Stabilizing an Unstable Economy: On the Choice of Proper Policy Measures

Toichiro Asada,
Department of Economics, Chuo University,
Tokyo, Japan

Carl Chiarella,
School of Finance and Economics, University of Technology,
Sydney,Australia

Peter Flaschel,
Department of Business Administration and Economics, Bielefeld University,
Bielefeld, Germany

Tarik Mouakil,
Centre for Financial Analysis and Policy, University of Cambridge,
Cambridge, United Kingdom

Christian Proaño*
Macroeconomic Policy Institute (IMK)
Düsseldorf, Germany

Willi Semmler
Department of Economics, New School University
New York, USA

November 3, 2009

*Corresponding author: christian-proano@BOECKLER.DE

Abstract

Currently, many monetary and fiscal policy measures are aimed at preventing the financial market meltdown that started in the US subprime sector and has spread world wide as a great recession. Although some slow recovery appears to be on the horizon, it is worthwhile exploring the fragility and potentially destabilizing feedbacks of advanced macroeconomies in the context of Keynesian macro models. Fragilities and destabilizing feedback mechanisms are known to be potential features of all markets– the product markets, the labor market, and the financial markets. In this paper we in particular focus on the financial market. We use a Tobin-like macroeconomic portfolio approach, and the interaction of heterogeneous agents on the financial market to characterize the potential for financial market instability. Though the study of the latter has been undertaken in many partial models, we focus here on the interconnectness of all three markets. Furthermore, we study the potential that labor market, fiscal and monetary policies have to stabilize unstable macroeconomies. Besides other stabilizing policies we in particular propose a countercyclical monetary policy that sells assets in the boom and purchases assets in recessions. Modern stability analysis is brought to bear to demonstrate the stabilizing effects of those suggested policies.

Keywords: Monetary Business Cycles, Portfolio Choice, (In-)Stability, Stabilizing Policy Measures

JEL classifications: E12 , **E24**, **E31**, **E52**.

As we approach the last decade of the twentieth century, our economic world is in apparent disarray. After two secure decades of tranquil progress following World War II, in the late 1960s the order of the day became turbulence - both domestic and international. Bursts of accelerating inflation, higher chronic and higher cyclical unemployment, bankruptcies, crunching interest rates, and crises in energy, transportation, food supply, welfare, the cities, and banking were mixed with periods of troubled expansions. The economic and social policy synthesis that served us so well after World War II broke down in the mid-1960s. What is needed now is a new approach, a policy synthesis fundamentally different from the mix that results when today's accepted theory is applied to today's economic system. (Minsky (1982), p.3)

1 Introduction

The financial crisis starting in the US subprime sector, has spread world wide as a great recession. A hyperactive monetary and fiscal policy since the end of 2007 has aimed at preventing a further financial meltdown in the advanced countries. Some observers maintain that a slow recovery appears to be on the horizon. Yet, it is worthwhile exploring the fragility and potentially destabilizing feedbacks of advanced macroeconomies in the context of Keynesian macro models. Further macroeconomic work is needed. As the history of macroeconomic dynamics and business cycles – which recently have been developed as boom - bust cycles – has taught us, fragilities and destabilizing feedbacks are known to be potential features of all markets – the product markets, the labor market, and the financial markets.

In this paper we in particular will focus on the financial market. We use a Tobin-like macroeconomic portfolio approach, coupled with the interaction of heterogeneous agents on the financial market, to characterize the potential for financial market instability. Though the study of the latter has been undertaken in many partial models, we focus here on the interconnectness of all three markets. Furthermore, we study what potential labor market, fiscal and monetary policies can have in stabilizing unstable macroeconomies. It was in particular Minsky (1982) who has put forward many ideas to stabilize an unstable economy. Beside other stabilizing policies we in particular propose a countercyclical monetary policy that sells assets in the boom and purchases assets in recessions. Modern dynamic and stability analysis are brought to bear to demonstrate the stabilizing effects of those suggested policies.

The paper builds on work by Asada, Flaschel, Mouakil, and Proano (2009) by using models of that research agenda as the starting point for the proper design of a macrodynamic framework, and labor market and fiscal and monetary policies in a framework which allows in general for large swings in financial and real economic activity. It builds on baseline models of the dynamic interaction of the labor market, the product market and financial markets with risky assets. We revive a framework of a macroeconomic portfolio approach that was suggested by Tobin (1969, 1980), but also builds on recent

work on the interaction of heterogeneous agents in the financial market.[1] We allow for heterogeneity in share and goods price expectations and study the financial, nominal and real cumulative feedback chains that may give rise to the potential of an unstable economy. The work connects to traditional Keynesian business cycle analysis as Tobin, Minsky, and Akerlof have suggested and this seems appropriate given that governments world-wide have resorted to Keynesian type policies to combat the current global financial crisis.

The remainder of the paper is organized as follows. Section 2 sketches the main modules of a portfolio approach to Keynesian business cycle theory. The portfolio approach can be stabilizing if gross substitution of assets is allowed for. Yet, it can also generate fragile dynamics and a destabilizing potential through expected asset price dynamics. Since the model exhibits growth, section 3 introduces the model in intensive form and section 4 explores the comparative statics of the asset markets. Steady state properties are explored in section 5 and the potential for fragility and destabilizing feedbacks are studied in section 6. Section 7 studies labor market and fiscal policies, possibly giving rise to stabilizing feedbacks. Section 7 proposes a new form of monetary policy that is not only concerned with interest rates, but in particular with countercyclical selling and buying of assets, a policy that the US Fed in fact has undertaken and which is, in spirit, close to the Minsky‘s (1982) ideas. The stabilizing effects of this policy are also explored.

2 Asset markets and Keynesian business cycles: A portfolio approach

In the tradition of Tobin (1969, 1980) we will depart from standard theory and provide the structural form of a growth model using a portfolio approach [2] and building in heterogeneous agents‘ behavior on asset markets. In order to discuss details we split the model into appropriate modules that refer to the sectors of the economy, namely households, firms, and the government (fiscal and monetary authority). Beside presenting a detailed structure of the asset market, we also represent the wage–price–interactions, and connect the financial market to the labor and product market dynamics.

2.1 Households

As discussed in the introduction we disaggregate the sector of households into worker households and asset holder households. We begin with the description of the behavior of workers:

[1] In recent work on behavioral finance the interaction of the fundamentalist and behavioral traders is seen as central in creating bubbles and crashes, see Brunnermeyer (2009).

[2] Flow-oriented equations for the prices of the assets were used in Chiarella, Flaschel, Groh, and Semmler (2000).

Worker households

$$\begin{aligned} \omega &= w/p, && (1)\\ C_w &= (1-\tau_w)\omega L^d, && (2)\\ S_w &= 0, && (3)\\ \hat{L} &= n = \text{const.} && (4) \end{aligned}$$

Equation (1) gives the definition of the real wage ω before taxation, where w denotes the nominal wage and p the actual price level. We operate in a Keynesian framework with sluggish wage and price adjustment processes. We follow the Keynesian framework by assuming that the labor demand of firms can always be satisfied out of the given labor supply[3]. Then, according to (2), real income of workers equals the product of real wages times labor demand, which net of taxes $\tau_w\omega L^d$, equals workers' consumption, since we do not allow for savings of the workers as postulated in (3).[4] No savings implies that the wealth of workers is zero at every point in time. This in particular means that the workers do not hold any assets and that they consume instantaneously their disposable income. As is standard in theories of economic growth, we finally assume in equation (4) a constant growth rate n of the labor force L based on the assumption that labor is supplied inelastically at each moment in time. The parameter n can be easily reinterpreted to be the growth rate of the working population plus the growth rate of labor augmenting technical progress.

The income, consumption and wealth of the asset holders are described by the following set of equations:

Asset holder households

$$\begin{aligned} r_k^e &= (Y^e - \delta K - \omega L^d)/K, && (5)\\ C_c &= (1-s_c)[r_k^e K + iB/p - T_c], \quad 0 < s_c < 1, && (6)\\ S_p &= s_c[r_k^e K + iB/p - T_c] && (7)\\ &= (\dot{M} + \dot{B} + p_e\dot{E})/p, && (8)\\ W_c &= (M + B + p_e E)/p, \quad W_c^n = pW_c. && (9) \end{aligned}$$

The first equation (5) of this module of the model defines the expected rate of return on real capital r_k^e to be the ratio of the currently expected real cash flow and the real stock of business fixed capital K. The expected cash flow is given by expected real revenues from sales Y^e diminished by real depreciation of capital δK and the real wage sum ωL^d. We assume that firms pay out all expected cash flow in the form of dividends to the asset holders. These dividend payments are one source of income for asset holders. The second source is given by real interest payments on short term bonds (iB/p) where i is the nominal interest rate and B the stock of such bonds. Summing up these types of

[3]We do not allow for regime switches as they are discussed in Chiarella, Flaschel, Groh, and Semmler (2000, ch.5)

[4]See Chiarella, Flaschel, Groh, and Semmler (2000) for the inclusion of workers' savings into a KMG framework.

interest incomes and taking account of lump sum taxes T_c in the case of asset holders (for reasons of simplicity) we obtain the disposable income of asset holder given by the terms in the square brackets of equation (6), which together with a postulated fixed propensity to consume $(1 - s_c)$ out of this income gives us the real consumption of asset holders.

Real savings of pure asset owners is real disposable income minus their consumption as exposed in equation (7). The asset owners can allocate the real savings in the form of money $\dot{M}$, or buy other financial assets, namely short-term bonds $\dot{B}$ or equities $\dot{E}$ at the price p_e, the only financial instruments that we allow for in the present reformulation of the KMG growth model. Hence, the savings of asset holders must be distributed to these assets as stated in equation (8). Real wealth of pure asset holders is thus defined in equation (9) as the sum of the real cash balance, real short term bond holdings and real equity holdings of asset holders. Note that the short term bonds are assumed to be fixed price bonds with a price of one, $p_b = 1$, and a flexible interest rate i.

Next we introduce portfolio holdings to be described as follows. Following the general equilibrium approach of Tobin (1969) we can express the demand equations of asset owning households for financial assets as:

$$
\begin{aligned}
M^d &= f_m(i, r_e^e) W_c^n, && (10)\\
B^d &= f_b(i, r_e^e) W_c^n, && (11)\\
p_e E^d &= f_e(i, r_e^e) W_c^n, && (12)\\
W_c^n &= M^d + B^d + p_e E^d. && (13)
\end{aligned}
$$

The demand for money balances of asset holders M^d is determined by a function $f_m(i, r_e^e)$ which depends on the interest rate on short run bonds i and the expected rate of return on equities r_e^e. The value of this function times the nominal wealth W^n gives the nominal demand for money M^d, so that f_m describes the portion of nominal wealth that is allocated to pure money holdings. Note that this formulation of money demand is not based on a transaction motive, since the holding of transaction balances will be the job of firms.

We do not assume that the financial assets of the economy are perfect substitutes, but make the assumption that financial assets are imperfect substitutes. This is implicit in the approach that underlies the above block of equations. But what is the motive for asset holders to hold a fraction of their wealth in form of money, when there is a riskless interest bearing asset? In our view it is reasonable to employ a speculative motive: Asset holders want to hold money in order to *be able* to buy other assets or goods with zero or very low transaction costs. This of course assumes that there are (implicitly given) transaction costs when fixed price bonds are turned into money.[5]

The nominal demand for bonds is determined by $f_b(i, r_e^e)$ and the nominal demand for equities by $f_e(i, r_e^e)$, which again are functions that describe the fractions that are

[5]Köper (2003), in his ch.7, modifies this framework by assuming that money holdings equal $M3$ and that bonds are flexprice or long-term bonds which give rise to capital gains or losses just as the equities of the present paper.

allocated to these forms of financial wealth. From equation (9) we know that actual nominal wealth equals the stocks of financial assets held by the asset holders. We assume, as is usual in portfolio approaches, that the asset holders do demand assets of an amount that equals in sum their nominal wealth as stated in equation (9). In other words, they just reallocate their wealth in view of new information on the rates of returns on their assets and take account of their wealth constraint.

