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**Monetary Policy, Multiple Equilibria and
Hysteresis Effect on the Labor Market**

by

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Abstract

In this paper we demonstrate that a standard loss function of a central bank may generate multiple equilibria which can contribute to hysteresis effects on the labor market. Multiple equilibria are feasible if the objective function of the central bank is non-quadratic. Such preferences may arise if the weights for output and inflation stabilization are state dependent, for example, if output stabilization is more important for low levels of actual output compared to higher levels (or inflation stabilization is more important for high inflation rates than for low rates). In the presence of multiple equilibria large shocks can give rise to history dependence and hysteresis effects. We also show how to solve the monetary control problem with non-quadratic preferences, multiple equilibria and history dependence.

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1 Introduction

The current research on monetary policy rules generally presumes that the central bank follows some discretionary monetary policy and displays a concern with output variability as well as inflation variability in its loss function. The research also has shown that central banks recently tend to give a strong weight to inflation targeting. Underlying the discretionary monetary policy of central banks is the Phillips-curve which is considered to hold true in the short run (but not necessarily in the long run). Mostly in this research the central bank is posited to minimize a loss function which is quadratic both in production and inflation or in unemployment and inflation respectively. The use of the linear quadratic control problem has been the tradition of modeling central bank's behavior. Here the steady state is uniquely determined. Examples of such central banks preferences can be found in Svensson (1997)¹ and the numerous contributions in the recently edited book by Taylor (1999a).

In other recent approaches a more general welfare function has been taken as starting point to evaluate policy actions of monetary authorities. This is pursued by Rotemberg and Woodford (1999) who postulate a household's welfare function² and undertake a second order Taylor series expansion about a steady state. This gives a quadratic loss function about possible steady states of the model. They then study the impact of different variants of monetary policy rules on the household's welfare. Naturally, under mild non-concavity of the households' welfare function³ there are likely to arise multiple steady states.

An explicit model with multiple steady-state equilibria is given in Benhabib, Schmitt-Grohé and Uribe (1998a,b). They also use a framework with a household's utility function

¹See also Cukierman and Lippi (1999)

²See also Christiano and Gust (1999).

³One can show that in case of assets entering the households' welfare or in case of the existence of externalities multiple steady states may emerge, see Semmler and Sieveking (2000).

where consumption and money balances affect household's welfare positively and labor effort and inflation rates negatively. In their model multiple steady-state equilibria arise due to a specific (but rather simple) policy rule (Taylor rule) and certain cross-derivatives between consumption and money balances in the household's utility function. Their inflation path is a perfect foresight path and thus they do not need to use a Phillips-curve as in the traditional literature that builds on the linear quadratic control problem. They study the local and global dynamics about the steady states but do not undertake a welfare evaluation (neither with respect to the equilibrium path nor for different policy rules).

In this paper we will not pursue the latter line of research but rather, because of heuristic reasons, stay in the tradition of the literature on quadratic loss functions that has been employed in monetary control models. We will, however, slightly depart from the quadratic loss function. There are other recent papers that have also departed from the standard quadratic objective function. Nobay and Peel (1998) for example suppose that a Linex function, that is a combination of a linear and exponential function, is more appropriate in order to model the deviation of inflation and output from their desired levels. This holds because a Linex functions implies that an inflation rate (output level) above (below) the desired level goes along with higher disutility compared to an inflation rate (output level) below (above) the desired value. Orphanides and Wilcox (1996) postulate non-quadratic preferences where output only is stabilized if the inflation rate is within a certain bound. Moreover, Orphanides and Wieland (1999) argue that the loss function is flat for a certain range of inflation rates and output levels. This implies that the central bank takes discretionary policy measures only when certain threshold levels are reached. As long as inflation and unemployment remain within certain bands the central bank will not become active.

