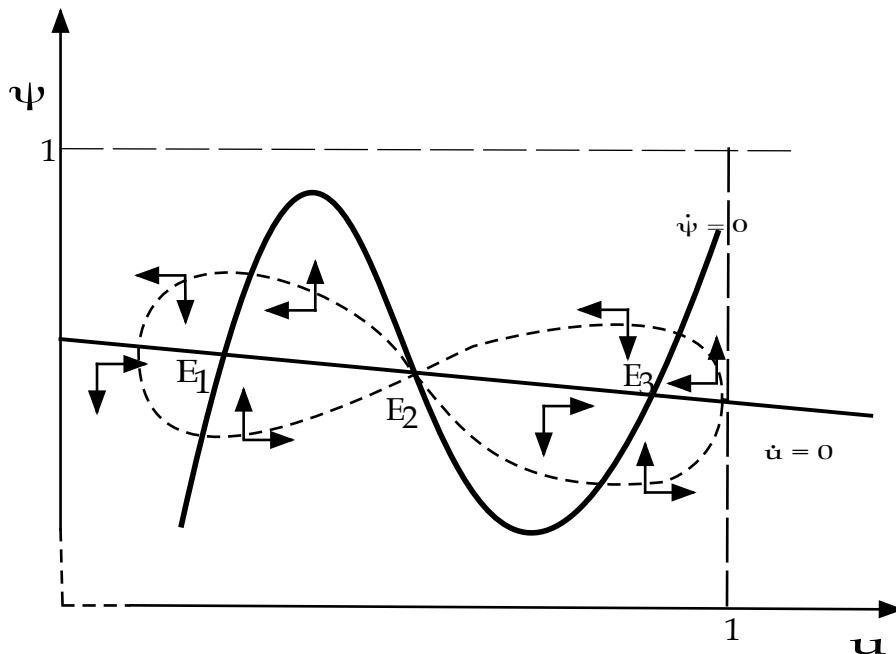


Figure 8: Phase Diagram for the Profit-Led Demand Regime

The dynamics depicted in Figure 8 not only sheds further light on the corresponding 2D case, but also parallels closely enough the one-dimensional profit-led scenario. Imposing that both variables stay in the $[0,1]$ interval, we have two areas in which the wage share and the capital utilization rate will rotate counterclockwise around some equilibrium value, eventually converging to it, unless the system sits on the unique saddle path: in this case it will converge to the intermediate equilibrium E_2 . Notice also that, since $\frac{\partial \hat{\psi}}{\partial \psi} = -\beta_{\psi\psi} < 0$, the equilibrium E_3 will correspond to the steady state 3 in Figure 2, whereas E_1 will be equivalent to equilibrium 1 in Figure 2.

Figure 8 can be seen as somewhat combining the two cases discussed in Barbosa-Filho and Taylor (2006) with regard to the US Economy (1948-2001)¹⁶: the economy studied displays

¹⁵Note that in contrast to figure 7 the equilibria E_1, E_3 in figure 8 are attracting ones and that the cycles are here clockwise in orientation (and the saddlepaths are attracting outside, not inside trajectories).

¹⁶Although in their paper, they have explicit time series thresholds for the change in slope. The explanation we provide is in terms of quantitative values of the two variables of interest.

generally a stabilizing profit squeeze effect, but in the period 1955-70 the forced saving switch in the distributive curve determines an unstable equilibrium. However, while in their paper they found a demand regime steeper than the distributive curve, here the opposite is true. Thus, our intermediate equilibrium E_2 is only saddle path stable. Of course, the slope of the demand regime matters, in what determines how many equilibria we will find in our system.¹⁷

Given the negative slope of the demand regime in this case, we find a trade-off between short run growth and egalitarian distribution, which is a traditional feature of a profit-led economy (see among others Naastepad, 2006)): in order to stimulate the economy toward a higher capital utilization rate over the business cycle, a reduction in the wage share is needed. Note finally with respect to the figure 8 that there is of course a fourth steady state at the origin of the phase space (that cannot be reached from the positive orthant). Moreover, due to the existence of three interior steady state positions it is no longer clear how the NAIRU can be defined in such a framework.