What remains to be modeled in the household sector is the expected rate of return on equities r_e^e which, as usual, consists of real dividends per unit of equity ($r_k^e pK/p_e E$), and expected capital gains, π_e, the latter being nothing other than the expected growth rate of equity prices. Thus we can write

$$r_e^e = \frac{r_k^e pK}{p_e E} + \pi_e. \tag{14}$$

In order to complete the modeling of asset holders' behavior, we need to describe the evolution of π_e. In the tradition of recent work on heterogeneous agents in asset markets, we here assume that there are two types of asset holders, who differ with respect to their expectation formation of equity prices.[6] There are behavioral traders, called *chartists*, who in principle employ an adaptive expectations mechanism

$$\dot{\pi}_{ec} = \beta_{\pi_{ec}}(\hat{p}_e - \pi_{ec}), \tag{15}$$

where $\beta_{\pi_{ec}}$ is the adjustment speed toward the actual growth rate of equity prices. The other asset holders, the *fundamentalists*, employ a forward looking expectation formation mechanism

$$\dot{\pi}_{ef} = \beta_{\pi_{ef}}(\eta - \pi_{ef}) \tag{16}$$

where η is the fundamentalists' expected long run growth rate of share prices. Assuming that the aggregate expected rate of share price increase is a weighted average of the two expected rates, where the weights are determined according to the sizes of the groups, we postulate

$$\pi_e = \alpha_{\pi_{ec}}\pi_{ec} + (1 - \alpha_{\pi_{ec}})\pi_{ef}. \tag{17}$$

Here $\alpha_{\pi_{ec}} \in (0, 1)$ is the ratio of chartists to all asset holders.

2.2 Firms

We consider the behavior of firms by means of two submodules. The first describes the production framework and their investment in business fixed capital and the second

[6] Brunnermeyer (2009) calls them behavioral and fundamentalist traders.

introduces the Metzlerian approach of inventory dynamics concerning expected sales, actual sales and the output of firms.

Firms: production and investment

$$
\begin{aligned}
r_k^e &= (pY^e - wL^d - p\delta K)/(pK), && (18)\\
Y^p &= y^p K, && (19)\\
u &= Y/Y^p, && (20)\\
L^d &= Y/x, && (21)\\
e &= L^d/L = Y/(xL), && (22)\\
q &= p_e E/(pK), && (23)\\
I &= i_q(q-1)K + i_u(u-\bar{u})K + nK, && (24)\\
\hat{K} &= I/K, && (25)\\
p_e\dot{E} &= pI + p(\dot{N} - \mathcal{I}) && (26)
\end{aligned}
$$

Firms are assumed to pay out dividends according to expected profits (expected sales net of depreciation and minus the wage sum), see the above module of the asset owning households. The rate of expected profits r_k^e is expected real profits per unit of capital as stated in equation (18). Firms produce output utilizing a production technology that transforms demanded labor L^d combined with business fixed capital K into output. For convenience we assume that the production takes place with a fixed proportion technology.[7] According to (19) potential output Y^p is given at each moment of time by a fixed coefficient y^p times the existing stock of physical capital. Accordingly, the utilization of productive capacities is given by the ratio u of actual production Y and the potential output Y^p. The fixed proportions in production give rise to a constant output-labor coefficient x, by means of which we can deduce labor demand from goods market determined output as in equation (21). The ratio L^d/L thus defines the rate of employment in the model.

The economic behavior of firms must include their investment decision with regard to business fixed capital, which is determined independently of the savings decision of households. We here model investment decisions per unit of capital as a function of the deviation of Tobin's q, see Tobin (1969), from its long run value 1[8], and the deviation of actual capacity utilization from a normal rate of capital utilization. We add an exogenously given trend term, here given by the natural growth rate n in order to allow this rate to determine the growth path of the economy in the usual way. We employ here Tobin's average q which is defined in equation (23). It is the ratio of the nominal value of equities and the reproduction costs for the existing stock of capital. Investment in business fixed capital is reinforced when q exceeds one, and is reduced when q is smaller then one. This influence is represented by the term $i_q(q-1)$ in equation (24). The

[7] See Chiarella, Flaschel, Groh, and Semmler (2000) (chapter 4) for the treatment of a production function with smooth factor substitution and a discussion as to why this assumption is not as restrictive as might be believed by many economists.

[8] This holds if there is no adjustment cost of capital.

term $i_u(u - \bar{u})$ models the component of investment which is due to the deviation of utilization rate of physical capital from its non accelerating inflation value $\bar{u}$. The last component, nK, takes account of the natural growth rate n which is necessary for steady state analysis if natural growth is considered as exogenously given. Equation (26) is the budget constraint of the firms. Investment in business fixed capital and unintended changes in the inventory stock $p(\dot{N} - \mathcal{I})$ must be financed by issuing equities, since equities are the only financial instrument of firms in this paper. Capital stock growth finally is given by net investment per unit of capital I/K in this demand determined model of the short–run equilibrium position of the economy.

Next we model the inventory dynamics following Metzler (1941) and Franke (1996). This approach is a very useful concept for describing the goods market disequilibrium dynamics with all of its implications.

Firms output adjustment:

$$
\begin{aligned}
N^d &= \alpha_{n^d} Y^e, && (27)\\
\mathcal{I} &= nN^d + \beta_n (N^d - N), && (28)\\
Y &= Y^e + \mathcal{I}, && (29)\\
Y^d &= C + I + \delta K + G, && (30)\\
\dot{Y}^e &= nY^e + \beta_{y^e}(Y^d - Y^e), && (31)\\
\dot{N} &= Y - Y^d, && (32)\\
S_f &= Y - Y^e = \mathcal{I}, && (33)
\end{aligned}
$$

where $\alpha_{n^d}, \beta_n, \beta_{y^e} \geq 0$.

Equation (27) states that the desired stock of physical inventories, denoted by N^d, is assumed to be a fixed proportion of the expected sales. The planned investments $\mathcal{I}$ in inventories follow a sluggish adjustment process toward the desired stock N^d according to equation (28). Taking account of this additional demand for goods, equation (29) writes the production Y as equal to the expected sales of firms plus $\mathcal{I}$. To explain the expectation formation for goods demand, we need the actual total demand for goods which in (30) is given by consumption (of private households and the government) and gross investment by firms.

From a knowledge of the actual demand Y^d, which is always satisfied, the dynamics of expected sales is given in equation (31). It models these expectations to be the outcome of an error correction process, that incorporates also the natural growth rate n in order take account of the fact that this process operates in a growing economy. The adjustment of sales expectations is driven by the prediction error $(Y^d - Y^e)$, with an adjustment speed that is given by β_{y^e}. Actual changes in the stock of inventories are given in (32) by the deviation of production from goods demanded.

The savings of the firms S_f is as usual defined by income minus consumption. Because firms are assumed to not consume anything, their income equals their savings and is given by the excess of production over expected sales, $Y - Y^e$. According to the production account in table 1 the gross accounting profit of firms finally is $r_k^e pK + p\mathcal{I} = pC + pI +$

$p\delta K + p\dot{N} + pG$. Substituting in the definition of r_k^e from equation (18), we compute that $pY^e + p\mathcal{I} = pY^d + p\dot{N}$ or equivalently $(Y - Y^e) = \mathcal{I}$ as stated in equation (33).

Uses	Resources
Production Account of Firms:	
Depreciation $p\delta K$	Private consumption pC
Wages wL^d	Gross investment $pI + p\delta K$
Gross accounting profits $\Pi = r_k^e pK + p\mathcal{I}$	Inventory investment $p\dot{N}$
	Public consumption pG
Income Account of Firms:	
Dividends $r_k^e p_y K$	Gross accounting profits Π
Savings $p\mathcal{I}$	
Accumulation Account of Firms:	
Gross investment $pI + p\delta K$	Depreciation $p\delta K$
Inventory investment $p\dot{N}$	Savings $p\mathcal{I}$
	Financial deficit FD
Financial Account of Firms:	
Financial deficit FD	Equity financing $p_e\dot{E}$

Figure 1: *The four Activity Accounts of the Firms*

2.3 Fiscal and monetary authorities

The role of the government in this paper is to provide the economy with public (non-productive) services within the limits of its budget constraint. Public purchases (and interest payments) are financed through taxes, through newly printed money, or newly issued fixed-price bonds ($p_b = 1$). The budget constraint gives rise to some repercussion effects between the public and the private sector.[9]

$$T = \tau_w \omega L^d + T_c, \tag{34}$$

$$T_c - iB/p = t_c K, \qquad t_c = \text{const.} \tag{35}$$

$$G = gK, \qquad g = \text{const.} \tag{36}$$

$$S_g = T - iB/p - G, \tag{37}$$

$$\hat{M} = \mu, \tag{38}$$

$$\dot{B} = pG + iB - pT - \dot{M}. \tag{39}$$

[9]See for example Sargent (1987, p.18) for the introduction of net of interest taxation rules.

We model the tax income consisting of taxes on wage income and lump sum taxes on capital income T_c. With regard to the real purchases of the government for the provision of government services we assume, again as in Sargent (1987), that these are a fixed proportion g of real capital, which taken together allows us to represent fiscal policy by means of simple parameters in the intensive form representation of the model and in the steady state considerations to be discussed later on. The real savings of the government, which is a deficit if it has a negative sign, is defined in equation (37) by real taxes minus real interest payments minus real public services. For reasons of simplicity the growth rate of money is given by a constant μ. Equation (38) is the monetary policy rule of the central bank and shows that money is assumed to enter the economy via open market operations of the central bank, which buys short-term bonds from the asset holders when issuing new money. Then the changes in the short-term bonds supplied by the government are given residually in equation (39), which is the budget constraint of the governmental sector. This representation of the behavior of the monetary and the fiscal authority clearly shows that the treatment of policy questions is not a central part of the paper.[10]

2.4 Wage-price interactions

We now turn to a module of our model that can be the source of significant centrifugal forces within the complete model. These are the three laws of motion of the wage-price spiral. Picking up the approach of Rose (1967)[11] of two short-run Phillips curves, i) the wage Phillips curve and ii) the price Phillips curve, the relevant dynamic equations can be written as

$$\hat{w} = \beta_w(e - \bar{e}) + \kappa_w \hat{p} + (1 - \kappa_w)\pi^c, \tag{40}$$

$$\hat{p} = \beta_p(u - \bar{u}) + \kappa_p \hat{w} + (1 - \kappa_p)\pi^c, \tag{41}$$

$$\dot{\pi^c} = \beta_{\pi^c}(\alpha\hat{p} + (1 - \alpha)(\mu - n) - \pi^c). \tag{42}$$

where $\beta_w, \beta_p, \beta_{\pi^c} \geq 0$, $0 \leq \alpha \leq 1$, and $0 \leq \kappa_w, \kappa_p \leq 1$. This approach makes use of the assumption that relative changes in money wages are influenced by demand pressure in the market for labor and price inflation (cost-pressure) terms. Price inflation in turn depends on demand pressure in the market for goods and on money wage (cost-pressure) terms. Wage inflation therefore is described in equation (40) on the one hand by means of a demand pull term $\beta_w(e - \bar{e})$, which states that relative changes in wages depends positively on the gap between actual employment e and its NAIRU value $\bar{e}$. On the other hand, the cost push elements in wage inflation is the weighted average of short-run (perfectly anticipated) price inflation $\hat{p}$ and medium run expected overall inflation π^c, where the weights are given by κ_w and $1 - \kappa_w$. The price Phillips curve is quite similar, it also displays a demand pull and a cost push component. The demand pull term is given by the gap between capital utilization and its NAIRU value, $(u - \bar{u})$, and

[10] See Köper (2003) for an explicit treatment of government interest payments.

[11] See also Rose (1990).

the cost push element is the κ_p and $1-\kappa_p$ weighted average of short run wage inflation $\hat{w}$ and expected medium run overall inflation π^c.

What is left to model is the expected medium run inflation rate π^c. We postulate in equation (42) that changes in expected medium run inflation are due to an adjustment process towards a weighted average of the current inflation rate and steady state inflation. Thus we introduce here a simple kind of forward looking expectations into the economy. This adjustment is driven by an adjustment velocity β_{π^c}.

The economy described here is detailed on the real, nominal and financial side. Yet, with respect to the government sector it is still rudimentary. This can be justified at the present stage of analysis by observing that many of the typical macrodynamic models have similar features.[12]

2.5 Capital markets: Gross substitutes and stability

We have not yet discussed the determination of the nominal rate of interest i and the price of equities p_e and thus have not yet formulated how capital markets are organized. Following Tobin's (1969) portfolio approach, and also Franke and Semmler (1999), we here simply postulate that the following equilibrium conditions

$$\begin{aligned} M &= M^d = f_m(i, r_e^e) W_c^n, \quad W_c^n = M + B + p_e E, && (43)\\ B &= B^d = f_b(i, r_e^e) W_c^n, && (44)\\ p_e E &= p_e E^d = f_e(i, r_e^e) W_c^n, \quad r_e^e = \frac{pY^e - wL^d - p\delta K}{p_e E} + \pi_e^e, && (45) \end{aligned}$$

always hold and thus determine the above two prices for bonds and equities as statically endogenous variables of the model. Note here that all asset supplies are given magnitudes at each moment in time and recall from (14) that r_e^e is given by $\frac{r_k^e pK}{p_e E} + \pi_e$ and thus varies at each point in time solely due to variations in the share price p_e. Our model thus supports the view that the secondary market is the market where the prices or interest rates for the financial assets are determined such that these markets are cleared at all moments in time. This implies that newly issued assets do not impact significantly on these prices.

