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**Estimating the Uzawa-Lucas Model for the U.S.
and Germany**

by

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Abstract

Numerous cross-sectional tests have been performed to evaluate the predictions of recent growth theories such as the Uzawa-Lucas growth model. In a series of papers and in his book, Jones (1995a, 1995b, 1997) has shifted the attention toward the time series predictions of endogenous growth models. By contrasting endogenous growth models with facts, one is frequently confronted with the prediction that levels of economic variables, such as education or human capital, imply lasting effects on the growth rate of an economy. As stylized facts show, measures of education or human capital in most advanced countries have dramatically increased, mostly more than the GDP. Yet, the growth rates have roughly remained constant or even declined. In this paper we modify the growth effects of education and human capital in our variant of the Uzawa-Lucas growth models and test the model using time series data for the U.S. and Germany from 1962.1 to 1996.4. We consider two versions: In the first, we treat the time spent for education as exogenously given and neglect the external effect of human capital. In the second version, the time spent for education is an endogenous variable and the external effect of human capital is taken into account. Our results demonstrate that the model is compatible with the time series for aggregate data in those countries. All parameters fall into a reasonable range.

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1 Introduction

The model we study in this paper is the two-sector model introduced by Uzawa (1965) and Lucas (1988), known as the Uzawa-Lucas model. It is constructed like the neoclassical model of growth by Solow (1956) yet total output depends on physical and human capital and the saving rate is not exogenous but endogenously determined by the preference and technology parameters. In particular, it is assumed that an infinitely-lived household maximizes the discounted stream of utility arising from consumption.

As to the empirical relevance of education and human capital as concerns economic growth, Barro and Sala-i-Martin (1995) present cross-section results in their book, chap. 12. These authors do not suppose a certain economic model by which growth rates are explained but simply undertake regressions with the growth rate of real per capita GDP as the dependent variable which is explained by various exogenous variables. Those cross-section studies demonstrate that schooling and human capital seem to have positive effects on the growth rates of countries. But it must also be stated that the variable schooling for example is not robust in the empirical cross-country study undertaken by Sala-i-Martin (1997). On the other hand, however, in Levine and Renelt (1992) the secondary school enrollment rate is not fragile but a robust variable.

Criticism has been raised on the cross-sectional econometric studies. It has been demonstrated that the cross-sectional studies, by lumping together countries of different stages of the development may miss the thresholds on development (Bernard and Durlauf, 1995). Moreover, the cross-sectional studies assume that preference and technology parameters are the same for all countries.

An alternative to cross-section studies is the use of time series methods to test modern growth models. In a series of papers, and in his book, Jones (1995a, 1995b, 1997) has shifted the attention toward the time series predictions of the endogenous growth models. From the time series perspective one is, however, confronted with the prediction of endogenous growth models that a rise in the level of an economic variable, like an increase in education, human capital or knowledge capital, implies strong and lasting effects on the growth rate of the economy. In fact, in Lucas (1988) the growth rate is predicted to monotonically increase with the level of education. Those permanent growth effects have been empirically contested by Jones (1995a, 1997). As stylized facts show, level variables such as education, human capital or research intensity in most advanced countries have dramatically increased, mostly more than the GNP. Yet, the growth rates of productivity,

for example computed as five years averages, have fallen. This gives rise to the question to what extent level variables of modern growth models have effects on growth rates. This indeed is a serious question since one would like to know if a country can expect a higher growth rate if it spends resources on the creation of human capital or if it builds up its stock of knowledge as a result of R&D spending. A possibility to reconcile the Uzawa-Lucas model with rising level variables is to assume that the higher the level of human capital the more difficult it is to generate additional human capital.

In this paper we also pursue a time series approach. By estimating the preference and technology parameters of the various models with time series data we attempt to give an answer to the questions of whether modified versions of the Uzawa-Lucas model are compatible with time series evidence.

In the next section, we present modified versions of the Uzawa-Lucas growth model with human capital. Section 3 briefly discusses how human capital can be measured in empirical studies. Section 4 presents the data we used and section 5 gives estimation results. The last section finally concludes.

2 The modified Uzawa-Lucas Model

As pointed out in the Introduction the Uzawa-Lucas model contains scale effects. There, if the time spent for education, $1 - u(t)$, rises the growth rate of human capital, $h(t)$, rises, too, and, thus, the balanced growth rate. Yet, empirical evidence does not seem to support those scale effects. This holds true for both the U.S. and Germany. In both countries the time spent for education rises whereas the growth rate of human capital slightly declines over time. Therefore, estimating the equation $\dot{h}(t)/h(t) = \kappa(1 - u(t))$ of the original Uzawa-Lucas models yields a negative κ which does not make sense since it would imply a negative marginal product of education in the process generating human capital. This shows that the original Uzawa-Lucas model will not be compatible with the time series. To take account of this fact we modify our two versions of the Uzawa-Lucas model.

To eliminate the scale effects present in the Uzawa-Lucas model we modify the equation describing the evolution of the stock of human capital. There are several possibilities how this can be achieved. We will consider two variants. First, we suppose that the equation

\dot{h}/h is given by

$$\frac{\dot{h}(t)}{h(t)} = h(t)^{p_1-1} \kappa (1 - u(t))^{p_2} - \delta_h, \quad (1)$$

with $p_1, p_2 \in (0, 1)$. This formulation implies that the higher the level of human capital the more difficult it is to generate additional human capital. The same holds for the time spent for education. The more time is already spent for education the smaller is the increase in the change of human capital as a result of more education. That is we assume decreasing returns. Further, we also allow for depreciation human capital, with δ_h giving the depreciation rate.