We too will slightly depart from a quadratic loss function and demonstrate that the

(intertemporal) optimization problem faced by a central bank may lead – or contribute⁴ – to history dependence and hysteresis effects on the labor market. We suppose an endogenous weighting function which makes output stabilization more important relative to inflation control for low levels of output compared to high levels. On the other hand inflation stabilization will become more important at high levels of inflation. Recent literature has introduced a nonlinearity in the interest rate feedback rule, the Taylor rule, to take account of such considerations. It has been stated that a central bank is likely to pursue an active monetary policy at high inflation rate and a passive policy at low inflation rates.⁵ As the model by Benhabib et al. (1998a,b) our model too is likely to give rise to multiple steady state equilibria but also to history dependence. We show that such a model can be solved by applying either Pontryagin’s maximum principle and the Hamiltonian as well as the Hamilton-Jacobi-Bellman (HJB) equation. In contrast to Rotemberg and Woodford (1999), Christiano und Gust (1999) and Benhabib et al. (1998a,b) our approach allows us to evaluate the welfare function also outside the steady state equilibria.

The remainder of the paper is organized as follows. Section 2 studies a simple control problem faced by the central bank and discusses in detail the effects of a non-quadratic objective function of the central bank. Section 3 demonstrates how hysteresis effects on the labor market may arise given our assumptions in section 2. Section 4 concludes the paper. The appendix contains a discussion of solution techniques for models with multiple equilibria, namely the Hamiltonian function and the HJB-equation, and derives the optimal Taylor rule for our model.

⁴The idea of hysteresis effects on the labor markets has originally been introduced by Blanchard and Summers (1986, 1988). In the recent literature, see Stiglitz (1997), this argument has been used to explain why the U.S. economy has experienced a persistent low level of unemployment and Europe a persistent high level of unemployment. We don’t want to argue that monetary policy is the sole cause for hysteresis but rather can significantly contribute to it. This is a line of research that also Blanchard (1997) has pursued. Further discussions are given below.

⁵In Benhabib, Schmitt-Grohé and Uribe (1998a,b) this takes the form of a state dependent interest rate feedback rule of the central bank.

2 The Central Bank's Control Problem

The monetary authority can control the level of aggregate output $x(t)$ by its policy variable $u(t)$ (in the short run). For simplicity we assume that the change in aggregate output is a linear function of $u(t)$,

$$\dot{x}(t) = u(t). \quad (1)$$

$u(t) > 0$ (< 0) implies that the central bank conducts an expansionary (contractionary) monetary policy. This means that it expands the money supply, for example, or that it reduces the interest rate in order to stimulate output. Appendix 2 studies the problem when the central bank sets the interest rate according to an interest rate reaction function.⁶

The deviation of the inflation rate $\pi(t)$ from its core level π^* depends on both $u(t)$ and the deviation of aggregate output $x(t)$ from the exogenously given long-run output level, x_n which implies a constant inflation rate, whereby x_n corresponds to be the NAIRU. If $x(t) > x_n$ we have an inflationary pressure tending to raise the inflation rate above its core level π^* and vice versa. This assumption implies that a high level of output corresponds to a high level of employment tending to raise inflation. Note that hereby we could view the core inflation as being given by the expected price change where the expectation is formed by economic agents that are forward looking, backward looking or representing a linear combination of both, see Romer (1996, ch. 5) and where also the central bank's desired inflation rate plays a role. Thus, the core inflation rate π^* could be viewed as summarizing medium run competitive pressures on the product and labor markets, medium run money growth and price expectations extracted from product and labor markets as well as financial and commodity markets.⁷ This inflation rate is also often taken as the medium-run target rate of inflation for the monetary authority.⁸ The

⁶Assuming a dynamic IS equation relating the change in output to the interest rate, one can derive a central bank interest rate reaction function, the Taylor rule, describing the optimal interest rate. This is undertaken in appendix 2, see also Svensson (1997), and Semmler and Greiner (1999) for more details.

⁷See also Stiglitz (1997).