A wage-led demand regime leads to some complications. Since the locus $\dot{u} = 0$ has a negative intercept on the ψ -axis, an equilibrium with a very low rate of both capital utilization and wage share could disappear, as it is shown in Figure 9. According to the slope of the demand regime curve, there is even the possibility that the only surviving steady state is an intermediate one.

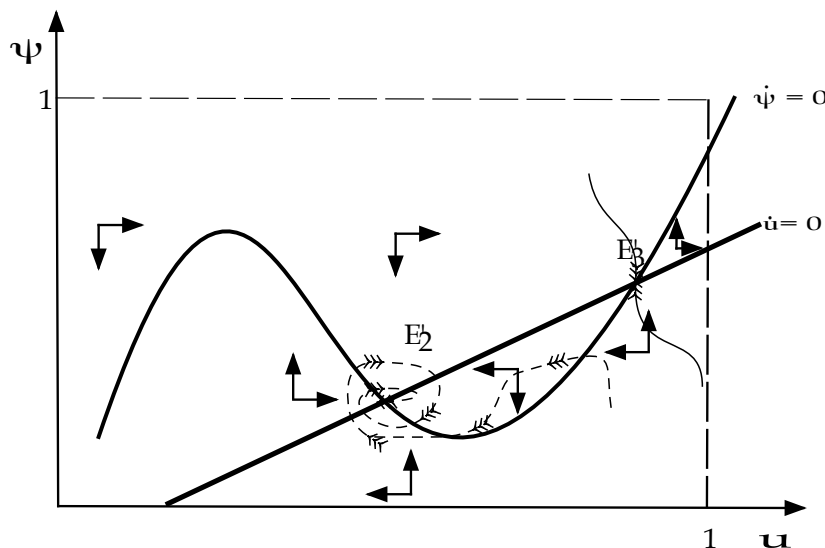


Figure 9: Phase Diagram for the Wage-Led Demand Regime

Even this situation, however, matches our 1D findings in Figure 3: looking at the Jacobian matrix, given the positive value of β_u , when the slope of the distributive curve is negative, the determinant will be positive, and the negative trace will ensure stability. Then, the

¹⁷Note however that the case in which the locus $\dot{u} = 0$ is so steep that there is only one intermediate equilibrium requires an intercept of the curve higher than 1, and this is a case we would like to rule out from the analysis. Naturally, the last word has to be said by the econometricians, but we proceed here assuming that the $\dot{u} = 0$ -isocline is sufficiently flat.

clockwise oscillations around an intermediate equilibrium like E'_2 will converge eventually to that steady state. Conversely, when $(1 - \kappa_p)\beta'_w(\cdot) > (1 - \kappa_w)\beta_p$, the corresponding equilibrium will be a saddle. We observe again that the wage-led case is counterfactual to the empirical situation found to characterize the US economy after World War II: not only the long-phase cycle found to exist, but also all business cycles have a counterclockwise orientation. On the basis of empirical observations about the economy we are studying, we go no further in analyzing a wage-led effective demand scenario.

Note also the difference between the two different scenarios in terms of policy implications. In a profit-led economy, starting from an equilibrium like E_1 , a demand-based stimulus (restriction) to economic activity needs to be very strong to be effective. If this is the case, it will lead to a situation of the type E_3 (respectively E_1), but will pay a price in terms of distributive conflict. Due to the stability features of both E_1 and E_3 , if the strength of the policy measure is not enough, a demand shock can be ineffective in reaching the policy makers' *desiderata*. On the other hand, in a wage-led scenario, if the economy is in (or around) equilibrium, a further stimulus to the economic activity can have a very hard time in achieving the desired effects, because of the uniqueness of the stable saddle path ensuring convergence to an equilibrium like E_3 . On the other hand, if policy makers acting in a wage-led economy deem it overheated at an equilibrium like E_3 they will find easy to sort the desire effects adopting restrictive policy measures, but they will pay the price of a lower wage share.

The analysis of these two different kinds of asymmetry in the effectiveness of demand policy¹⁸ deserves further attention, and is left for future research here, together with deeper considerations about the mechanisms behind the agents' expectations.