Second, we consider a version which is basically the same as in the original Uzawa-Lucas model with the major exception that the exogenous variable κ is now a function depending on time. Thus, we also want to take into account that other variables, like physical capital or subsidies from the government for example, which are not explicitly taken into account may play a role in the process generating human capital. In this case, the function $\dot{h}(t)/h(t)$ is written as

$$\frac{\dot{h}(t)}{h(t)} = \kappa(t)(1 - u(t))^{p_2} - \delta_h, \quad (2)$$

As in the original Solow growth model we would postulate here that there are additional exogenous factors affecting the economy. However, in contrast to the Solow growth model, in our case these exogenous factors are not posited to influence the aggregate production function for final output but the growth rate of human capital. We propose this as an alternative formulation of the Uzawa-Lucas model since there are likely to be, as cross-sectional studies have revealed, other forces affecting the growth rate of effective human capital and, thus, the growth rate of the economy.

Moreover, to be more realistic we will also allow for depreciation of physical capital, K , with $\delta_K \in (0, 1)$ denoting the depreciation rate. But, of course, this has nothing to do with the scale effects mentioned above. Further, we take into account that the labour input, L , is not constant but may vary over time which seems to be realistic. The growth rate of labour supply is denoted by n .

The equations describing the economy are given by the result of the following optimization problem:

$$\max_{c, u} \int_0^\infty L(t) \frac{c(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \quad (3)$$

subject to

$$\dot{K}(t) = AK(t)^{1-\alpha}(u(t)h(t)L(t))^\alpha h_a^\zeta - L(t)c(t) \quad (4)$$

$$\dot{h}(t) = h(t)^{p_1} \kappa (1 - u(t))^{p_2} - \delta_h h(t) \quad (5)$$

$$K(0) \geq 0, h(0) \geq 0,$$

with ρ the subjective discount rate, $1/\sigma > 0$ the intertemporal elasticity of substitution of consumption between two points in time (or the inverse of the coefficient of relative risk aversion) and $c = C/L$ consumption per capita.¹ A is a constant technology level or the level of technology, $(1 - \alpha) \in (0, 1)$ is the share of capital and $\zeta \geq 0$ is the parameter given the external effect of human capital.

In this paper we consider two versions of the model: In a first simplified version of the model we neglect the external effect (so that $\zeta = 0$) and the second control variable u , that is we assume that u is not chosen optimally but given exogenously. We call this version the modified Uzawa-Lucas I model. The optimization problem then is to solve the problem (3) subject to (4), with (5) given exogenously. Solving this optimization problem, the modified Uzawa-Lucas I model is described by the following four equations²:

$$\frac{\dot{K}}{K} = A \left(\frac{1}{y}\right)^{-\alpha} (uL)^\alpha - Lxy - \delta_K \quad (6)$$

$$\frac{\dot{C}}{C} = \frac{-(\rho + \delta_K) + (1 - \alpha)A \left(\frac{1}{y}\right)^{-\alpha} (uL)^\alpha}{\sigma} \quad (7)$$

$$\frac{\dot{h}}{h} = h^{p_1-1} \kappa (1 - u)^{p_2} - \delta_h \quad (8)$$

$$\frac{\dot{L}}{L} = n, \quad (9)$$

with $y = h/K$ and $X = c/h$.

In the second version which we call the modified Uzawa-Lucas II model we treat the time spent for education, u , as the second control variable and take into account positive externalities of human capital, i.e. $\zeta > 0$. The equations describing the economy are obtained by solving the optimization problem (3) subject to (4) and (5) as:

$$\frac{\dot{k}}{k} = Ak^{-\alpha} h^{\alpha+\zeta} u^\alpha - \frac{c}{k} - n - \delta_K \quad (10)$$

¹In the following we omit the time argument.

²The derivation is straightforward. It is available on request.

$$\frac{\dot{h}}{h} = h^{p_1-1} \kappa (1-u)^{p_2} - \delta_h \quad (11)$$

$$\frac{\dot{c}}{c} = \frac{A(1-\alpha)}{\sigma} k^{-\alpha} h^{\alpha+\zeta} u^\alpha - \frac{\rho + \delta_K}{\sigma} \quad (12)$$

$$\begin{aligned} \frac{\dot{u}}{u} = & \frac{\delta_K \alpha - \delta_h}{1 + (1-\alpha) - p_2} + \frac{h^{p_1-1} \kappa (1-u)^{p_2-1} p_2 u}{1 + (1-\alpha) - p_2} + \frac{\alpha n}{1 + (1-\alpha) - p_2} + \\ & \frac{p_1 h^{p_1-1} \kappa (1-u)^{p_2}}{1 + (1-\alpha) - p_2} + \frac{\alpha + \zeta - p_1}{1 + (1-\alpha) - p_2} \left(h^{p_1-1} \kappa (1-u)^{p_2} - \delta_h \right) - \\ & \frac{1-\alpha}{1 + (1-\alpha) - p_2} \left(\frac{c}{k} \right) \end{aligned} \quad (13)$$

$$\frac{\dot{L}}{L} = n \quad (14)$$

Subsequently, we will estimate the modified systems Uzawa-Lucas I and Uzawa-Lucas II.