The trade between the asset holders induces a process that makes asset prices fall or rise in order to equilibrate demands and supplies. In the short run (in continuous time) the structure of wealth of asset holders, W_c^n is, disregarding changes in the share price p_e, given to them and for the model. This implies that the functions $f_m()$, $f_b()$, and $f_e()$, introduced in equations (10) to (12) must satisfy the well known conditions

$$\begin{aligned} f_m(i, r_e^e) + f_b(i, r_e^e) + f_e(i, r_e^e) &= 1, && (46)\\ \frac{\partial f_m(i, r_e^e)}{\partial z} + \frac{\partial f_b(i, r_e^e)}{\partial z} + \frac{\partial f_e(i, r_e^e)}{\partial z} &= 0, \quad \forall\, z \in \{i, r_e^e\}. && (47) \end{aligned}$$

These conditions guarantee that the number of independent equations is equal to the number of statically endogenous variables (i, p_e) that the asset markets are assumed to determine at each moment in time.

[12] See also the basic model by Sargent (1987).

We postulate that the financial assets display the gross substitution property

$$\frac{\partial f_b(i, r_e^e)}{\partial i} > 0, \quad \frac{\partial f_m(i, r_e^e)}{\partial i} < 0, \quad \frac{\partial f_e(i, r_e^e)}{\partial i} < 0, \tag{48}$$

$$\frac{\partial f_e(i, r_e^e)}{\partial r_e^e} > 0, \quad \frac{\partial f_m(i, r_e^e)}{\partial r_e^e} < 0, \quad \frac{\partial f_b(i, r_e^e)}{\partial r_e^e} < 0, \tag{49}$$

which means that the demand for all other assets increases whenever the price of one asset rises.[13] The above discussion concentrates on stocks and their impact on asset prices, including the so-called Walras' law of stocks.[14]

2.6 Capital markets: Fundamentalists, chartists and asset price dynamics

Next we consider again, as final closure of our portfolio approach to the business cycle suggested here, the potentially stabilizing and destabilizing capital gains expectations of fundamentalists and chartists. The addition of such expectations may be treated in two steps, first the fairly tranquil fundamentalists' expectations and then the chartists' expectations coming from the behavioral traders that tend to be destabilizing if they adjust with sufficient strength. This last feature of the model, the by and large formation of capital gains expectations, is the most demanding aspect (as far as stability analysis is concerned) of the dynamical system that we are considering and is mainly left to future research as far as exact stability proofs are concerned.[15]

The laws of motion governing the expectations about the equity prices are not changed by the transformation to intensive form and thus continue to read as

$$\dot{\pi}_{ef} = \beta_{\pi_{ef}}(\eta - \pi_{ef}), \tag{50}$$

$$\dot{\pi}_{ec} = \beta_{\pi_{ec}}(\hat{p}_e - \pi_{ec}). \tag{51}$$

In the following only the value of aggregate capital gains expectations is needed, but its computation requires the historical values of the actual appreciation of equity prices $\hat{p}_e$. However we lack a law of motion for this latter quality, because the general equilibrium portfolio approach only provides us with $\hat{p}_e$ by taking the time derivative of the equilibrium conditions. This leads to very complicated expressions for equity price appreciation that are here only considered implicitly.

Before we come to a consideration of the intensive form of the model, its steady state and its stability properties, as well as among other things the potentially destabilizing role of chartist-type capital gains expectations, we discuss the full structure of our model by means of what is shown in figure 2. This figure highlights the destabilizing role of

[13] For a formal definition see for example Mas-Colell, Whinston, and Green (1995, p. 611).

[14] Köper (2003) shows in addition that the Walras' law of flows also holds, representing an important consistency check of the model.

[15] Brunnermeyer (2009) shows that instabilities, bubbles and crashes are overwhelmingly due to the fact that there are heterogeneous agents in the asset market, giving rise to heterogeneous information, heterogeneous beliefs and limits to arbitrage, see also Abreu and Brunnermeyer (2003).

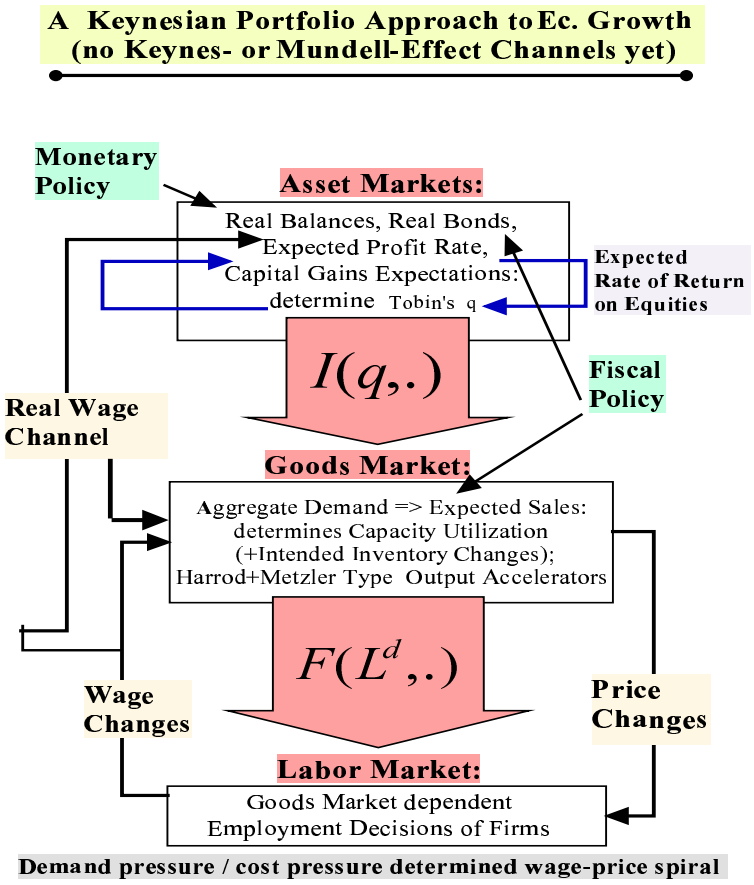

Figure 2: *Keynes' causal downward nexus (from self-contained financial markets dynamics to economic activity), repercussive feedback chains (from economic activity to expected returns on equities), supply side dynamics (the wage-price spiral) and policy rules in a Keynesian model with portfolio dynamics*

the wage-price spiral, where now – due to the assumed investment behavior – we always have a positive impact of real wages on aggregate demand and thus the result that wage flexibility will be destabilizing (if not counteracted by its effects on expected profits and their effect on financial markets and Tobin's q). We have already indicated that financial markets adjust towards their equilibrium in a stable manner as long as we disregard the expectations dynamics on the financial market. Monetary policy, whether money supply oriented and thus of type $i(M, p)$ or of a Taylor type $M(i, \hat{p})$, should – via the gross substitution effects – also contribute to the stability of financial markets. Fiscal policy impacts on the goods and the financial markets and may be of an orthodox type or of a Keynesian countercyclical kind. Due to the very intertwined, dynamical structure that we are now facing, it is however not clear how fiscal policy in detail might contribute to the shaping of the business cycle, a topic that here will be left to future

research. There remains the discussion of the self-reference within the asset markets (that is the closed loop structure between capital gains expectations and actual capital gains) which must also be the most difficult part of the considered dynamical system, the details of which must also be left to future research.

3 The model in intensive form

Next we derive the intensive form of the model. We will express all stock and flow variables in per unit of capital terms in the laws of motion and also in the associated algebraic equations (that need to be inserted into the laws of motion in order to obtain an autonomous dynamical system). We thus divide nominal stock and flow variables by the nominal value of the capital stock pK and all real ones by K, the real capital stock. This allows the determination of a (unique) economic steady state solution as an interior point of rest of the resulting nine state variables.

We begin with the intensive form of some necessary definitions or identities, which we need to represent the dynamical system in a sufficiently comprehensible form. Note here that the function q used in this block of equations will be determined and discussed later on, in section 4, where the comparative statics of the portfolio part of the model is investigated. Thus we set

$$
\begin{aligned}
Y/K &= y &&= (1+\alpha_{n^d}(n+\beta_n))y^e - \beta_n\nu,\\
Y^e/K &= y^e,\\
N/K &= \nu,\\
L^d/K &= l^d &&= y/x,\\
L/K &= l,\\
&\quad e &&= l^d/l,\\
&\quad u &&= y/y^p,\\
&\quad r_k^e &&= y^e - \delta - \omega l^d,\\
C/K &= c &&= (1-\tau_w)\omega l^d + (1-s_c)(y^e - \delta - \omega l^d - t_c),\\
I/K &= i(\cdot) &&= i_q(q-1) + i_u(u-\bar{u}) + n,\\
Y^d/K &= y^d &&= c + i(\cdot) + \delta + g,\\
p_eE/(pK) &= q &&= q(m, b, r_k^e, \pi_e),\\
&\quad r_e^e &&= r_k^e/q + \pi_e,\\
&\quad \pi_e &&= \alpha_{\pi_e}\pi_{ec} + (1-\alpha_{\pi_e})\pi_{ef}.
\end{aligned}
$$

The above equations describe output and employment per unit of capital, the rate of utilization of the existing stock of labor and capital, the expected rate of return on capital, consumption, investment and aggregate demand per unit of capital, Tobin's average q, and the expected rate of return on equities (including expected capital gains π_e).

Now we translate the laws of motion of the dynamically endogenous variables into capital intensive form. The law of motions for the nominal wages and price level stated in equations (40) and (41) interact instantaneously and thus depend on each other.

Solving these two linear equations for $\hat{w}$ and $\hat{p}$ gives[16]

$$\hat{w} = \kappa\left(\beta_w(e-\bar{e}) + \kappa_w\beta_p(u-\bar{u})\right) + \pi^c, \tag{52}$$
$$\hat{p} = \kappa\left(\beta_p(u-\bar{u}) + \kappa_p\beta_w(e-\bar{e})\right) + \pi^c, \tag{53}$$

with $\kappa = (1-\kappa_w\kappa_p)^{-1}$. From these two inflation rates one can compute the growth law of real wages $\omega = w/p$ by means of the definitional relationship $\hat{\omega} = \hat{w} - \hat{p}$, from which

$$\hat{\omega} = \kappa[(1-\kappa_p)\beta_w(e-\bar{e}) + (\kappa_w - 1)\beta_p(u-\bar{u})]. \tag{54}$$

Next we obtain the set of equations that explains the dynamical laws of the expected rate of inflation, the labor capital ratio, the expected sales, and the stock of inventories in intensive form, which are

$$\dot{\pi^c} = \alpha\beta_{\pi^c}\kappa[\beta_p(u-\bar{u}) + \kappa_p\beta_w(e-\bar{e})] + (1-\alpha)\beta_{\pi^c}(\mu - n - \pi^c), \tag{55}$$
$$\hat{l} = n - i(\cdot) = -i_q(q-1) - i_u(u-\bar{u}), \tag{56}$$
$$\dot{y}^e = \beta_{y^e}(y^d - y^e) + (n - i(\cdot))y^e, \tag{57}$$
$$\dot{\nu} = y - y^d - i(\cdot)\nu. \tag{58}$$

Equation (55) is almost the same as in the extensive form model, but here the term $\hat{p}-\pi^c$ is substituted by use of equation (53). Equation (56), the law of motion of relative factor endowment, follows from (4) and (25) and is given by the (negative) of the investment function as far as its dependence on asset markets and the state of the business cycle are concerned. Equation (57) is obtained by taking the time derivative of y^e, so that

$$\dot{y}^e = \tfrac{d(Y^e/K)}{dt} = \tfrac{\dot{Y}^eK - Y^e\dot{K}}{K^2} = \tfrac{\dot{Y}^e}{K} - y^e i(\cdot) = \beta_{y^e}(y^d - y^e) + y^e(n - i(\cdot)).$$

In essentially the same way one obtains equation (58).

The laws of motion governing the expectations about the equity prices are not changed in the intensive form model and thus again read

$$\dot{\pi}_{ef} = \beta_{\pi_{ef}}(\eta - \pi_{ef}), \tag{59}$$
$$\dot{\pi}_{ec} = \beta_{\pi_{ec}}(\hat{p}_e - \pi_{ec}). \tag{60}$$

The aggregate expectation of equity price inflation continues to be given by (17).