Remark: If the economy is fluctuating around the saddlepaths and comes closer to it, a small shock may suffice to move it into one of the basins of attraction of the steady states E_1 or E_2 so that it will then change its course and converge either do to depressed or a boom situation. Convergence into the depressed basin may for example be the situation experienced in Germany, while the US economy seems to fluctuate outside the basins of attraction of the investigated dynamics, see figure 10.

6.2 A brief empirical comparison

In the empirical phase plots shown in figure 10, we depict an estimated long phase cycles as against the six business cycles that were observed in the considered time span in the US economy (bottom right and left).¹⁹ As shown in Kauermann et al. (2008), all business cycles have by and large the same counterclockwise orientation, as the long-phase cycle. The

¹⁸Monetary policy in particular, given the absence of both public consumption and taxes in the present framework.

¹⁹See Kauermann et al. (2008) for the econometric methodology that allows to separate endogenously long phase cycles from cycles of business cycle frequency and for empirical applications that are closely related to the ones shown in figure 10.

bottom right figure seems to suggest that the depressed business cycles are at most two in number. In the figures top right and left we show in addition (by dots) the time series that was used in the estimations as well as the long phase fluctuations and the business fluctuations around them.

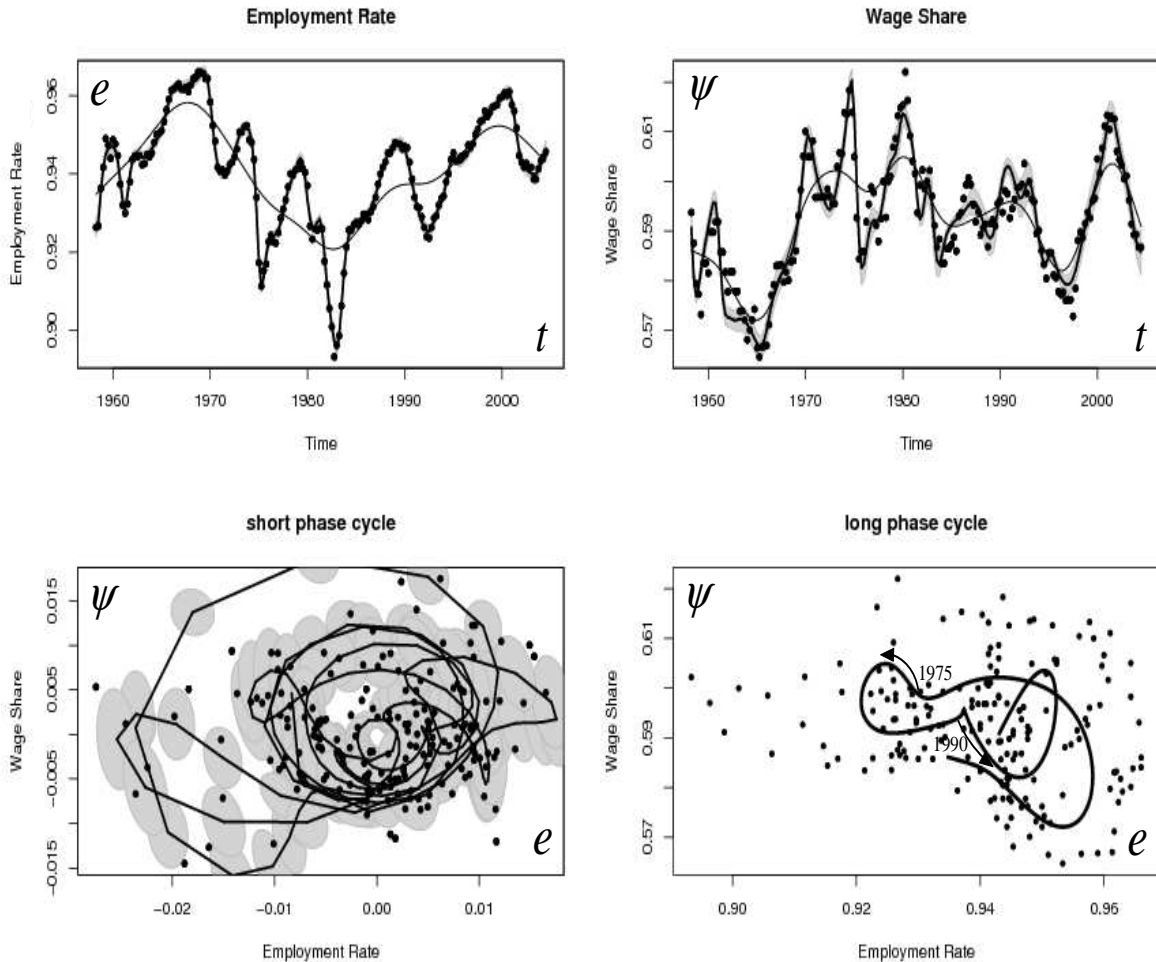


Figure 10: Long phase and business cycles in the U.S. economy after World War II (see the appendix for the details of the applied econometric technique)

Inspecting the measured long phase cycle for the US economy in more detail shows – in contradiction to the phase plots one relates with the Goodwin (1967) growth cycle model – that (top left in this figure) the wage share can increase again during phases of significant unemployment of both labor and capital, in fact after a Goodwin-like phase of decline due to the rise in unemployment (fall in the employment rate). This further Goodwin loop – which is not possible in the many conventional studies of Goodwin’s growth cycle model is perfectly in line with what we have derived in figure 8 and it shows that the depressed area in this figure is indeed relevant for one episode in the evolution of the distributional conflict in the US economy.

We stress again that the situations depicted in figure 10 reject the case of a wage-led economy, but support in the just discussed form the intermediate occurrence of goods-market led regimes which make the distributive conflict more prolonged.²⁰

We conclude that this simple model of a wage-price spiral is already rich enough to allow for somewhat persistent periods of booms, and for considerably long depressions that may need economic policy interventions aimed at avoiding economic breakdown along some of the trajectories of the dynamics.