However, before we do that, we want to briefly address the question of how the balanced growth path (BGP) looks like for our modified systems. In this case, we define a BGP as a path on which the output to capital ratio, Y/K , is constant and all variables grow at constant but not necessarily equal growth rates, with the exception of u and $1-u$ which are constant on a BGP. A constant output to capital ratio implies $\dot{Y}/Y = \dot{K}/K$ and, together with $\dot{K} = Y - cL$, a constant consumption to capital ratio, $C/K \equiv cL/K$. Thus, we can state that on a BGP the following equalities hold,

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{C}}{C} = \frac{\dot{c}}{c} + n.$$

Further, the requirement that the growth rates are constant implies that the time derivatives of the growth rates equal zero, i.e. $d/dt(\dot{h}/h) = d/dt(\dot{K}/K) = 0$. Differentiating \dot{h}/h with respect to time and setting the left hand side equal to zero gives

$$\frac{\dot{h}}{h} = \frac{p_2}{1-p_1} \frac{d/dt(1-u)}{1-u} \equiv \frac{p_2}{1-p_1} n_{1-u},$$

where we have defined $d/dt(1-u)/(1-u) \equiv n_{1-u}$. Doing the same for \dot{K}/K yields

$$\frac{\dot{K}}{K} = \frac{p_2}{1-p_1} n_{1-u} + n_u + n,$$

with $\dot{u}/u \equiv n_u$. Recalling that $n_{1-u} = n_u = 0$ holds on a BGP,³ we can state that for the

³For the Uzawa-Lucas II model with an endogenous u it can be shown that $\dot{u}/u = \text{constant}$ implies $\dot{u} = 0$.

modified Uzawa-Lucas models a BGP is given by

$$\frac{\dot{Y}}{Y} - n = \frac{\dot{K}}{K} - n = \frac{\dot{c}}{c} = 0 = \frac{\dot{h}}{h} = n_{1-u} = n_u.$$

This shows that the growth rate of aggregate variables equals the growth rate of the labour supply, n , in the long run. Thus, the modified Uzawa-Lucas systems do not generate sustained per capita growth in the long run.⁴ Consequently, the modified models are not endogenous growth models any longer but belong to the class of exogenous growth models. In this case, aggregate variables grow at the same rate as the labour input implying that per capita quantities remain constant. The reason for that outcome is that in the long run the time spent for the formation of human capital is constant, i.e. $d/dt(1-u) = 0$ holds, implying that the growth of human capital ceases, that is $\dot{h} \rightarrow 0$. A constant level of time spent for education cannot generate a positive growth rate of human capital in the long run given the concavity of \dot{h} in h . This concavity assumption implies that a higher level of human capital requires an increase in the time spent for education in order for the change of human capital to be positive. Only along the transition path, that is as long as the time spent for the formation of human capital is not constant, positive per capita growth are predicted to be observed.

If the equation describing the growth rate of human capital is given by (2) the function $\kappa(t)$ is crucial as to the question of whether the modified Uzawa-Lucas models can generate sustained per capita growth or not. For a constant value of $\kappa(t)$ there may be positive or zero per capita growth in the long run depending on the other parameters of the model. This holds because in the long run the time spent for education is constant implying that the right hand side is constant, too. If the right hand side is positive, human capital monotonically rises over time which may make sustained per capita growth feasible. However, it is also feasible that the economy ends up with zero per capita growth in the long run if the level of education is just high enough to compensate depreciation of human capital.

3 Proxies for Human Capital in Empirical Studies

In order to explain the enormous diversity across countries in measured per capita income levels, most empirical studies indeed show that human capital plays an important role.

⁴It should be noted that this holds independent of whether p_2 is smaller or equal to one provided $p_1 < 1$.

Yet the measurement of human capital is not unique across different studies.

Generally, human capital comprises a person's stock of knowledge and abilities the increase of which raises the productivity of the person. The stock of knowledge and the abilities of a person may be acquired by schooling but it may also be acquired outside the formal education system. For example, abilities may arise from on-the-job training. Therefore, in a broad definition the measurement of human capital should cover formal and informal education, on-the-job-training, physical and mental fitness, nutrition and social services affecting quality of work. Yet factors like physical and mental health are not easy to measure. Instead, often proxies for human capital are constructed, which include such variables as enrollment rates or average years of schooling. Further, if one intends to test the Uzawa-Lucas model outlined above only that type of human capital should be taken into account. This holds because in this model human capital is merely the result of time dedicated to the formation of knowledge or abilities. Other forms of human capital formation are not taken into account in this model.

Nevertheless, there is no generally accepted way of how the stock of human capital is correctly constructed. One possibility to define human capital per capita, h , is the following:⁵

$$h_t = \int_0^{\infty} \theta_t(s) \eta_t(s) ds$$

where $\eta_t(s)$ is the share of population with s years of schooling and $\theta_t(s)$ are efficiency parameters. These efficiency parameters denote the mapping from a person's years of schooling s to his or her human capital. Mulligan and Sala-i-Martin (1993) use the wage - schooling relationship in order to identify $\theta_t(s)$ up to a constant:

$$\frac{w_t(s)}{w_t(0)} = \frac{\theta_t(s)}{\theta_t(0)}$$

In their study on the U.S. they assume that $\theta_t(s)$ does not vary across states so that the efficiency parameters can be identified from the slope of the wage - schooling relationship of any particular state.

Barro and Lee (1993) or Psacharopoulos and Arriagada (1986) replace for $\theta_t(s)$ the years of schooling s . They take the number of years of schooling as a proxy for human capital:

$$h_t = \int_0^{\infty} s \eta_t(s) ds$$

⁵The coefficients in this section are independent of those of the other sections, they are not involved in any estimations.