Finally, the laws of motion for real balances and real bonds per unit of capital have to be derived. Based on the knowledge of the laws for inflation $\hat{p}$ and investment $i(\cdot)$ we can derive the differential equation for bonds per unit of capital shown in equation (61) from

$$\dot{b} = \frac{d(B/pK)}{dt} = \frac{\dot{B}}{pK} - b(\hat{p} + i(\cdot))$$

[16]For details of the calculations involved see Chiarella and Flaschel (2000) and Köper (2003).

where $\dot{B}$ is given by equation (39). The same idea is used for the changes in the money supply. We thus finally obtain the two differential equations

$$\begin{aligned} \dot{b} &= g - t_c - \tau_w \omega l^d - \mu m \\ &\quad - b\left(\kappa[\beta_p(u-\bar{u}) + \kappa_p \beta_w (e-\bar{e})] + \pi^c + i(\cdot)\right), \qquad (61) \\ \dot{m} &= m\mu - m(\kappa[\beta_p(u-\bar{u}) + \kappa_p \beta_w (e-\bar{e})] + \pi^c + i(\cdot)). \qquad (62) \end{aligned}$$

According to the above, the dynamics in extensive form can therefore be reduced to nine differential equations, where however the law of motion for share prices has not yet been determined, or to seven differential and one integral equation which is easier to handle than the alternative representation, since there is then no law of motion for the development of future share prices to be calculated. Note with respect to these dynamics that economic policy (fiscal and monetary) is still represented in very simple terms here, since money supply is growing at a given rate and since government expenditures and taxes on capital income net of interest payments per unit of capital are given parameters. This makes the dynamics of the government budget constraint (see (61) the law of motion for bonds per unit of capital b) a very trivial one as in Sargent (1987, ch. 5), and thus leaves the problems associated with these dynamics a matter for future research. The advantage is that fiscal policy can be discussed in a very simple way here by means of just three parameters.

A comparison of the present dynamics with those of the previous models of the authors[17] reveals that there are now two variables from the financial sector that feed back to the real dynamics in this extended system, the bond to capital ratio b representing the evolution of government debt and Tobin's average q. The first (dynamic) variable however only influences the real dynamics since it is one of the factors that influences the statically endogenous variable q which in turn enters the investment function as a measure of the firms' performance.

Government bonds do not influence the economy in other ways, since there are not yet wealth effects in consumption and since the interest income channel to consumption has been suppressed by the particular assumption about tax collection concerning capital income. In addition, the interest rate channel of the earlier approaches of the authors, where the real rate of interest as compared to the real profit rate entering the investment function, is now absent from this function. The nominal interest rate as determined by portfolio equilibrium thus does not matter in the present formulation of the model, where Tobin's q in the place of this interest rate now provides the channel by which investment behavior is reacting to the results brought about by the financial markets.

A feature of the present dynamics is that there are no laws of motion left implicit. The model contains now a completely formulated dynamics, but still one where the real financial interaction is represented in very basic terms. Price inflation (via real balances and real bonds) and the expected rate of return on capital (via the dividend rate of return) influence the behavior of asset markets via their laws of motion such as gross substitution of assets and expectation dynamics for asset prices, while the reaction of

[17] See Chiarella and Flaschel (2000) and Chiarella, Flaschel, Groh, and Semmler (2000).

asset markets feeds back into the real part of the economy instantaneously through the change in Tobin's q that they (and the dynamics of expected capital gains) bring about.

4 The comparative statics of the asset markets

After having specified both the extensive and intensive forms of the model and having shown the existence and uniqueness of an interior economic steady state solution of the intensive form we now focus on the short–run comparative statics of the financial markets module of the system. We derive in particular the function $q = q(m, b, r_k^e, \pi_e)$ of which we have already made use in the intensive form presentation of the model, and which will be needed to investigate the stability properties of the model around its steady state position in the next section.

We assume that the asset demand functions display the properties which guarantee a unique interior steady state solution; see lemma 3 in the following section. We now approximate these demand functions by linear functions in a neighborhood of the steady state in order to derive the local stability properties of the next section. These *linearized versions of the asset demand functions* can be written as (with $r_e^e = r_k^e/q + \pi_e$):[18]

$$\begin{aligned} f_m^l(i, r_e^e) &= \alpha_{m0} - \alpha_{m1} i - \alpha_{m2}(r_k^e/q + \pi_e), \\ f_b^l(i, r_e^e) &= \alpha_{b0} + \alpha_{b1} i - \alpha_{b2} r_e^e, \\ f_e^l(i, r_e^e) &= \alpha_{e0} - \alpha_{e1} i + \alpha_{e2} r_e^e, \end{aligned}$$

where the superscript l denotes the linearized form and where

$$\alpha_{ij} \geq 0 \quad \forall \quad i \in \{b, m, e\}, j \in \{0, 1, 2\}.$$

Because of Walras' Law of Stocks it is sufficient to focus on the first two asset market equilibrium conditions in all subsequent equilibrium considerations. For money and bonds these two equilibrium conditions now read

$$m = (\alpha_{m0} - \alpha_{m1} i - \alpha_{m2}(r_k^e/q + \pi_e))(m + b + q), \tag{63}$$

$$b = (\alpha_{b0} + \alpha_{b1} i - \alpha_{b2}(r_k^e/q + \pi_e))(m + b + q). \tag{64}$$

Solving (63) and (64) for the interest rate i we obtain, respectively

$$i_{\mathrm{LM}} = \frac{\alpha_{m0} - \alpha_{m2}(r_k^e/q + \pi_e) - m/(m + b + q)}{\alpha_{m1}}, \tag{65}$$

$$i_{\mathrm{BB}} = \frac{-\alpha_{b0} + \alpha_{b2}(r_k^e/q + \pi_e) + b/(m + b + q)}{\alpha_{b1}}. \tag{66}$$

The LM–subscript denotes the interest rate that equates demand for real balances and real money supply and the BB–subscript denotes the interest rate that equates real bond

[18] We remark here that the parameters of such functions must be chosen (in particular in numerical investigations) such that a meaningful relationship between the interest rate i^o and the rate of return on equities r_e^{eo} is established in the steady state.

demand and supply. Figure 3 displays examples of these two functions as a function of q. The intersection of the LM–curve and the BB–curve then provides the equilibrium values for the short-term interest rate i and Tobin's q. The figure only shows examples of such functions and as we know that the functions are not linear in q we do not know yet whether the equilibrium exists and is unique. Note however that we are only considering a neighborhood of the steady state solution for the variables i, q, m, b, r_k^e, π_e. In order to show that i and q exist and are uniquely determined for all m, b, r_k^e, π_e sufficiently close to the steady state solution we therefore have to show that the assumptions of the implicit function theorem are valid at the steady state.

Lemma 1. *The assumptions of the lemma 3 on the unique existence of a steady state solution of the considered dynamics are assumed to hold, see the following section. There is then also a unique solution (i, q) to the equations (80) and (81), which thus clears the asset markets, for all values of m, b, r_k^e, π_e in an appropriately chosen neighborhood of the interior steady state solution of the dynamics (54) to (62).*

Proof: We have to show that the Jacobian of the system

$$\begin{aligned} f_m(i, r_k^e/q + \pi_e)(m + b + q) - m &= 0, \\ f_b(i, r_k^e/q + \pi_e)(m + b + q) - b &= 0, \end{aligned}$$

is regular with respect to the variables i and q, which means that

$$\begin{vmatrix} \frac{\partial}{\partial i}(f_m(i, r_k^e/q + \pi_e)(m + b + q) - m) & \frac{\partial}{\partial q}(f_m(i, r_k^e/q + \pi_e)(m + b + q) - m) \\ \frac{\partial}{\partial i}(f_b(i, r_k^e/q + \pi_e)(m + b + q) - b) & \frac{\partial}{\partial q}(f_b(i, r_k^e/q + \pi_e)(m + b + q) - b) \end{vmatrix} \neq 0$$

must hold true. We can readily calculate that the sign configuration of the entries in this Jacobian is

$$\begin{pmatrix} - & + \\ + & + \end{pmatrix}$$

which immediately implies the regularity of this Jacobian. ∎

We have thus shown that the financial markets can always be cleared through adjustments of the short-term interest rate and Tobin's q. But how do these two variables react in the short-run as the above given statically exogenous variables change over time? We consider this question first on the level of the partial equilibrium curves shown in figure 3. We can derive for the dependence of the two interest functions i_{LM} and i_{BB} on the variables r_k^e, π_e, q and m the following:

$$i_{\mathrm{LM}}(\underset{-}{r_k^e}, \underset{-}{\pi_e}, \underset{-}{m}, \underset{+}{b}, \underset{+}{q}) \quad \text{and} \quad i_{\mathrm{BB}}(\underset{+}{r_k^e}, \underset{+}{\pi_e}, \underset{-}{m}, \underset{+}{b}, \underset{-}{q}). \tag{67}$$

These results follow directly by taking the respective partial derivatives of the functions in equations (65) and (66).

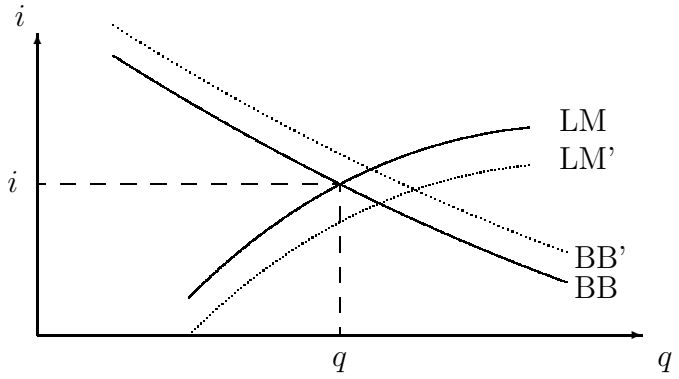

Figure 3: *The LM and BB Curves: The dashed lines show how these curves simultaneously shift when one of the statically exogenous variables r_k^e, π_e, q, m rises or b falls.*

Equations (65) and (66) together through the equilibrium condition by $i_{\rm LM} = i_{\rm BB}$ yield

$$\begin{aligned} &\frac{\alpha_{m0} - \alpha_{m2}(r_k^e/q + \pi_e) - m/(m+b+q)}{\alpha_{m1}} \\ &- \frac{-\alpha_{b0} + \alpha_{b2}(r_k^e/q + \pi_e) + b/(m+b+q)}{\alpha_{b1}} \;=\; 0. \end{aligned} \tag{68}$$

Application of the implicit function theorem then gives the following qualitative dependencies of Tobin's average q:-

$$\begin{aligned} &q(\underset{+}{r_k^e}, \underset{+}{\pi_e}, \underset{+}{m}, \underset{+}{b}) \qquad \forall \quad q > (\frac{\alpha_{b1}}{\alpha_{m1}} - 1)m, \\ &q(\underset{+}{r_k^e}, \underset{+}{\pi_e}, \underset{+}{m}, \underset{-}{b}) \qquad \forall \quad q < (\frac{\alpha_{b1}}{\alpha_{m1}} - 1)m. \end{aligned} \tag{69}$$

The first situation in (69) must apply locally around the steady state if $(\frac{\alpha_{b1}}{\alpha_{m1}} - 1)m^o < 1$ holds true while the other one holds in the opposite case.[19] We thus get the result that an increase in r_k^e, the basis for the dividend rate of return, unambiguously increases Tobin's q, as does an increase in the expected capital gains π_e. Furthermore, an increase in m also pushes q upwards and thus increases investment, just as an increase in m would do in the presence of a negative dependence of the rate of investment on the rate of interest, the Keynes effect in traditional models of the AS-AD variety. The positive influence of m on q thus mirrors the Keynes effect of traditional Keynesian short-run equilibrium analysis. The nominal rate of interest is however no longer involved in the real part of the model as it is here formulated which allows us to ignore the comparative statics of this interest rate in the current analysis.

[19]We do not pay attention here to the border case where $(\frac{\alpha_{b1}}{\alpha_{m1}} - 1)m^o = 1$ holds true. Note here also that the α_{ij} sum to one for $j = 0$ and to zero for $j = 1, 2$ which implies that $\frac{\alpha_{b1}}{\alpha_{m1}} - 1$ is always nonnegative.