7 Conclusions

In this paper, we studied the effects of a non-linearity in the demand pressure term of a wage Phillips curve into a structuralist macro model considering the dynamic interaction between the distributive curve and the demand regime of an economy. To do so, we borrowed both from Flaschel and Krolzig (2006) and Barbosa-Filho and Taylor (2006), simplifying the analysis in assuming no productivity growth. Considering a simplified 1D case in which a dynamic demand regime equation is inserted into the reduced form wage-price spiral, we found that: i) in a profit-led scenario there are two stable extreme equilibria, corresponding respectively to a low level of the wage share and to a high one, and one unstable intermediate equilibrium; ii) in a wage-led economy there is a single stable steady state for an intermediate level of the wage share and two saddle-path stable equilibria featuring respectively a low or a high level of the wage share.

The explicit consideration of the demand regime into a 2D system, featuring only the cross-dual interactions between ψ and u , led to a more complex characterization of the dynamic properties of the economy. We found that, in the profit-led case, the two extreme, stable equilibria display counterclockwise dynamics, while the intermediate steady state is only saddle-path stable. In the wage-led case, the intermediate stable steady state features clockwise dynamics, the (one or two, depending on the slope of the demand regime) extreme one(s) being saddle point(s). However, in this case, considering only the cross-dual dynamics, it is not possible, in principle, to determine whether the Goodwin-style equilibria are attractors, because of the purely imaginary roots of the characteristic polynomial of the Jacobian matrix.

Finally, allowing not only the cross-dual effects but also own effects on the two variables whose dynamic behavior we have studied permits, on one hand, to better characterize otherwise somewhat foggy dynamics, and to better qualify and assess the findings of the simple 1D case we considered as a starting theoretical point of this paper.

²⁰It must however be noted, in this respect, that we used the employment rate in place of capacity utilization. This is because the assumed one-to-one relation between the two variables assumed throughout has no parallel in actual data.

In order to keep the analysis as simpler as possible, the productivity rate of growth has been assumed to be zero throughout all the discussion. As already remarked (see footnote 4), this assumption has not so strong effects in the conclusions of this paper, because we have been working in the (u, ψ) plane throughout all the discussion. Nevertheless, it would be of extreme interest to consider explicitly the role played by a growing productivity within the model, considering the interaction capacity utilization - labor productivity growth as it is done for example in Naastepad (2006), and it is left for future research.