But using years of schooling as a definition of human capital may be problematic. It is subject to errors in cross-country analysis because the number of days and hours of schooling per year can vary substantially across countries. Also different educational systems are a reason why years of schooling may not be a good approximation for the stock of human capital. Another problem of measuring human capital in this way is that the true value of s is not known for those who completed only part of each schooling level. Dropouts and repeaters are not accounted for.⁶

Because of this argument, Nehru, Swanson and Dubey (1995) define the stock of human capital H_{gt} as the sum of person-school years:

$$H_{gt} = \sum_g \sum_t s_{gt}$$

where s_{gt} is the addition to the stock of human capital as a result of one year of education in grade g at year t . s_{gt} is measured as the enrollment rate in grade g at time t without dropouts and repeaters.

It should also be mentioned that the data collection for different countries poses some problems since data sets are often not available or incomplete. Another difficulty is that reports on schooling data tend to become more accurate with economic development. It is easier to find data sets for industrialized countries like e.g. the U.S. or Germany than for developing countries. A weakness of all of the above mentioned constructions of human capital is that they do not measure the quality of education and this makes intertemporal as well as cross-country comparisons difficult to interpret.

As to the quality of schooling, several measures are conceivable:⁷ private school attendance, teacher salaries, expenditures per pupil, or teachers per pupil. In general, all of these variables positively affect the quality of education and, thus, the formation of human capital. For example, in private schools the standards may be higher than in public schools which has a positive effect on education. A similar argument holds as concerns the salaries of teacher. Badly paid teachers will lack motivation which, for its part, may have negative repercussions on instructions at schools and, as a result, for human capital formation. Therefore, human capital could plausibly be approximated using expenditures for education.

All these measures have in common that they are so-called input-based indices which

⁶The update of educational attainment in Barro and Lee (2000) takes into account repeaters. The data, however, are available only for five-year intervals.

⁷See also Mulligan and Sali-i-Martin (1993), Nehru, Swanson and Dubey (1995).

approximate human capital by looking at the input in a person's abilities. Besides these input-based measures there are also output-based proxies for human capital.

One attempt to construct such an index was made by Hanushek and Kimko (2000). There, human capital is approximated by the results of internationally comparable mathematics and science test scores and by measures of average years of schooling. The tests were taken at different points in time and each test had a different number of countries which participated. The tests were designed in a way such that they allow comparisons between different countries. But the tests changed over time so that they were not comparable across time.

Another possibility to measure human capital by an output-based index is to compute labor income as a function of years of schooling.⁸ Labor income l_t is defined as the sum of the earnings of all residents:

$$l_t = \int_0^{\infty} w_t(s)u_t(s)\eta_t(s)ds,$$

where the participation rate $u_t(s)$ is the ratio of persons with s years of schooling to total population with s years of schooling. w_t is again the wage rate depending on years of education and η is also, as above, the share of population with s years of schooling.

To summarize, there basically exist two measurement approaches: On the one hand human capital can be approximated by input-based measures based on educational data like enrollment rates and years of schooling or expenditures for education, on the other hand, human capital can be approximated through output-based measures like wages or tests.⁹

4 Measuring the Variables

To estimate the Uzawa-Lucas model we need data for the capital stocks h and K as well as data for aggregate consumption and labor input, C , L . Further, the model needs an approximation for the time spent to build up human capital accumulation, $(1 - u)$.

The human capital stock and the physical capital stock are computed according to the perpetual inventory method with a constant depreciation rate as in Coe and Helpman (1994) or Park (1995). For the physical capital stock we use investment data and to

⁸See also Jorgenson, Gollop and Fraumeni (1987).

⁹Strictly speaking the indices by Sala-i-Martin and Mulligan (1993) and Jorgenson et al. (1987) are both input- and output-based because they combine both approaches.

approximate human capital we use total government and private educational expenditures instead of enrollment rates or schooling years. The advantage of our approach is that we do not need to deal with differences of educational systems and we do not need to convert years of schooling into capital. Further, the data are available for all countries (for more details see the appendix).

In constructing the series for $(1 - u)$ we had to make a compromise. Though we know that the time devoted to human capital accumulation includes many years of schooling, training on the job, etc., we only use the earned university degrees as a fraction of the employment. The reason for this decision is that university degrees are comparable while the lower degrees are very different across the countries. Therefore, we define $(1 - u)$ as follows:

$$1 - u = \frac{\text{university degrees}}{\text{employees}} \cdot s$$

with $s = 6$ as approximated time (years) at university.¹⁰

Table 1 presents a survey of how we construct the data sets:

Table 1: Data Sets

variables	U.S.	Germany
C	private consumption	
K	accumulated gross fixed capital formation	
h	accumulated educational expenditures per capita	
$1 - u$	weighted shares of uni.-degrees on employees	
L	total employees	

The data for consumption, investment and total employees for the U.S. and Germany are from OECD (1998, 1999). The data for consumption, investment and labour were available quarterly. The data for educational expenditure and university degrees were only available annually. In this case, quarterly data were constructed by linear interpolation. The data for educational expenditure and university degrees are from National Science Board (2000) and from Office for National Statistics (1965)-(1998) for the U.S. and from Statistisches Bundesamt (1977), (1991)-(1996) for Germany. Further, all data are real data.

¹⁰University degrees include diplomas and doctoral degrees.