Results with respect to the influence of bonds b on a change in Tobin's q are however ambiguous and depend on the steady state value of real balances m as well as on the parameters that determine the interest rate sensitivity of money and bonds demand. We can get more insights into the formation of Tobin's q by means of the following lemma:

Lemma 2. *In a neighborhood around the steady state, the partial derivative of Tobin's q with respect to cash balances exceeds the partial derivative of q with respect to bond holdings:*

$$\frac{\partial q}{\partial m} > \frac{\partial q}{\partial b}$$

Proof: We can rewrite the inequality of the lemma as[20]

$$-\frac{\det \frac{\partial(F_1,F_2)}{\partial(i,m)}}{\det \frac{\partial(F_1,F_2)}{\partial(i,q)}} > -\frac{\det \frac{\partial(F_1,F_2)}{\partial(i,b)}}{\det \frac{\partial(F_1,F_2)}{\partial(i,q)}},$$

the denominator of which we know that is negative, so we get equivalently the condition

$$\begin{aligned} & \det \tfrac{\partial(F_1,F_2)}{\partial(i,m)} > \det \tfrac{\partial(F_1,F_2)}{\partial(i,b)} \\ \Leftrightarrow \quad & -\alpha_{m1}b + \alpha_{b1}(b+q) > \alpha_{m1}(m+q) - \alpha_{b1}m \\ \Leftrightarrow \quad & \alpha_{b1}(m+b+q) > \alpha_{m1}(m+b+q) \\ \Leftrightarrow \quad & \alpha_{b1} > \alpha_{m1}. \end{aligned}$$

which is true, because this inequality is an implication of equation (83). ∎

This lemma tells us that an open market policy of the government, which means that the central bank buys bonds by means of issuing money ($dm = -db$), indeed has an expansionary effect on Tobin's q since

$$\frac{\partial q}{\partial m}dm + \frac{\partial q}{\partial b}(-dm) > 0. \tag{70}$$

5 Steady state considerations

In this section we show the existence of a steady state in the economy under consideration. We here stress that this can be done independently of the analysis given in the preceding section on the comparative statics of the asset market equilibrium system, since Tobin's q is given by 1 in the steady state via the real part of the model and since the portfolio equations can be uniquely solved in conjunction with the government budget constraint for the three variables i, m, b which they then determine. Note that m and b are data in the short-run analysis of the behavior of asset markets of the preceding section (where q and i are determined through them as the variables that bring the asset

[20]Note we use the notation $\det \frac{\partial(F_1,F_2)}{\partial(i,x)}$ to denote the determinant with elements $\frac{\partial F_1}{\partial i}$, $\frac{\partial F_1}{\partial x}$, $\frac{\partial F_2}{\partial i}$, $\frac{\partial F_2}{\partial x}$ for $x \in (m, b, q)$.

markets into equilibrium), while m and b are variables in the long run that are derived from asset market equilibrium conditions and the government budget constraint.

As the model is formulated we have the following nine state variables

$$m, b, y^e, \omega, l, \nu, \pi^c, \pi_{ef}, \pi_{ec}$$

in the considered dynamical system, We have written these state variables in the order they will used in the stability analysis in a following section. This order is generally not the same as in the steady state analysis of the model where 'causalities of a different type (than in stability analysis) is involved.

Lemma 3. *Assume $s_c > \tau_w$ and $s_c r^{e0} > n + g - t_c$. Assume furthermore that the parameter $\bar{\phi}$ used below has a positive numerator, so that the government runs a primary deficit in the steady state, (and thus between zero and one if the money supply is growing). The dynamical system given by equations (54) to (62) possesses a unique interior steady state solution $(\omega^o, l^o, m^o > 0)$ with equilibrium on the asset markets, if the fundamentalists long run reference of the increase in equity prices equals the steady state inflation rate of goods prices*

$$\eta = \hat{p}^o,$$

and

$$\begin{aligned} & \lim_{i \to 0} (f_m(i, r^{e0} + \pi_e^o) + f_b(i, r^{e0} + \pi_e^o)) \;\; < \;\; \bar{\phi}, \\ \text{and} \quad & \lim_{i \to \infty} (f_m(i, r^{e0} + \pi_e^o) + f_b(i, r^{e0} + \pi_e^o)) \;\; > \;\; \bar{\phi}, \end{aligned}$$

holds true with $\bar{\phi} = \frac{g - t_c - \tau_w \omega l^{do}}{g - t_c - \tau_w \omega l^{do} + \mu}$.[21]

Proof: If the economy rests in a steady state, then all intensive variables stay constant and all time derivatives of the system become zero. Thus by setting the left hand side of the system of equations (54) to (62) equal to zero, we can deduce the steady state values of the variables.

From equation (56) we can derive that $i(\cdot)^o = n$ holds, from (57) we get $y^{eo} = y^{do}$, and from (62) that $\mu = (\kappa[\beta_p(u - \bar{u}) + \kappa_p \beta_w (e - \bar{e})] + \pi^c + i(\cdot))$. Substituting the last relation into equation (42) and using $i(\cdot)^o = n$ we obtain with $\alpha\beta_\pi \neq -(1 - \alpha)\beta_\pi^c$ that $\mu - n - \pi^c = 0$ and $\kappa[\beta_p(u - \bar{u}) + \kappa_p \beta_w (e - \bar{e})] = 0$. Thus we have for $u - \bar{u}$ and $e - \bar{e}$ the two equations

$$\begin{aligned} u - \bar{u} &= -\kappa_p \beta_w (e - \bar{e}) / \beta_p, \\ u - \bar{u} &= (1 - \kappa_p)\beta_w (e - \bar{e}) / [(1 - \kappa_w)\beta_p]. \end{aligned}$$

[21] Note with respect to this part of the lemma that the steady state values used in the above assumption are calculated before this assumption is applied to a determination of the steady state value of the nominal rate of interest.

By assumption we have $\beta_p, \beta_w > 0$ and $0 \leq \kappa_p, \kappa_w \leq 1$, so $e - \bar{e}$ must equal zero in order that the last two equations be fulfilled. When $e = \bar{e}$, then according to (54) we know that $u = \bar{u}$. Then equation (56) leads to $q^o = 1$.

With these relations one can easily compute the unique steady state values of the variables y^e, l, π^c, ν, ω as

$$
\begin{aligned}
y^{eo} &= \frac{y^o}{1 + n\alpha_{n^d}}, \quad \text{with } y^o = \bar{u} y^p, && (71)\\
l^o &= y^o/(\bar{e}x), && (72)\\
\pi^{co} &= \mu - n, && (73)\\
\nu^o &= \alpha_{n^d} y^{eo}, && (74)\\
\omega^o &= \frac{y^{eo} - n - \delta - g - (1 - s_c)(y^{eo} - \delta - t_c)}{(s_c - \tau_w) l^{do}}, && (75)\\
r^{e0} &= y^{eo} - \delta - \omega^o l^{do}. && (76)
\end{aligned}
$$

All these values are determined on the goods and labor markets. The steady state value of the real wage has in particular been derived from the goods market equilibrium condition that must hold in the steady state and it is positive under the assumptions made in lemma 3.

We next take account of the asset markets, which determine the values of the short-term interest rate i (which is now bears the burden of clearing the asset markets), but now in conjunction with the determination of the steady state for m and b, where $m + b$ is determined through the government budget constraint. This is the case because the steady state rate of return on equities relies, on the one hand, solely on r^{e0} (since q has been determined through the condition $i(\cdot) = n$ and shown to equal one in steady state) and, on the other hand, on the expected inflation rate of share prices

$$
r_e^{e0} = r^{e0} + \pi_e^o,
$$

which equals the goods price inflation rate in the steady state as will be shown below.

The steady state values of the two kinds of expectations about the inflation rate of equity prices (of chartists and fundamentalists) are

$$
\pi_{ef}^o = \eta, \qquad \pi_{ec}^o = \eta \tag{77}
$$

from which one can derive that $\pi_e^o = \eta = \hat{p}^o = \pi^{co} = \mu - n$ must hold. We have seen that, in the steady state, Tobin's q equals one and its time derivative equals zero, so that we can derive

$$
\begin{aligned}
\dot{q} &= 0\\
\Rightarrow \quad \frac{(\dot{p}_e E + p_e \dot{E}) pK - p_e E(\dot{p}K + p\dot{K})}{p^2 K^2} &= 0\\
\Rightarrow \quad \frac{\dot{p}_e E + p_e \dot{E}}{pK} &= \hat{p} + n.
\end{aligned}
$$

According to equation (26) we have $p_e\dot{E} = pI + p(\dot{N} - \mathcal{I})$ we thus get in the steady state that $p_e\dot{E} = pI$. Inserting this into the last implication shown we get $\hat{p}_e = \hat{p}$ and thus as an important finding that $\eta = \mu - n$ must hold in order to allow for a steady state.

Now we determine the steady state values of the stocks of real cash balances and the stock of bonds. These values have to be determined in conjunction with the steady state interest rate i^o which is now solely responsible for clearing the asset markets, because the result that Tobin's $q = 1$ has already been determined on the real markets.

The budget constraint of the government is given in intensive form by

$$\dot{b} + \dot{m} \;=\; g - t_c - \tau_w \omega l^d - (b + m)(\hat{p} + i(\cdot)). \tag{78}$$

One therefore obtains in the steady state that

$$b^o + m^o \;=\; (g - t_c - \tau_w \omega l^d)/\mu. \tag{79}$$

Furthermore, consider the asset demand functions (10) and (11), namely

$$m \;=\; f_m(i, r_e^e)(m + b + q), \quad q = 1, \tag{80}$$

$$b \;=\; f_b(i, r_e^e)(m + b + q), \quad q = 1. \tag{81}$$

The left side of the last two equations are the supplied amounts and the right sides represent the demand for the assets m, b.

Using now equation (79) in the form

$$\mu(m^o + b^o) \;=\; g - t_c - \tau_w \omega l^d, \tag{82}$$

the system of three linear independent equations (80) to (82) can be used to deduce the three unique steady state values i^o, b^o, and m^o which we will show below.

Beginning with the steady state interest rate we sum equations (80) and (81) and multiplying by μ obtain

$$\mu(m^o + b^o) \;=\; (f_m^o + f_b^o)\mu(m^o + b^o + 1),$$

where f_m^o and f_b^o denote the values of $f_m(i^o, r^{e0} + \pi_e^o)$ and $f_b(i^o, r^{e0} + \pi_e^o)$ respectively. Substituting in the budget constraint in the form of equation (82) we get

$$f_m^o + f_b^o \;=\; \bar{\phi},$$

with $\bar{\phi} = \dfrac{g - t_c - \tau_w \omega^o l^{do}}{g - t_c - \tau_w \omega^o l^{do} + \mu}$. From property (47) and (49) we can conclude that

$$\frac{\partial(f_m + f_b)}{\partial i} > 0, \tag{83}$$

which implies that the cumulated demand for money and bonds is a strictly increasing function in the variable i.

If $lim_{i\to 0}(f_m(i, r^{eo}+\pi_e^o)+f_b(i, r^{eo}+\pi_e^o)) < \bar{\phi}$ and $lim_{i\to\infty}(f_m(i, r^{eo}+\pi_e^o)+f_b(i, r^{eo}+\pi_e^o)) > \bar{\phi}$ then by monotonicity and continuity there must be a value of i, which equilibrates the asset markets in the above aggregated form. Then, steady state supplies of m and b can be calculated by equations (80) and (81) in a unique way, based on the steady state interest rates $i = i^o$ and $r_e^{e0} = r^{e0} + \pi_e$. This concludes the derivation of the uniquely determined steady state values for our dynamical system (54) to (62) which in turn when inserted into this system indeed imply that the dynamics is at a point of rest in this situation. ■

Note that inflation rates are uniform throughout in this model type (also for stock prices) and that government debt B is growing with the same rate as money supply μ in the steady state, while the real sector is growing with the natural rate n (which is also the growth rate of equity supply). We observe finally that the calculation of the steady state value of the rate of wage and the rate of return on capital can be simplified when it is assumed that government expenditures are given by $g + \tau_w \omega l^d$ in place of only g.

6 Potential Sources of Instability

Next we want to study the potential source of instability. We hereby will use eigenvalue analysis as well as simulation.

Lemma 4. *The steady state of the dynamic system (88) loses its stability by way of a Hopf bifurcation, i.e., in a cyclical fashion. Such Hopf bifurcations in particular occur when the parameters we assume in the next section as being sufficiently small are made sufficiently large.*

Proof: The proof basically rests on the fact that the determinant of the Jacobian of steady state of the dynamic system (88) is always negative, so that eigenvalues have to cross the imaginary axis (excluding zero) when stability gets lost. With respect to the actual loss of stability one has to study however the minors of order 1, 2 and more of the Jacobian of the dynamics at the steady state or use numerical methods (such as eigenvalue diagrams, see below) in order to get the result that significant flexibilities in the wage-price spiral or in the financial markets (including high money demand elasticities) will indeed lead to loss of stability by way of persistent or explosive business fluctuations. ■

As numerical simulations have shown, the range where such local Hopf-bifurcation matter is a very limited one. This implies the need for global changes (regime switches) in behavior if the economy is locally explosive and departs too much from its steady state. There is indeed at least one important example for such a behavioral switch that in many situations (as far as the real markets are concerned) is sufficient to restrict the trajectories of the dynamics to an economically meaningful domain of their whole phase space. This nonlinearity concerns the fact, already observed by Keynes (1936) that money wages may be flexible in an upward direction, but are rigid (or at least considerably less flexible) in the downward direction.