References

- ASADA, T., CHIARELLA, C., FLASCHEL, P. and R. FRANKE (2003): *Open Economy Macrodynamics. An Integrated Disequilibrium Approach* Heidelberg: Springer.
- BARBOSA-FILHO, N. and L. TAYLOR (2006): Distributive and Demand Cycles in the US Economy - A Structuralist Goodwin Model. *Metroeconomica*, 57 issue 3, pp. 389-411.
- BLANCHARD, O.J. and L. KATZ (1999): Wage dynamics: Reconciling theory and evidence. *American Economic Review. Papers and Proceedings*, 69 – 74.
- FLASCHEL, P. and H.-M. KROLZIG (2006): Wage-price Phillips curves and macroeconomic stability: Basic structural form, estimation and analysis. In: C. Chiarella, P. Flaschel, R. Franke and W. Semmler (eds.): *Quantitative and Empirical Analysis of Nonlinear Dynamic Macromodels*. Contributions to Economic Analysis, Elsevier, Amsterdam.
- FLASCHEL, P., KAUERMANN, G. and W. SEMMLER (2007): Testing wage and price Phillips curves for the United States. *Metroeconomica*, 58, 550 – 581.
- GOODWIN, R.M. (1967): A growth cycle. In: C.H. Feinstein (ed.): *Socialism, Capitalism and Economic Growth*. Cambridge, UK: Cambridge University Press, 54 – 58.
- HIRSCH, M. and S. SMALE (1974): *Differential Equations, Dynamical Systems, and Linear Algebra*. New York: Academic Press.
- KAUERMANN, G., TEUBER, T. and P. FLASCHEL (2008): Estimating loops and cycles using penalized splines. Bielefeld: CEM working paper.
- KRIVOBOKOVA, T. and G. KAUERMANN (2004): A short note on penalized spline smoothing with correlated errors Bielefeld University: Technical report.
- NAASTEPAD, C.W.M. (2006): Technology, Demand and Distribution: A Cumulative Growth Model with an Application to the Dutch Productivity Slowdown. *Cambridge Journal of Economics*, 30, 403 - 434.
- OKUN, A.M. (1970): *The Political Economy of Prosperity*. Washington, D.C.: The Brookings Institution.
- PROAÑO, C., FLASCHEL, P., ERNST, E. and W. SEMMLER (2007): Gradual Wage-Price Adjustments in Keynesian Macrodynamics. Evidence from the U.S. and the Euro Area. New School University: Scepta working paper.
- ROSE, H. (1967): On the non-linear theory of the employment cycle. *Review of Economic Studies*, 34, 153 – 173.
- RUPPERT, D., WAND, M. and R. CARROLL (2003): *Semiparametric Regression*. Cambridge: Cambridge University Press.
- TAYLOR, L. (2004): *Reconstructing Macroeconomics. Structuralist Proposals and Critique of the Mainstream* Cambridge, MA: Harvard University Press.

Appendix: Penalized Spline Estimates

For the estimation of the (Wage) Philips Curve a Penalized Spline approach has been used, see for instance Ruppert et al. (2003), such that the (penalized) log likelihood for normal errors can be written as

$$l(\theta) = -(y - C\theta)^2 / \sigma^2 - \lambda_1 \theta^T D_1 \theta - \dots - \lambda_m \theta^T D_m \theta \quad (6)$$