5 Estimation of the Model for the U.S. and Germany

In a first step, we estimate the equations (6) to (8) and (10) to (13) using quarterly data for the modified Uzawa-Lucas I and II versions. In a second step, we replace (1) by equation (2) and reestimate the modified Uzawa-Lucas I model. As to the estimation strategy see the appendix to this chapter. For the U.S. we examine the period from 1962.1 to 1996.4 and for Germany the period from 1962.1 to 1991.4.¹¹ As concerns the parameter μ_h we have tried several different values. Finally, μ_h was chosen such that the theoretical models (Uzawa-Lucas I and II) best match the empirical data (see also the appendix to this chapter). For the U.S. μ_h is set to $\mu_h = 0.62$ and for Germany we take $\mu_h = 0.15$. As to the parameter δ_K we take the predetermined value $\delta_K = 0.025$ in the equations. The parameters to be estimated then are $((1 - \alpha), A, \rho, \sigma, \kappa, p_1, p_2, \delta_h)$. Tables 2 and 3 present the obtained estimation results (standard errors in parenthesis):

Table 2: Estimation of the Uzawa-Lucas I model

	U.S.	Germany
Parameter	value (std. err.)	value (std. err.)
$(1 - \alpha)$	0.39 (0.0262)	0.346 (0.0952)
A	0.0716 (0.003)	0.066 (0.0057)
ρ	0.025 (0.0027)	0.008 (0.0008)
σ	1.576 (0.3198)	0.6812 (0.0925)
κ	0.04 (0.0003)	0.028 (0.0006)
p_1	0.321 (0.0587)	0.0002 (0.1118)
p_2	0.05 (0.0034)	0.059 (0.0042)
δ_h	0.032 (0.0003)	0.018 (0.0004)

Though we neglect the external effect of human capital and the second control variable, u , the estimations of the Uzawa-Lucas I model are reasonable. The capital shares in the U.S. and Germany are significant and about 40 and 35 percent which are reasonable values. The same holds for the rate of time preferences which are about 10 percent for the U.S. when considered for one year and 3 percent for Germany. The estimated coefficient for the inverse of the intertemporal elasticity of substitution of consumption are also statistically

¹¹Data for Germany are for West Germany.

significant. The value for the U.S. is reasonable while the value for Germany seems a bit low implying that the intertemporal elasticity of substitution is very high.

As concerns the parameters of the production function for human capital we see that all parameters are significant except p_1 for Germany which is not significant. Further, one realizes that p_2 is very low both for the U.S. and Germany. This results from fact that the time series of $1 - u$ and \dot{h}/h are inversely related. κ is also very small but positive. As mentioned above setting p_1 and p_2 equal to one, as in the original version of the Uzawa-Lucas model, would imply a negative κ which would not make sense economically. Thus, the Uzawa-Lucas model is compatible with U.S. and German time series only after taking account of scale effects. In our variants this was done by introducing the parameters p_1 and p_2 . Next, we will estimate the Uzawa-Lucas II model.

The difference between the Uzawa-Lucas I and the Uzawa-Lucas II model is the presence of the external effect of human capital, ζ in (12), and equation (13) which gives the growth rate of the time spent for education. It is this latter additional equation which makes this extended version definitely more complicated. The estimation of the Uzawa-Lucas II model yields the following results.

Table 3: Estimation of the Uzawa-Lucas II model

	U.S.	Germany
Parameter	value (std. err.)	value (std. err.)
$(1 - \alpha)$	0.39 (0.0517)	0.327 (0.1882)
A	0.082 (0.0127)	0.106 (0.0498)
ρ	0.027 (0.0124)	0.027 (0.001)
σ	5.611 (3.3261)	0.998 (0.1349)
κ	0.047 (0.0006)	0.014 (0.0004)
p_1	0.29 (0.7583)	0.4553 (0.7032)
p_2	0.108 (0.0062)	0.006 (0.0006)
δ_h	0.033 (0.0005)	0.01 (0.0004)
ζ	4.4e-0.006 (0.052)	0.005 (10.6)

The parameter values α , ρ , g_L and σ of the second estimation are very similar to the results of the Uzawa-Lucas - I model. So, the capital share, the rate of time preference and the inverse of the intertemporal elasticity of substitution (for Germany) still take values which are generally considered as plausible. For the U.S. economy, however, σ becomes

very large and the standard deviation also rises so that statistical significance of this parameter must be doubted. The parameter p_1 is now statistically insignificant for both Germany and the U.S. As concerns the external effect of human capital, ξ , this parameter is not statistically significant neither for the U.S. nor for Germany. Thus, this model does not allow to conclude that human capital is associated with positive externalities.

As mentioned above, a different opportunity to deal with scale effects is to assume that there is an exogenously determined trend. We do this by positing that κ is a function depending on time t giving equation (2). As to the function $\kappa(t)$ we assume that it is given by

$$\kappa(t) = \frac{\kappa_0}{\theta(t)} \quad (15)$$

with

$$\theta(t) = \beta\theta(t-1) + \nu \quad (16)$$

and with the initial condition $\theta(0) = 1$.

Thus, we estimated the modified Uzawa-Lucas I model consisting of equations (6), (7) and (2) where $\kappa(t)$ is determined by (15) and (16). The results show that the parameters $((1-\alpha), A, \rho, \sigma)$ do not take different values. This is due to the fact that the parameters in equation (2) are not included in equations (6) and (7). The estimated parameters for equation (2) are given in table 4 with standard errors again in parenthesis.