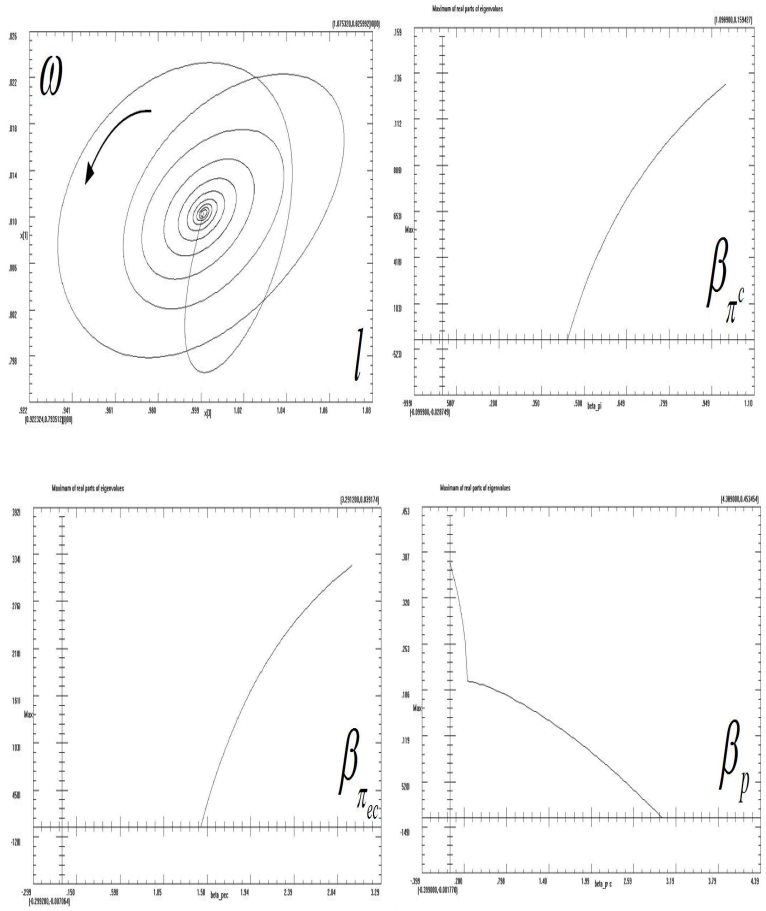

Figure 4: Damped oscillations (top left) and the loss of local stability via Hopf-bifurcations with respect to β_{π^c}, $\beta_{\pi_{ec}}$ and β_p

Let us assert without proof that the normal or adverse Rose effect of changing real wages leading to changing aggregate demand and thereby to further changes in money wages, the price level and the real wage. This holds for the baseline model, with no explicit financial market[22], but will also be present in the currently considered model with portfolio choice and heterogeneous agents on the asset market. Either wage or price flexibility will, through their effects on the expected rate of return on capital, and from there on asset markets, be destabilizing and lead to Hopf- bifurcations, limit cycles or (locally) purely explosive behavior eventually. The Mundell or real rate of interest effect is not so obviously present in the considered dynamics as there is no long real rate of interest involved in investment (or consumption) behavior. Increasing expected price inflation does not directly increase aggregate demand, economic activity and thus the actual rate of price inflation. This surely implies that the model needs to be extended in order to take account of the role that is generally played by the real rate

[22]See Chiarella and Flaschel (2000) and Chiarella, Flaschel, Groh, and Semmler (2000).

of interest in macrodynamic models. There are finally two accelerator effects involved in the dynamics, the Metzlerian inventory accelerator mechanism and the Harrodian fixed business investment accelerator. We therefore expect that increasing the parameters β_n and i_u will also be destabilizing and also lead to Hopf bifurcations and other complex dynamic behavior.

We finally provide two numerical examples, concerning damped oscillations, loss of stability via Hopf-bifurcation, the generation of limit cycles as business fluctuations from the global perspective by the addition of downward money wage rigidity to the money wage Phillips curve and finally – through this kinked wage Phillips curve – the generation of complex dynamics if increases in certain adjustment speeds make the steady state strongly repelling. We refer the reader to Chiarella and Flaschel (2000), Asada, Flaschel, Mouakil, and Proano (2009) for more detailed numerical studies of the implications of kinked money wage Phillips curves.

The simulations in the top-left of figure 4 show damped oscillations when the parameter choices of our stability propositions are applied. The other three figures show eigenvalue diagrams that plot the maximum real part of eigenvalues against crucial parameters of the dynamical system under consideration namely β_{π^e}, $\beta_{\pi_{ec}}$ and β_p. These show the expected results that increasing speeds of adjustments in the movements of the inflationary climate and the capital gain expectations of chartists will be destabilizing, while price flexibility is stabilizing (and correspondingly: wage flexibility is destabilizing).

In figure 5 we show an example of a period (cycle) doubling route to complex dynamics (but not chaos) from the economic point of view, since the cycles that are generated are fairly similar to each other. We increase the speed of adjustment of money wages from $\beta_w = 1.4$ to $\beta_w = 2.0$ and from there to $\beta_w = 2.82$ and then to $\beta_w = 3.0$. The first thing to note is that the dynamics remain viable over such a broad range of adjustment speeds for money wages, due to the kink in the money wage Phillips curve and despite a strong local instability around the steady state described above. To the right of the shown attractors the trajectories are of a fairly smooth type, yet top left they are going through some turbulence which makes the attractor more and more complex with the increasing adjustment speed of money wages.

We do not go into the details of such simulations any further here, but only present them as evidence that the considered model type is capable of producing various dynamic outcomes and is thus a very open one with respect to possible business cycle implications. We might also need some empirical estimation of parameter values in order to get more specific results from our instability analysis. Yet overall we could demonstrate that the high dimension dynamics may have many sources of instability.

7 Dampened business cycles: Labor market and fiscal policies

Next we want to raise the question of what might stabilize our macroeconomic dynamics. Let us first suppose that all assumptions stated in lemma 3 hold. What is left to analyze then is the dynamical behavior of the system, when it is displaced from its steady state

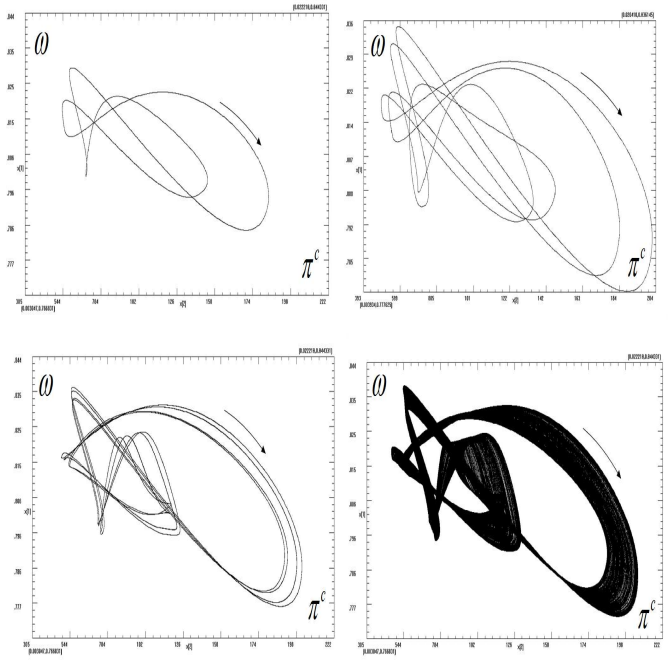

Figure 5: *A period doubling route to complex dynamics through an accelerating wage-price spiral, augmented by downward money wage rigidity (with financial market accelerators still tranquil)*

position, but still remains in a neighborhood of the steady state. In the following we provide propositions, which in sum imply that there must be a locally stable steady state, if some sufficient conditions that are very plausible from a Keynesian perspective are met.

We begin with an appropriate subsystem of the full dynamics for which the Routh–Hurwitz conditions can be shown to hold. Setting $\beta_p = \beta_w = \beta_{\pi_{ef}} = \beta_{\pi_{ec}} = \beta_n = \beta_{\pi^c} = 0$, $\beta_{y^e} > 0$, and keeping $\pi^c, \pi_e, \omega, \nu$ thereby at their steady state values we get the following subdynamics of state variables m, b and y^e which are then independent of the rest of the system:[23]

$$\begin{aligned} \dot{m} &= m(\mu - (\pi_o^c + i(\cdot))), \\ \dot{b} &= g - t_c - \tau_w \omega \frac{y}{x} - \mu m - b(\pi_o^c + i(\cdot)), \\ \dot{y}^e &= \beta_{y^e}\left[c + i(\cdot) + \delta + g - y^e\right] + y^e(i(\cdot) - n). \end{aligned} \tag{84}$$

Proposition 1. *The steady state of the system of differential equations (93) is locally asymptotically stable if β_{y^e} is sufficiently large, the investment adjustment speed i_u concerning deviations of capital utilization from the normal capital utilization is sufficiently small and the partial derivatives of desired cash balances with respect to the interest rate*

[23]Note that l may vary, but does not feed back into the presently considered subdynamics.

$\partial f_m/\partial i$ and the rate of return on equities $\partial f_m/\partial r_e^e$ are sufficiently small. Moreover the equity market must be in a sufficiently tranquil state, i.e., the partial derivative $\partial f_e/\partial r_e^e$ must also be sufficiently small.

Proof: See Köper (2003), also with respect to all other following propositions of this section. ■

The proposition asserts that local asymptotic stability at the steady state of the considered subdynamics holds when, the demand for cash is not very much influenced by the rates of return on the financial asset markets,[24] the accelerating effect of capacity utilization on the investment behavior is sufficiently small, and the adjustment speed of expected sales towards actual demand is fast enough. Moreover, and this is an important condition, the stock markets must be sufficiently tranquil in the reaction to changes in the rate of return on equities, i.e., they are in particular not close to a liquidity trap.

In order to show how policy can enforce the validity of this situation we need some preliminary observations first. In the given structure of financial markets it is natural to assume that even $\partial f_m/\partial r_e^e = 0$ and $\partial f_e/\partial i = 0$ holds true, since fixprice bonds are equivalent to saving deposits and thus form together with money M just what is named M_3 in the literature. The internal structure of M_3 is however just a matter of proper cash management and should therefore imply that the rate of return r_e^e on equities does not matter for it. The latter only concerns the demand for equities versus the demand for the aggregate M_3 which both solely then depend on the rate of return for equities, since the dependence on the rate of interest cancel when M_3 is formed.

Moreover, since the transaction costs for reallocations within M_3 can be assumed as being fairly small and the speed of adjustment of the dynamic multiplier (which is infinite if IS-equilibrium is assumed) may be assumed to be large, we have only one critical parameter left in the above proposition which may be crucial for the stability of the considered subsystem of the dynamics, the investment parameter i_u, potentially representing and accelerator of Harrodian type. This suggests that fiscal policy should be used to counteract the working of this accelerator mechanism which leads from higher capacity utilization to higher investment to higher goods demand and thus again to higher capacity utilization.

The following proposition formulates how fiscal policy should be designed in order to create damped oscillations around the balanced growth path of the model (if they are yet present).

Theorem 1. *Assume an independent fiscal authority solely responsible for the control of business fluctuations (acting independently from the business cycle neutral fiscal policy of the government) which implements the following two rules for its activity oriented expenditures and their funding:*

$$g^u = -g_u(u - \bar{u}), \quad t^u = g_u(u - \bar{u})$$

[24] This would correspond to a strong Keynes effect in the corresponding working model of Chiarella and Flaschel (2000, ch. 6).

The budget of this authority is always balanced and we assume that the tributes t^u are paid by asset holding households. The stability condition on i_u is now extended to the consideration of the parameter $i_u - g_u$. Then: An anti-cyclical policy g^u that is chosen in a sufficiently active way will enforce damped oscillations in the considered subdynamics if the savings rate s_c of asset holders is sufficiently close to one (and if stock markets are sufficiently tranquil).

Therefore: An anti-cyclical policy that is chosen in a sufficiently active way will enforce damped oscillations in the considered subdynamics (1) if the savings rate of asset holders is sufficiently close to one and if stock markets are sufficiently tranquil. Note that neither the steady state nor the laws of motions are changed through this introduction of such a self -determined business cycle authority, if $s_c = 1$ holds true, which we assume to hold true in the following for reasons of simplicity.