⁸The core inflation could also be perceived as forward looking target rate of inflation that is consis-

German Bundesbank, for example, has in its π^* concept defined such a core inflation rate which it then attempted to adhere to. Recently, the concept of core inflation has been restated in Deutsche Bundesbank (2000). Inflation expectations is captured in π^* in the sense that the central bank's view on acceptable or desirable rates of inflation play an important role for private inflation expectations, see Gerlach and Svensson (2000). Of course, the inflation rate π^* might be seen to be influenced by actual inflation rates as well, for example as Gerlach and Svensson (2000) argue, if the private agents are disappointed by the central bank's target and thus the rate π^* might be assumed to move over time.⁹ For analytical purposes, however, in order to avoid a two state variable model, it is here presumed to be fixed. Note that by using the concept of core inflation we refrain from explicitly modeling the private sector expectations formation as in Sargent (2000) in order to simplify the model.¹⁰

As concerns the functional form for $\pi(t) - \pi^*$ we assume the following Phillips-curve equation

$$\pi(t) - \pi^* = \alpha u(t) + \beta(x(t) - x_n), \quad \alpha, \beta \geq 0. \quad (2)$$

The higher¹¹ u the higher the inflation rate and its deviation from the core value π^* . This implies that an expansionary monetary policy raises actual inflation. Further, the more the aggregate output level exceeds the natural output the greater is the inflationary pressure.¹²

tent with saddle path dynamics of our optimal control model. To avoid those complicated numerical computations we keep the core rate π^* fixed.

⁹Note that we could define a moving core inflation rate as, for example, the German Bundesbank had proposed during the high inflationary period of the 1970s or the disinflation period of the 1980s and 90s. This would not change our below developed results. For empirical estimates on the inflation target of the Bundesbank for the period 1980-1994, see Clarida et al. (1998). A moving target rate could be viewed as corresponding to a moving rate consistent with the stable branch of the saddle path.

¹⁰Sargent (2000) uses an adaptive learning scheme in a two agents' model – private agents and the central bank – in which the private agents update their beliefs about future inflation rates through adaptive learning.

¹¹In the following we suppress the time argument t .

¹²The fact that the control u appears here in the Phillips-curve can be derived from the assumption that lags of output, as in Svensson (1997), are relevant for wage bargaining or price setting by firms.

The objective of the monetary authority is composed of two parts: First, as usual, it wants to keep the inflation rate π as close to the exogenously given core rate π^* . This is achieved by assuming a quadratic penalty function $h_1(\pi)$ which attains its minimum at $\pi = \pi^*$.

Second, the monetary authority wants to stabilize aggregate output around the natural output. Deviation from the natural output are penalized by a quadratic penalty function $h_2(x)$ with the minimum given at $x = x_n$. Assuming an intertemporal perspective, the objective functional of the monetary authority is described as

$$\min_u \int_0^\infty e^{-\delta t} (h_1(\pi) + h_2(x)) dt, \quad (3)$$

subject to (1) and (2), with δ denoting the discount rate.

The solution to this intertemporal optimization problem is unique, if the objective function is a quadratic function in x . However, if the objective function of the central bank is non-quadratic, a more complex dynamic outcome can be observed as we will demonstrate in detail in section 3. Let us first elaborate on some economic reasons which motivate the introduction of a non-quadratic objective function.

One possible justification for departing from a quadratic objective function is to assume a weighting function. As pointed out in a number of empirical studies, to be discussed below, and as suggested in the model by Svensson (1997), we can assume that the goal of output stabilization should obtain a weight determining its significance relative to the goal of keeping inflation close to the core π^* . However, in contrast to what is frequently assumed, we posit that the weight on output stabilization is not a constant but a function depending on the level of actual output. For high values of aggregate output the goal of raising output is less important compared to a situation when aggregate output is low. On the other hand the weight for the inflation rate becomes more important when output rises and the inflation rate is high. We model this idea in a simple way by fixing the

weight for the inflation rate equal to 1 and assume a weighting function $w(x)$ of the form

$$w(x) = \begin{cases} a_1, & \text{for } x \in [0, x_1) \\ a(x), & \text{for } x \in [x_1, x_2) \\ a_0, & \text{for } x \geq x_2, \end{cases}$$

with $a_1 \in \mathbb{R}_{++}$, $a_0 < a_1 \leq a_2$ and $a(x) : \mathbb{R}_{++} \rightarrow \mathbb{R}_{++}$, with $da(x)/dx < 0$. x_j , $j = 1, 2$.