Table 4: Estimation of the Uzawa-Lucas I model with equation (2)

	U.S.	Germany
Parameter	value (std. err.)	value (std. err.)
κ_0	0.036 (0.001)	0.034 (0.0015)
p_2	0.20 (0.0107)	0.13 (0.0092)
δ_h	0.007 (0.0003)	0.003 (0.0004)
ν	0.039 (0.0016)	0.003 (0.0022)
β	0.989 (0.0009)	1.017 (0.0014)

As can be seen from table 4 all parameters are statistically significant. As concerns the function $\kappa(t)$ one also realizes that this function negatively depends on time. Of course, this was to be expected since \dot{h}/h declines over time while $(1-u)$ rises. Comparing the estimated parameter p_2 in table 2, where equation (1) was assumed, with the one in table

4, where equation (2) was estimated, one realizes that this parameter is larger in the latter case. Probably, this is due to the strongly negative exogenous time trend which tends to reduce the growth rate such that the (positive) effect of education on the formation of human capital is now larger.

Overall although we have obtained reasonable parameter values the preference or technology parameters are of course sensitive to the model specification and the empirical measurement of the variables involved. Moreover, there might be variables missing like R&D spending, public investment in infrastructure, openness of the economy, efficiency of the financial sector, political stability and so on. As we have seen in the cross-sectional studies there are posited numerous variables to affect economic growth. The impact of some of those variables on economic growth, which may change over time, are captured by our estimated time trend $\kappa(t)$. In the next step we will specify some further, possibly important, variables and estimate their impact.

6 Conclusion

This paper has presented an empirical estimation of the Uzawa-Lucas model. Looking at the time series of the growth rate of human capital and at the time series of education showed that these two series are inversely related, i.e. the growth rate of human capital slightly declines while education rises. To take account of this phenomenon we modified the production function for human capital. So, in a first step we posited that the growth rate of human capital negatively depends on the level of human capital, which can be justified by referring to satiation, and that there are decreasing returns to education. The estimation of this modified Uzawa-Lucas model demonstrated that it is compatible with the time series of the U.S. and German economies. Most of the structural parameters take values which are generally accepted as plausible in the economics literature.

In a second step, the major modification of the original Uzawa-Lucas model was the assumption of an exogenous time trend which is negatively correlated with the growth rate of human capital. This can be justified by variables affecting the growth rate of human capital but which are not explicitly accounted for in our model. The estimation of this modified Uzawa-Lucas model produced statistically significant coefficients for the production function of human capital with the parameters of the other equations left unchanged. This version then would imply that the effect of spending time for education on the growth rate of human capital is also affected by other forces and their impact has

changed over time.

The latter modification of the Uzawa-Lucas model, however, has also consequences for the analytical model. In case of an exogenously given time trend the form of this function is of crucial importance as to the question of whether long run per capita growth can be observed or not. In case this function is constant in the long run the emergence of positive per capita growth depends on the other parameters of the function in the model.

From a theoretical point of view, the first modification without an exogenous time trend is more appealing because in this case the outcome does not depend on exogenous factors. But this modification has far reaching consequences. It implies that the model does not generate endogenous growth at the steady state any longer but becomes an exogenous growth model with the long run per capita growth rate equal to zero, unless exogenous variables have a stimulating growth effect. Consequently, positive per capita growth can only be observed along the transition path to the long run steady state which may of course take long to be reached. This implies that both the U.S. and the German economy must be on the transition path and cannot be described as economies on the BGP. It may be correct that the economies are indeed on the transition path if one resorts to the Uzawa-Lucas model in order to describe these economies. This may hold true because education has risen over the time period we are considering. However, this also implies that per capita growth ceases because the economies will reach a situation where the time spent for education reaches a constant value implying zero per capita growth.

So, the overall message of the first modification is that in the long run economies reach a situation without positive per capita growth at least if one does not rely on exogenous factors to eliminate the scale effects present in the original Uzawa-Lucas model. But, recalling chapter 2, this is in contrast to the stylized fact of sustained growth with no tendency for a decline in the growth rates. Therefore, it can be concluded that the Uzawa-Lucas model may be a good approximation for economies over a certain time period where education rises, implying that the economies are on the transition path to the long run steady state. However, this model is at odds with the stylized fact of positive long run growth rates if it is modified such that it does not reveal scale effects. This is a simple consequence of the fact that the time spent for education cannot grow without an upper bound although advanced economies such as the U.S. and Germany may still be far away from such an upper bound.

Appendix

1. Data Construction and Preliminary Estimation

The modified Uzawa-Lucas I Model

To estimate the model, we first have to construct the data series K and H . We compute these data series from the expenditure flows by using the perpetual inventory method. Specifically,

$$K(t+1) = (1 - \delta_K)K(t) + I(t)$$

$$h(t+1) = (1 - \delta_h)h(t) + e(t)^{\mu_h}$$

where I is aggregate investment in physical capital and $e(t)$ is education expenditure per capita. It should be noted that we have raised the power of $e(t)$ to μ_h . This is more general than setting the power equal to 1. Apparently, to construct these data series, we need to specify the following parameters δ_K , δ_h and μ_h . We set these parameters to the following values:

U.S.	Germany
$\delta_K = 0.025$	$\delta_K = 0.025$
$\delta_h = 0.025$	$\delta_h = 0.025$
$\mu_h = 0.62$	$\mu_h = 0.15$

Setting δ_K to 0.025 (recall that we use quarterly data) is very standard and has been frequently used in the RBC literature.¹² The other parameters δ_h and μ_h are chosen based on our preliminary estimation to match \dot{K}/K and \dot{C}/C , whose moments are expressed by equations (6) and (7). For these preliminary estimation, we set $(1 - \alpha)$, ρ and σ to 0.4, 0.01 and 2 respectively, which we believe to be economically reasonable. A is an additional parameter to be estimated along with the parameters δ_h and μ_h . The estimation method we employ is the Generalized Methods of Moments (GMM). The optimization algorithm used is the simulated annealing (see below).