Next we consider the same system but allow β_p to become positive, though only small in amount. This means that ω which had previously entered the m, b, y^e–subsystem only through its steady state value now becomes a dynamic variable, giving rise to the 4D dynamical system

$$\begin{array}{rcl} \dot{m} &=& m(\mu - (\kappa\beta_p(\frac{y}{y^p} - \bar{u}) + \pi_o^c + i(\cdot))), \\ \dot{b} &=& g - t_c - \tau_w\omega\frac{y}{x} - \mu m - b(\kappa\beta_p(\frac{y}{y^p} - \bar{u}) + \pi_o^c + i(\cdot)), \\ \dot{y^e} &=& \beta_{y^e}\left[c + i(\cdot) + \delta + g - y^e\right] + y^e(i(\cdot) - n), \\ \dot{\omega} &=& \omega\kappa(\kappa_w - 1)\beta_p(\frac{y}{y^p} - \bar{u}). \end{array} \tag{85}$$

Proposition 2. *The interior steady state of the dynamical system (85) is locally asymptotically stable if the conditions in proposition 1 are met and β_p is sufficiently small.*

Note here that the implication of this new condition for the considered subdynamics is also obtained by the assumption $\kappa_w = 1$, i.e., workers and their representatives should always demand for a full indexation of their nominal wages to the rate of price inflation. This implies:

Theorem 2. *Assume that the cost-push term in the money wage adjustment rule is given by the current rate of price inflation (which is perfectly foreseen). Then: the considered 4D subdynamics implies damped oscillations around the given steady state position of the economy.*

This type of a scala mobile thus implies stability instead of - -as might be expected – instability, since it simplifies the real wage channel of the model considerably. It needs however the following theorem in addition in order to really tame the wage-price spiral of the model.

Enlarging the system (85) by letting β_w become positive we get the subsystem

$$\begin{array}{rcl} \dot{m} &=& m(\mu - (\kappa\left[\beta_p(\frac{y}{y^p} - \bar{u}) + \kappa_p\beta_w(\frac{y}{xl} - \bar{e})\right] + \pi_o^c + i(\cdot))), \\ \dot{b} &=& g - t_c - \tau_w\omega\frac{y}{x} - \mu m - b(\kappa\left[\beta_p(\frac{y}{y^p} - \bar{u}) + \kappa_p\beta_w(\frac{y}{xl} - \bar{e})\right] + \pi_o^c + i(\cdot)), \\ \dot{y^e} &=& \beta_{y^e}[c + i(\cdot) + \delta + g - y^e] + y^e(i(\cdot) - n), \\ \dot{\omega} &=& \omega\kappa[(1 - \kappa_p)\beta_w(\frac{y}{xl} - \bar{e}) + (\kappa_w - 1)\beta_p(\frac{y}{y^p} - \bar{u})], \\ \dot{l} &=& l[-i_q(q - 1) - i_u(\frac{y}{y^p} - \bar{u})]. \end{array} \tag{86}$$

Proposition 3. *The steady state of the dynamical system (86) is locally asymptotically stable if the conditions in proposition 2 are met and β_w is sufficiently small.*

Theorem 3. *We assume that the economy is a consensus based one, i.e., labor and capital reach agreement with respect to the scala mobile principle in the dynamic of money wages. Assume that they also agree on this background that additional money wage increases should be small in the boom ($u - \bar{u}$) and vice versa in the recession. This makes the steady state of the considered 5D subdynamics asymptotically stable.*

Based the described consensus between capital and labor, they therefore can both benefit from it (also with respect to a simplification of negotiations about the general level of money wages).

We now enlarge the system further by letting β_n become positive to obtain

$$\begin{array}{rcl}
\dot{m} &=& m(\mu - (\kappa \left[\beta_p(\frac{y}{y^p} - \bar{u}) + \kappa_p\beta_w(\frac{y}{xl} - \bar{e})\right] + \pi^c_o + i(\cdot))),\\
\dot{b} &=& g - t_c - \tau_w\omega\frac{y}{x} - \mu m - b(\kappa \left[\beta_p(\frac{y}{y^p} - \bar{u}) + \kappa_p\beta_w(\frac{y}{xl} - \bar{e})\right] + \pi^c_o + i(\cdot)),\\
\dot{y}^e &=& \beta_{y^e}[c + i(\cdot) + \delta + g - y^e] + y^e(i(\cdot) - n),\\
\dot{\omega} &=& \omega\kappa[(1 - \kappa_p)\beta_w(\frac{y}{xl} - \bar{e} + (\kappa_w - 1)\beta_p(\frac{y}{y^p} - \bar{u})],\\
\dot{l} &=& l[-i_q(q - 1) - i_u(\frac{y}{y^p} - \bar{u})],\\
\dot{\nu} &=& y - (c + i(\cdot) + \delta + g) - \nu i(\cdot).
\end{array} \tag{87}$$

Proposition 4. *The steady state of the dynamical system (87) is locally asymptotically stable if the conditions in proposition 3 are met and β_n is sufficiently small.*

Theorem 4. *The Metzlerian feedback between expected sales and output is given by*

$$y = (1 + \alpha_{n^d}(n + \beta_n))y^e - \beta_n\nu.$$

This static relationship implies that lean production α_{n^d} or cautious inventory adjustment β_n (or both) can tame the Metzlerian output accelerator.

We here do not introduce any regulation of this Metzlerian sales-inventory adjustment process, but simply assume that this inventory accelerator process is of a secondary nature in the business fluctuations generate by the dynamics, in particular if the control of the Harrodian goods market accelerator is working properly.

We now let β_{π^c} become positive so that we then are back at the differential equation system

$$\begin{array}{rcl}
\dot{m} &=& m\mu - m(\kappa[\beta_p(u - \bar{u}) + \kappa_p\beta_w(e - \bar{e})] + \pi^c + i(\cdot)),\\
\dot{b} &=& g - t_c - \tau_w\omega l^d - \mu m\\
&& - b\left(\kappa[\beta_p(u - \bar{u}) + \kappa_p\beta_w(e - \bar{e})] + \pi^c + i(\cdot)\right),\\
\dot{y}^e &=& \beta_{y^e}(y^d - y^e) + (i(\cdot) - n)y^e,\\
\dot{\omega} &=& \omega\kappa[(1 - \kappa_p)\beta_w(e - \bar{e}) + (\kappa_w - 1)\beta_p(u - \bar{u})],\\
\hat{l} &=& n - i(\cdot) = -i_q(q - 1) - i_u(u - \bar{u}),\\
\dot{\nu} &=& y - y^d - i(\cdot)\nu,\\
\dot{\pi^c} &=& \alpha\beta_{\pi^c}\kappa[\beta_p(u - \bar{u}) + \kappa_p\beta_w(e - \bar{e})] + (1 - \alpha)\beta_{\pi^c}(\mu - n - \pi^c).
\end{array} \tag{88}$$

Proposition 5. *The steady state of the dynamic system (88) is locally asymptotically stable if the conditions in proposition 4 are met and β_π^c is sufficiently small.*

Theorem 5. *Assume that the business cycle is controlled in the way we have described it so far and that this implies that the fundamentalist expectations of inflation become dominant in the adjustment rule for the inflationary climate:*

$$\dot{\pi^c} = \beta_{\pi^c}(\alpha\hat{p} + (1-\alpha)(\mu - n) - \pi^c).$$

Choosing α sufficiently small guarantees the applicability of the preceding proposition.

The economy will thus exhibit damped fluctuations if the parameter α in the law of motion the inflationary climate expression π^c is chosen sufficiently small, which is a reasonable possibility if the business cycle is damped and actual inflation, here only generated by the market for goods:

$$\hat{p} \sim \beta_p(u - \bar{u})/(1 - \kappa_p) + \pi^c$$

is moderate. A stronger orientation of the change in the inflation climate on a return to the steady state rate of inflation thus helps to stabilize the economy.

Note here that the consideration of expectation formation on financial markets are still ignored (assumed as static). It is however obvious that an enlargement of the dynamics by these expectations does not destroy the shown stability properties if only fundamentalists are active, since this enlarges the Jacobian by a negative entry in its diagonal solely. Continuity then implies that a portion of chartists that is relatively small as compared to Fundamentalists will also admit to preserve the damped fluctuations we have shown to exist in the above sequence of propositions.

Proposition 6. *The steady state of the dynamic system (88) is locally asymptotically stable if the parameter α_{π_e} is sufficiently small.*

In order to get this result enforced by policy action, independently of the size of the chartist population, we introduce the following type of a Tobin tax on the capitals gains of equities:

$$\dot{\pi}_{ef} = \beta_{\pi_{ef}}(\eta - \pi_{ef}), \tag{89}$$

$$\dot{\pi}_{ec} = \beta_{\pi_{ec}}(\tau_e\hat{p}_e - \pi_{ec}). \tag{90}$$

Such a tax may be monitored through a corresponding tax declaration scheme which not only taxes capital gains, but also subsidizes capital losses (and thus is not entirely to the disadvantage of the asset holders of the model).

Theorem 6. *The Tobin tax parameter τ_e implies that damped business fluctuations remain damped for all tax rates chosen sufficiently large (below 100%).*

The financial market accelerator can therefore be tamed through the introduction of an appropriate level of a Tobinian capital gain taxation rule.

Note here however that this rule introduces a new sector to the economy which accumulates or deccumulates reserve funds R according to the rule

$$\dot{R} = \tau_e \dot{p}_e E.$$

In order to keep again the laws of motion of the economy unchanged (to allow the application of the above stability propositions) we thus assume that sector is independent from the other public institutions. Ro the steady state value ρ^o of these funds of this new sector we get when expressed per value unit of capital pK:

$$\rho^o = (R/pK)^o = \tau_e(\mu - n)/\mu < 1.$$

This easily follows from the law of motion

$$\hat{\rho} = \hat{R} - \hat{p} - \hat{K} = \frac{\dot{R}}{R}\frac{R}{pK} - \hat{p} - \hat{K}$$

since there holds $\hat{p} - \hat{K} = \mu$ and $\hat{E} = n, q = 1, \hat{p}_e = \hat{p}$ in the steady state. It is assumed that the reserves of this institution are sufficiently large so that they will not become exhausted during the damped business fluctuations generated by the model.

The stability results of the propositions are intuitively very appealing in view of what we know about Keynesian feedback structures and from what has been discussed in the preceding sections and in various chapters of Asada, Flaschel, Mouakil, and Proano (2009), since it basically states that the wage-spiral must be fairly damped, the Keynesian dynamic multiplier be stable and not too much distorted by the emergence of Metzlerian inventory cycles, that the Harrodian knife-edge growth accelerator is weak, that and inflationary and capital gains expectations are fundamentalist in orientation and money demand subject to small transaction costs and fairly unresponsive to rate of return changes on financial assets (that is money demand is not close to a liquidity trap). Such assumptions represent indeed fairly natural conditions from a Keynesian perspective.

On this basis we then obtained in the above theorems the result that independently conducted countercyclical fiscal policy can limit the fluctuations on the goods market, that an appropriate consensus between capital and labor can tame the wage-price spiral and that a Tobin tax can tame the financial market accelerator. Metzlerian inventory dynamics and fluctuations in the inflationary climate that is surrounding the economy may then also be weak and thus not endanger asymptotic stability. But what about monetary policy?

8 Dampened business cycles: Monetary policy

We so far have presumed that in the baseline model traditional monetary policy, as money supply and interest rate policy, is ineffective in the control of the economy between

the short and the medium run. As it is set up it only effects the cash management process of asset holders, but leaves $M_3 = M + B$ invariant.[25] Note however that such a monetary policy can be dangerous in the case of the liquidity trap, since this model allows for the equity owners attempt to a large degree to sell their equities against the fully liquid assets $M, B,$. This would imply – as in the current financial crisis – that the public could end up sitting on the bad assets.

The alternative is to suggest that the central bank buys the bad assets and drives up asset prices again. This is a demanding policy option that must be investigated and discussed in more detail. Yet this policy seems to have been pursued in the current financial market meltdown and this variant of monetary policy has recently come to the forefront in the discussion. Details may be beyond the scope of the present paper but we might make, as to this policy, some important observations.

The fiscal authorities, the US-Treasury, has extensively purchased equity, for example by taking over Funnie Mae and Freddie Mac, and taking over shares of automobile companies. The Fed has purchased, in order to clean up banks's balance sheets, a large amount of complex securities (MBS and CDOs) to avoid a fire sales of bad assets and a downward spiral. It also undertook extensive lending to the private sector by accepting bad assets as collaterals. This extensive purchase, or acceptance, of equity assets was a new policy variant coming to the forefront as the financial meltdown evolved in the years 2008/9. This attempt to rescue the financial and banking sectors, through the purchase of securities, was widely viewed as a step to prevent a system wide breakdown.[26] Next we want to build into our macro model some elements of this new policy.