This function implies that output stabilization is always less important than inflation control because the weight on output stabilization is always lower than the weight on inflation control (which is equal to 1). We set the maximum weight for output control equal to a_2 . In order to model a simple situation we assume that the weight for the output stabilization decreases (the relative weight for inflation stabilization increases) as output increases. Once a certain threshold level, x_2 , is reached the relative weight of output stabilization remains constant and equal to a_0 .¹³ Overall, we have formulated the change of the relative weight for output and inflation by solely making it depending on output.¹⁴

Inference on the size and change of the weights for inflation and output in the objective function can be made from numerous recent empirical studies on central banks' interest rate reaction functions.¹⁵ Empirical research on the central bank's interest rate reaction function, the Taylor rule, for the U.S. and some European countries suggest the following range and variation of weights.

Table 1: Studies with One Regime Change*)

¹³In the numerical example below we set $a_0 = 0.1$ and $a_2 = 0.5$.

¹⁴In order to obtain an analytical tractable model we have refrained from assuming more complicated weighted functions, for example, we might have considered weighting functions depending on both the output as well as inflation gap. Although this appears to be more realistic as our empirical estimates suggest, we have refrained from modeling this more complicated case.

¹⁵Svensson (1997) and Clarida et al. (1999) show that the above intertemporal central bank objective function, with weight $w(x)$, can be transformed into an optimal central bank interest rate reaction function. There then the weight affects inversely the reaction coefficient of inflation gap. Thus, the relative increase in the weight of the inflation gap in the interest rate reaction function is equivalent to the decreasing weight for output in the intertemporal objective function. We discuss this problem in appendix 2.

Study	Time period	w_π	w_x	w_r
U.S. ¹⁾	1960.1-1979.4	0.81	0.25	-
U.S. ¹⁾	1987.1-1997.3	1.53	0.76	-
U.S. ²⁾	1961.1-1979.2	0.83	0.27	0.68
U.S. ²⁾	1979.3-1996.4	2.15	0.93	0.79
U.S. ³⁾	1960.1-1979.2	0.13	0.02	-0.17
U.S. ³⁾	1979.3-1995.1	0.44	0.01	-0.28

*) Estimates are given here for two sub-periods. Some of the estimates included also a term for interest rates smoothing, denoted by w_r . The study by Flaschel et al. (1999) refers to unemployment gap instead of the output gap where the natural rate is measured solely as average unemployment over the time period considered. The coefficient for interest rate smoothing is negative, since the estimate is undertaken with a first differenced interest rate. Both features of the estimate may explain the low coefficients for w_x . For ¹⁾ see Taylor (1999); for ²⁾ see Clarida et al. (1999); for ³⁾ see Flaschel et al. (2001).

Table 2: Studies with Two Regime Changes*)

Study	Time period	w_π	w_x	w_r
U.S.	1970.1-1979.1	0.74	0.12	-
U.S.	1979.1-1989.1	0.74	-0.66	-
U.S.	1989.2-1998.10	1.05	1.12	-
Germany	1970.1-1979.12	0.88	0.81	-
Germany	1979.12-1989.12	0.91	0.32	-
Germany	1989.12-1998.12	0.36	0.87	-
France	1970.1-1979.12	0.66	0.55	-
France	1979.12-1989.12	0.82	-0.80	-
France	1989.12-1998.12	0.99	1.26	-

*) Estimates are given here for three subperiods. Estimates¹⁶ are undertaken by the authors with monthly data; data are from Eurostat (2000). The weight w_x represents the coefficient on an employment gap. The negative sign for w_x indicates interest rate increases in spite of the negative employment gap. Even if a term of interest rate smoothing was included in the regressions the relative weight of the coefficients for inflation and employment approximately remained the same. For Germany: r is 3-month libor; π is the consumer price change. For U.S.A.: r is "Federal Funds Rate" which is used as in the article by Clarida et al. (1998); π is consumer price change. For France: r is the call money rate (since some data for libor are unavailable); π is consumer price change.