Given the constructed data K and h , along with the observation of L , C and u , we are now able to estimate the parameters $(1 - \alpha)$, A , ρ , σ , κ , p_1 , p_2 and δ_h . We estimate these parameters by matching equations (6), (7) and (8).

¹²A quarterly depreciation rate of physical capital between 0.0125 and 0.025 is generally considered as plausible.

It should be noted that $(1 - \alpha)$ and A appear in both (6) and (7), while ρ and σ appear only in (7), and k, p_1, p_1 and δ_h only in (8). Therefore, our estimation strategy is as follows. We first estimate $(1 - \alpha)$ and A by matching both \dot{K}/K and \dot{C}/C simultaneously. The estimation method is again GMM. In this estimation, we set ρ and σ again to 0.01 and 2 respectively. Given the estimated parameters $(1 - \alpha)$ and A , we then estimate ρ and σ by matching \dot{C}/C only. Since this is a single equation estimation, we now use the method of nonlinear least square (NLLS) estimation. The rest of the parameters are estimated by matching (8). Again here we use the NLLS method where we use Newton-Raphson Algorithm as optimization algorithm. The estimation results are reported in table 2.

The modified Uzawa-Lucas II Model

The data were constructed as for the modified Uzawa-Lucas I Model. In addition to the parameters that appear in the modified Uzawa-Lucas Model I, we now have the parameter ζ that needs to be estimated. We estimate these parameters by matching equations (10) - (13).

The growth rate of labour, n , is set to its sample means. The estimation strategy is quite similar to the estimation of modified Uzawa-Lucas I Model. Specifically, we first estimate $(1 - \alpha)$, ζ and A by matching \dot{k}/k and \dot{c}/c simultaneously. Again, we use GMM for this estimation. The parameters ρ and σ , which appear in \dot{c}/c , are again set to the value 0.01 and 2 respectively. Given the estimated $(1 - \alpha)$, ζ and A , we then estimate ρ and σ by matching \dot{c}/c . The estimation method here is NLLS. The rest of the parameters are estimated by simultaneously by matching \dot{h}/h and \dot{u}/u . The estimation method is again GMM, where we use the Newton-Raphson algorithm. The results are reported in table 3.

2. The Generalized Methods of Moments (GMM) and the Simulated Annealing

The Generalized Methods of Moments

The GMM estimation employed here starts with a set of orthogonal conditions, representing the population moments established by a theoretical model:

$$E [g(y_t, \psi)] = 0 \tag{17}$$

where y_t is a $p \times 1$ vector of observed variables at date t ; ψ is a $q \times 1$ vector of unknown parameters to be estimated and $g(\cdot)$ is a $r \times 1$ vector mapping from R^{p+q} . Let T denote the sample size. The sample moments of $g(\cdot)$ can then be written as

$$g_T(\psi) = \frac{1}{T} \sum_{t=1}^T g(y_t, \psi). \quad (18)$$

The idea of GMM estimator is to choose an estimated ψ that matches the sample moments $g_t(\psi)$ and the population moments given by (17) as closely as possible. To achieve this, one needs to define a distance function by which that closeness can be judged. Hansen (1982) suggested a distance function:

$$J(\psi) = [g_T(\psi)]' W_T [g_T(\psi)], \quad (19)$$

where W_T , called the weighting matrix, is $r \times r$, symmetric and positive definite. Thus, the GMM estimator is the value of ψ , denoted as $\hat{\psi}$, that minimizes (19). From the results established in Hansen (1982), a consistent estimator of the variance-covariance matrix of $\hat{\psi}$ is given by

$$\text{Var}(\hat{\psi}) = \frac{1}{T} (D_T)^{-1} W_T (D_T')^{-1},$$

where $D_T = \partial g_T(\hat{\psi}) / \partial \psi'$. There is great flexibility in the choice of W_T for constructing a consistent and asymptotically normal GMM estimator. In this paper, we adopt the method by Newey and West (1987), where it is suggested that

$$W_T^{-1} = \hat{\Omega}_0 + \sum_{j=1}^m w(j, m) (\hat{\Omega}_j + \hat{\Omega}_j'), \quad (20)$$

with $w(j, m) \equiv 1 - j/(1 + m)$, $\hat{\Omega}_j \equiv (1/T) \sum_{t=j+1}^T g(y_t, \hat{\psi}^*) g(y_{t-j}, \hat{\psi}^*)$ and m to be a suitable function of T .⁵ Here $\hat{\psi}^*$ is required to be a consistent estimator of ψ . Thus two-step estimation is suggested as in Hansen and Singleton (1982). First, one chooses a sub-optimal weighting matrix to minimize (19) and hence obtains a consistent estimator $\hat{\psi}^*$. One then uses the consistent estimator obtained in the first step to calculate the optimum W_T through which (19) is re-minimized.

For example, for the preliminary estimation of the modified Uzawa-Lucas I model the set of orthogonal conditions (18) for our GMM estimation is given by the two equations (6) and (7).