In our baseline portfolio approach to Keynesian macrodynamics we sofar have first formulated a really tranquil monetary policy as far as the long-run is concerned, i.e., we assumed a constant growth rate of the money supply $\mu > n$. This policy was oriented towards the long run and implied in our model a positive inflation rate in the steady state. This rate should be chosen high enough to allow to avoid deflationary situations where the above described compromise between capital and labor may break down – since labor may be very opposed to money wage reductions (as Keynes (1936) already noted as a behavioral rule, a fact ignored by those economists who disregard the psychology of workers).

We take this as a starting point for our result that a monetary policy only oriented towards the short-term rate of interest is ineffective in our type of portfolio model, as we have presented it here, – unless it impacts capital gain expectations on the stock market. This holds for money supply steering as well as for the now fashionable interest rate policy rules, since such policy only effect the cash management process within the given stock of money $M_3 = M + B$. This result is a limit case of what Keynes already observed in the General Theory, where he wrote:

> Where, however, (as in the United States, 1933-1934) open-market operations have been limited to the purchase of very short-dated securities, the

[25]Note that we have not introduced here into our model long bonds and yield spreads between bonds of different maturity. To do so might be subject of future research)

[26]This policy was actually anticipated by Bernanke, Reinhard, and Sack (2004).

> effect may, of course, be mainly confined to the very short-term rate of interest and have but little reaction on the much more important long-term rates of interest. (Keynes (1936), p.197)

We have not yet long-term bonds present in our model type[27] and also no debt of firms, but only equities as means of financing their investment.[28] The following proposal of Keynes must here therefore be applied to the stock market in order to discuss his implications.

> If the monetary authority were prepared to deal both ways on specified terms in debts of all maturities, and even more so if it were prepared to deal in debts of varying degrees of risk, the relationship between the complex of rates of interest and the quantity of money would be direct. (Keynes (1936), p.205)

We do this in addition to the above monetary policy that concerns the long-run by assuming in extension of the rule $\dot{M} = \mu M,\ \mu = const.$ as integration of the long- as well as short- and medium-run orientation of monetary policy as follows:[29]

$$\hat{M} = \mu - \beta_{mq}(q - q^o), \quad \text{with} \quad \mu M = \dot{B}_c, \dot{M} - \mu M = -\beta_{mq}(q - q^o)M = p_e \dot{E}_c \tag{91}$$

This additional policy of the Central Bank takes the state of the stock market as measured by the gap between Tobin's q and its steady state value $q^o = 1$ as reference point in order to increase money supply above its long-run rate in the bust, by purchasing equities, by selling stock and decreasing therewith money supply below its long-run trend value in the boom. The opposite policy should be pursued in a recession.

This is clearly a monetary policy that attempts to control the fluctuations in equity assets and security prices generating, since it for example, buys stocks when the stock market is weak and sells stocks in the opposite case. We stress that this policy is meant to be applied under normal conditions on financial markets and may not be so easily available in the cases where a liquidity trap is in operation.

Transferred to the intensive form level this rule, which we call a Tobin rule in the following, now gives rise to the law of motion for real balances per value unit of capital:

$$\hat{m} = \mu - \beta_{mq}(q - q^o) - (\hat{p} + \hat{K}) \tag{92}$$

We already know that the trend increase in money supply by the Central Bank through open market operations in short term bonds simply implies that part of government debt is purchased by the CB such that the change in government debt is exactly given by the

[27] This could be included in future work , see also Köper (2003, ch.7)

[28] To include debt issuance of firms would amplify the bubbles and bursts, since the interaction of asset price movements and leveraging is rather destabilizing, see Semmler and Bernard (2009).

[29] which makes Central Bank money now endogenous in a pronounced way. Note however that we do not yet consider commercial banks and the endogeneity of the money supply that they are creating.

actual change in M_3. In addition to holding government bonds it is now also assumed that the CB holds equities in a sufficient amount in order to pursue its short-run oriented stock market policy. This policy is sustainable in the long-run, since the CB buys stock when cheap and sells it when expensive. It gives a new law of motion for real balances the differential equation

$$\dot{m} = \mu m - \beta_{mq}(q(m+b, r_k^e + \pi_e) - 1) - (\kappa[\beta_p(u-\bar{u}) + \kappa_p\beta_w(e-\bar{e})] + \pi^c + i(\cdot))m$$

It thus implies a significant change in the complexity of the dynamics to be investigated. We therefore only conjecture here that the above propositions and theorems can again be formulated and proved and will show that such a policy adds to the stability of the steady state of the dynamics:

Theorem 7. *The initially considered, now augmented 3D subdynamics of the full 9D dynamics:*

$$\begin{aligned} \dot{m} &= m(\mu - \beta_{mq}(q-q^o) - (\pi_o^c + i(\cdot))), \\ \dot{b} &= g - t_c - \tau_w\omega\frac{y}{x} - \mu m - b(\pi_o^c + i(\cdot)), \\ \dot{y}^e &= \beta_{y^e}\left[c + i(\cdot) + \delta + g - y^e\right] + y^e(i(\cdot) - n). \end{aligned} \tag{93}$$

can be additionally stabilized (by increasing the parameter range where damped oscillations are established and by making the originally given damped oscillations even less volatile) by an increasing parameter value β_{mq} of the new term $-\beta_{mq}(q-q^o)m$ in the law of motion for real balances, if anticyclical fiscal policy is sufficiently active to make the dynamic multiplier process a stable one (by neutralizing the Harrodian investment accelerator) and if the savings rate s_c of asset holders is sufficiently close to one (which allows to ignore effects from taxation on the consumption of asset holders).

Sketch of Proof:[30] Under the conditions assumed to hold on the asset markets we can solve for Tobin's q explicitly and get:[31]

$$q = \frac{f_e(r_e^e)}{1 - f_e(r_e^e)}(m+b) = q(r_e^e, m+b), \quad i.e., \quad \frac{\partial q}{\partial r_e^e} = \frac{f_e'(r_e^e)}{(1-f_e(r_e^e))^2}(m+b) > 0$$

The Routh-Hurwitz polynomial of the Jacobian matrix of the given extension of the original model is augmented through the stock market policy by the principal minors to be obtained from the additional matrix:

$$\begin{aligned} \dot{m} &= -\beta_{mq}(q-q^o)m \\ \dot{b} &= g - t_c - \tau_w\omega\frac{y}{x} - \mu m - b(\pi^c + i(\cdot)) \\ \dot{y}^e &= \beta_{y^e}\left[c + i(\cdot) + \delta + g - y^e\right] + y^e(i(\cdot) - n) \end{aligned}$$

[30] The full proof is in preparation, see Asada, Flaschel, Mouakil, and Proano (2009).

[31] Note that the rate of change $\dot{p}_e$ can in principle be calculated from such an expression, but will involve a large number of further laws of motion of the dynamics and is therefore not easily predictable in the given model type.

which only differs from the original one in its first row. This row can be used to eliminate the $i_q(\cdot)$ term in the $i(\cdot)$ function when calculating the principal minors of this additional Jacobian matrix. From this simplification one then easily gets that the Routh-Hurwitz coefficients a_1, a_2, a_3 of the characteristic polynomial of the augmented Jacobian exceed the originally given ones (note that according to lemma 2 we have $q_m > q_b$), while the determinant of the Jacobian in the final Routh-Hurwitz condition $a_1 a_2 - a_3$ is dominated by the additions to a_1, a_2. □

The important means to stabilize the economy or to make it at least less volatile are therefore given here by Keynesian anticyclical demand management, consensus based wage management, and Tobin type management of the financial market accelerating processes and – hopefully – also by the above willingness of the CB to trade not only in bonds, but also in equities. We stress here briefly that this extension is based on the following stock-flow relationships:

$$\begin{aligned}
\dot{B} &= pG + iB - pT - \mu M \\
\dot{E} &= \dot{E}_f - \dot{E}_c \\
p_e \dot{E}_c &= -\beta_{mq}(q - q^o)M = \dot{M}_q \\
\dot{M} &= \mu M + \dot{M}_q \\
\dot{\Pi}_c &= r_k^e p K p_e E_c / (p_e E + p_e E_c) + \dot{p}_e E_c \\
\dot{B}_c &= \mu M
\end{aligned}$$

Note that we now have to use "f" for firms and "c" for central bank as indices in order to distinguish their stock-flow contributions from the one of asset holders where we continue to use no index at all. Note also that interest payments on B_c are here assumed to be transferred back to the government so that part of the government deficit is just money financed. Note also that equity prices are determined by current stocks solely and thus independent on the inflow of now assets. Note finally that the central bank accumulates (or deccumulates) government bonds B_c, equities E_c and dividend payments and capital gains in terms of Π.

9 Conclusions

Summing up, it is not so much the individual behavior of economic agents (firms, households, institutions), but rather the interconnectedness of agents and markets which produced the stabilizing or destabilizing feedback effects within the dynamical system we have considered in this paper. The behavior of the agents was by and large a fairly simple one, while the dynamics they generated was subject to Harrodian and Metzlerian quantity accelerators, concerning the capacity utilization rate of firms and their inventory holdings. Moreover, such centrifugal forces were also present in the financial part of the model, there concerning the interaction of capital gains and capital gain expectations, operating in an otherwise stable portfolio model which was characterized by gross-substitutability. Finally, the real-financial market interaction between these two

accelerating mechanisms was also strongly impacted by a wage-price spiral, also characterized by centrifugal dynamical forces under certain assumptions on its adjustment parameters.

Left to itself, the macroeconomy thus experienced large boom -bust cycles, with extensive externalities when asset market bubbles burst.

In the context of our proposed model we then argued that such boom-bust cycles can be dampened. More specifically, in terms of policy of fiscal and monetary institutions, we have shown that countercyclical labor market and fiscal policies, with a tranquilized wage-price spiral, a Tobin tax on capital gains and the implementation of a Tobin rule in place of a Taylor rule could – taken together – be powerful means to make the business cycle not only less volatile, but in fact damped and maybe also monotonically converging to the balanced growth path of the economy.

Besides demand management by a fiscal authority, wage management through cooperation between capital and labor in a corporative system, we must also have monetary policies that here concentrate on financial markets in order to dampen business cycles on the macro level by means of new policies of buying and selling equity (and other risk-bearing securities).

References

Abreu, D., and M. Brunnermeyer (2003): "Bubbles and Crashes," *Econometrica*, 71, 173–204.

Asada, T., P. Flaschel, T. Mouakil, and C. Proano (2009): *Macroeconomic Stock-Flow Interaction and the Business Cycle*. book manuscript.

Bernanke, B., V. R. Reinhard, and B. Sack (2004): "Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment," Discussion Paper 48, Federal Reserve Board, Washington. D.C.

Brunnermeyer, M. (2009): "Bubbles," Forthcoming as entry in Palgrave, `http://www.princeton.edu/~markus/research/papers/bubbles_survey.pdf`.

Chiarella, C., and P. Flaschel (2000): *The Dynamics of Keynesian Monetary Growth. Macro Foundations*. Cambridge University Press, Cambridge, UK.

Chiarella, C., P. Flaschel, G. Groh, and W. Semmler (2000): *Disequilibrium, Growth and Labor Market Dynamics. Macro Perspectives*. Springer, Heidelberg.

Franke, R. (1996): "A Metzlerian Model of Inventory Growth Cycles," *Structural Change and Economic Dynamics*.

Franke, R., and W. Semmler (1999): "Bond Rate, Loan Rate and Tobin's q in a Temporary Equilibrium Model of the Financial Sector," *Metroeconomica*, 50(3), 351–385.

Keynes, J. M. (1936): *The General Theory of Employment, Interest, and Money*. MacMillan, London.

Köper, C. (2003): *Real-Financial Interaction in Contemporary Models of AS-AD Growth*. Peter Lang, Bern.

Mas-Colell, A., M. D. Whinston, and J. R. Green (1995): *Microeconomic Theory*. Oxford Univ. Press, New York.

Metzler, L. A. (1941): "The Nature and Stability of Inventory Cycles," *Review of Economic Statistics*, 23, 113 – 129.

Minsky, H. P. (1982): *Stabilizing an Unstable Economy*. McGraw Hill, New York, 2 edn.

Rose, H. (1967): "On the Non–Linear Theory of the Employment Cycle," *Review of Economic Studies*, 34, 153–173.

——— (1990): *Macroeconomic Dynamics: A Marshallian Synthesis*. Blackwell, Cambridge.

Sargent, T. J. (1987): *Macroeconomic Theory*. Academic Press, New York, 2nd edn.

Semmler, W., and L. Bernard (2009): "Boom-Bust Cycles: Leveraging, Complex Securities and Asset Prices," Discussion paper, New York: The New School University.

Tobin, J. (1969): "A General Equilibrium Approach to Monetary Theory," *Journal of Money, Credit, and Banking*, 1, 15–29.

——— (1980): *Asset Accumulation and Economic Activity*. Oxford; Basil Blackwell.