with the combined design matrix $C = (X Z)$ containing the fixed effect design matrix $X = (1 x_{1i} \dots x_{1i}^{q_1} x_{2i} \dots x_{2i}^{q_2} \dots x_{mi} \dots x_{mi}^{q_m})_{i=1, \dots, n}$ and the truncated spline basis $Z = (Z_1 \dots Z_m)$ with the j -th truncated spline basis defined by $Z_j = ((x_{ji} - \tau_{1,j})_+^{q_j} + \dots + (x_{ji} - \tau_{K,j})_+^{q_j})_{i=1, \dots, n}$ which are constructed with the truncation function $(x)_+^q := \max\{0, x\}^q$ and the K knots $\tau_{1,j}, \dots, \tau_{K,j}$ for the j -th dependent variable x_{j1}, \dots, x_{jn} . We have chosen different orders of the truncated polynomial, i.e. q_1, \dots, q_n , just to ensure that the structure for the unknown functions for some dependant variables have not been chosen to be too complex and for some variables we need to choose a higher order to visualize the first derivative of the estimated function in a smooth way. In the same way we could have choose different numbers of knots for each variable but to keep things simple we have used the same number of knots for all variables. The main diagonal of the penalty matrix D_l contain a one if the index belong to the truncated spline basis Z_l and otherwise the element contain a zero, i.e. $D_l = (d_{ij})_{i=1, \dots, mK+q+1}^{j=1, \dots, mK+q+1}$ with $q = \sum_{i=1}^m q_i$ and $d_{ij} = 1_{\{i=j\}} 1_{\{i \in \{q+2+(l-1)K, \dots, q+1+lK\}\}}$. The smoothing parameters $\lambda_1, \dots, \lambda_m$ control the complexity of the structure for the unknown functions and should be chosen carefully. No penalization $\lambda_j = 0$ result in a too complex function with $q_j + K$ degrees of freedom and a highly penalized function ($\lambda_j \rightarrow \infty$) result in a function of order q_j . We are following the suggestion of Krivobokova and Kauermann (2004) to use the REML estimator for smoothing parameters to avoid misleading parameters because of misspecified autocorrelated errors.

For the Wage Philips Curve we are describing the wage inflation (y) by the variables price inflation (x_1), the log of the wage share (x_2),²¹ the employment rate (x_3) and the price inflation climate (x_4). In a first step we have set the order of the truncated splines to one, i.e. $q_1 = \dots = q_4 = 1$, to avoid misleading estimations because of too complex functional relationships. The resulting estimating show, that the price inflation and the wage share are linear related with the wage inflation. The employment rate and the price climate are in a non-linear way related with the wage inflation, and even more the functional form for employment rate uses more than three degrees of freedom, such that a higher polynomial order could be used. In our second step, setting $q_3 = 2$, the resulting estimation is nearly similar to our first one such that the same smoothing parameters λ_1 and λ_2 and the same shape of functions for the employment rate and the price climate are estimated.

Similarly, for the Price Philips Curve we are describing the price inflation (y) by the variables wage inflation (x_1), the log of the wage share (x_2), the utilization rate (x_3) and the price inflation climate (x_4). But in contrast to the Wage Philips Curve the functional

²¹as in Blanchard and katz (1999).

shape of the Price Phillips Curve with respect to the utilization rate is not distinctively different from a linear curve (as was the functional shape of the WPC with respect to the employment rate), which gives the reason why we have omitted the visualization of the PPC estimation.

For the joint estimation of the employment rate (y_1) and the log of the wage share (y_2) we are distinguishing between long term and short term trends which is usually done when estimating business cycles. But instead of treating the deviations from the long term trend as errors we assume that the business cycles can be described by a functional form. Following Kauermann et al. (2008) we are assuming that the observations $y_t := (y_{1t}, y_{2t})^T$ can be described by a long term trend $c(t) := (c_1(t), c_2(t))^T$ and a short term trend $g(t) := (g_1(t), g_2(t))^T$, i.e. $y_t = c(t) + g(t) + \epsilon_t$ with normal residuals $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t})^T \sim \mathcal{N}(0, \sigma)$. The structure of the short term trend is even more specified by setting $g(t) := (\rho(t) \cos \phi(t), \rho(t) \sin \phi(t))^T$ with $\rho(t)$ representing the radius and $\phi(t)$ the angle around the center $c(t)$. For the estimation of the short term trend $g(t)$ polar coordinates are preferred because we assume that the speed and the direction of the trajectory for the detrended time series $y_t - c(t)$ are smooth functions over the time. The unknown functional forms of the radius $\rho(t)$, the angle $\phi(t)$ and the long term trends $c_1(t)$ and $c_2(t)$ are captured by a Penalized Spline approach such that the structure and the degree of complexity has to be estimated with the data at hand. But instead of estimating the short and long term functions simultaneously a hybrid version has been used because of numerical reasons. At the first stage, the long term trend is fitted by a given pair of long term penalty parameters. At the second stage, the resulting detrended observations $y_t - \hat{c}(t)$ are used to get the estimations for the short term functions using the REML estimation for choosing the optimal amount of smoothing for the radius and the angle. The optimal pair of long term smoothing parameters has been chosen by the Akaike Information Criterion, see for justification Kauermann et al. (2008).