⁵According to our sample size, we choose $m = 3$. This is based on the consideration that $m(T)$ takes the form of $T^{1/5}$, which satisfies the requirement on $m(T)$ as indicated in Theorem 2 of Newey and West (1987).

Generally, the estimation is undertaken in two steps. In a first step the weighting matrix (20) used in the distance function (19) will be approximated by an arbitrary weighting matrix in order to get an initial estimate of the parameter set ψ . In a second step the the estimated parameter set from the first step is employed and the weighting matrix (20) and the function (19) are recomputed. For our applications, a computer algorithm, written in ‘Gauss’, is designed to solve the optimization problem (19) with the simulated annealing. A sketch of the simulated annealing is given below. The computer program consists of an outer algorithm that computes the endogenous variable for a given parameter set and an inner algorithm, the simulated annealing, that searches for the parameter set in order to minimize the distance function (19). Both algorithms are iteratively connected.

A Sketch of the Simulated Annealing

The subsequently introduced global optimization algorithm, the simulated annealing, moves uphill and downhill and operates with a varying step size and a random search so as to escape local optima. The step size is narrowed and the random search confined to an ever smaller region when the global maximum is approached in the computation of the parameter set. The algorithm is applied to solve the GMM estimation as described above. The procedure amounts to the search of a set of parameters that minimizes the distance function (19). The problem is to search for that set in an appropriate order and within an appropriate space. Conventional algorithms¹³ for such groping are generally suited for cases where there is only one optimum. Given the fact that the model to be estimated is nonlinear in parameters, this is unlikely in our case. We thus employ the simulated annealing algorithm, since it is particularly suitable to escape local optima. A detailed description of its mathematical features are discussed in Corana et al. (1987) and Goffe et al. (1994).

Let $f(x)$, for example, be a function that is to be maximized and $x \in S$, where S is a subspace in R^n , where $n = 5$ in our case. This subspace S should be defined from the economic viewpoint and by computational convenience. In our case, we assume $-5 < x_i < 5$ for all i ($i = 1, 2, \dots, 5$). The algorithm starts with an initial parameter vector x^0 . Its function value $f^0 = f(x^0)$ is calculated and recorded. One sets the optimum x and $f(x)$, denoted by x_{opt} and f_{opt} respectively, equal to x^0 and $f(x^0)$. Other initial

¹³An extensive review of traditional algorithms can be found in Judge et al. (1985).

conditions include the initial step-length (a vector with the same dimension as x) denoted by v^0 and an initial temperature (a scalar) denoted by T^0 .

The new variable, x' , is chosen by varying the i th element of x^0 such that

$$x'_i = x_i^0 + r \cdot v_i^0 \quad (21)$$

where r is a uniformly distributed random number in $[-1, 1]$. If x' is not in S , repeat (21) until x' is in S . The new function value $f' = f(x')$ is then computed. If f' is larger than f^0 , x' is accepted. If not, the Metropolis criteria,¹⁴ denoted as p , is used to decide on acceptance, where

$$p = e^{(f' - f)/T^0} \quad (22)$$

This p is compared to p' , a uniformly distributed random number from $[0, 1]$. If p is greater than p' , x' is accepted. Besides, f' should also be compared to the updated f_{opt} . If it is larger than f_{opt} , both x_{opt} and f_{opt} are replaced by x' and f' .

The above steps (starting with (21)) should be undertaken and repeated N_S times¹⁵ for each i . Subsequently, the step-length is adjusted. The i th element of the new step-length vector (denoted as v'_i) depends on its number of acceptances (denoted as n_i) in its last N_S times of the above repetition and is given by

$$v'_i = \begin{cases} v_i^0 [1 + c_i(n_i/N_S - 0.6)/0.4] & \text{if } n_i > 0.6N_S; \\ \frac{v_i^0}{[1 + c_i(0.4 - n_i/N_S)/0.4]} & \text{if } n_i < 0.4N_S; \\ v_i^0 & \text{if } 0.4N_S \leq n_i \leq 0.6N_S \end{cases} \quad (23)$$

where c_i is suggested to be 2 as by Corana et al. (1987) for all i . With the new selected step-length vector, one goes back to (21) and hence starts a new round of iteration. Again after another N_S times of such repetitions, the step-length will be re-adjusted. These adjustments as to each v_i should be performed N_T times.¹⁶ We then come to adjust the temperature. The new temperature (denoted as T') will be

$$T' = R_T T^0 \quad (24)$$

with $0 < R_T < 1$.¹⁷ With this new temperature T' , we should go back again to (22). But this time, the initial variable x^0 is replaced by the updated x_{opt} . Of course, the

¹⁴Motivated by thermodynamics.

¹⁵ N_S is suggested to be 20 as by Corana et al. (1987)

¹⁶ N_T is suggested to be 100 by Corana et. al. (1987).

¹⁷ R_T is suggested to be 0.85 by Corana et. al. (1987).

temperature will be reduced further after one additional N_T times of adjusting the step-length of each i .

For convergence, the step-length in (21) is required to be very small. In (23), whether the new selected step-length is enlarged or not depends on the corresponding number of acceptances. The number of acceptance n_i is not only determined by whether the new selected x_i increases the value of objective function, but also by the Metropolis criteria which itself depends on the temperature. Thus a convergence will ultimately be achieved with the continuous reduction of the temperature. The algorithm will end by comparing the value of f_{opt} for the last N_ϵ times (suggested to be 4) when the temperature is attempted to be re-adjusted.

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