In line with our assumption above, the empirical studies, overwhelmingly reveal (with some minor exception) a higher weight for inflation than for output (or employment) gap

¹⁶ For a study on the central bank's interest rate reaction function in other OECD countries, see Clarida et al. (1998).

for both the studies with one regime change (pre- and post-Volcker periods), Table 1, and with two regime changes, Table 2. The studies with one regime change show that in the second period (the post-Volcker time period) the weight on inflation has increased. This, however, as Table 2 shows, has mainly occurred during the 1980's when most central banks have engineered a process of disinflation. On the other hand, as Table 1 also shows, when there was a secular rise in unemployment in Europe, the weight on the employment gap increased again (absolute and relative to the inflation gap). Note, that even in the U.S. the weight on the employment gap has increased again in the third period.

The study by Boivin (1998) undertaken for U.S. time series data estimates time varying weights on inflation and employment gap (with alternative measures for the NAIRU) Also here there are roughly three regimes visible. In a first regime, from 1973-1979 the weight on the employment gap is roughly 0.65 and on inflation 0.25. From 1979 to 1989 the weight switches for the employment gap from 0.65 to 0.2 and for the inflation gap from 0.25 to 0.5. In the last period, from 1988 to 1993 the weight for employment remains roughly unchanged and for inflation it increases to 0.6. We want to note that the Boivin study captures also indirectly the influence of a possible change in the slope of the Phillips-curve on the coefficients of the inflation gap and employment gap.

Overall, in summarizing the above results we can say that, first, in most of the studies, for the U.S. as well Europe, inflation stabilization has, most of the time, a higher weight than output stabilization. Second, the weights for the inflation and output (employment) gaps have undergone significant changes over time. Third, the weight for the output gap does not appear to solely depend on the output but also the inflation gap.¹⁷

The latter empirical fact might complicate our model. Yet, for our purpose it suffices to consider a simple model where the weights on output and price stabilization solely depend on the output gap. Employing this presumption on the central bank's interest

¹⁷Further evidence of state dependent weights in central banks' interest rate feedback rule is given in Semmler and Greiner (2001) where the Kalman filter is used to empirically estimate state dependent reaction functions of central banks in some OECD countries.

rate reaction function permits us to construct a welfare function with changing relative weight for inflation and output stabilization in the central bank's loss function. The exact relationship between the weights in the central bank's loss function and the central bank's interest rate reaction function is derived in appendix 2.

Figure 1 shows a numerical example, with the function $h_2(x)$ displayed in the upper panel given by the following assumed functional form $h_2(x) = -100 - 10(x - 50) + 3(x - 50)^2$. Note that we here assume certain functional forms in order to undertake numerical computations. The weighting function is shown in the middle panel of figure 1. The lower panel of figure 1, finally, gives the function $w(x) \cdot h_2(x)$, with a_1 set to $a_1 = 0.5$. This function displays two minima. The function $w(x)h_2(x)$ can be approximated by a polynomial of a higher degree which displays the same qualitative features¹⁸ as the function¹⁹ $w(x)h_2(x)$.

Figure 1: Central Bank's Welfare Function with State Dependent Weights

Thus, in order to obtain continuous function we choose an approximation given by a function such as $g(\cdot) = -10 - (x - 50) - 0.3(x - 50)^2 + 0.33(x - 50)^3 + 0.1(x - 50)^4$. This function is shown in figure 2.²⁰ These considerations demonstrate that our assumption of an endogenous weighting function – here solely depending on the output gap – may give rise to an objective function which can simply be described by a convex-concave-convex function.

Figure 2: Approximation of the Central Bank's Welfare Function

¹⁸Note that this approximation is undertaken solely for computational reasons.

¹⁹Note if we start from a representative household's preference as in Rotemberg and Woodford (1999) the change of the weight $w(\cdot)$ would be determined by a change of the structural parameters.

²⁰We do not need to attempt to find parameter values which give a more exact approximation of the function $w(x)h_2(x)$ since the basic message would remain unchanged.

We would also like to point out that a less conservative central bank, i.e. a central bank which puts an even higher weight on output stabilization, would not change the basic message described above. One could even take a situation where output stabilization and inflation control are equivalent goals if output levels exceed a certain threshold, but output stabilization becomes more important when output is lower than this threshold. One also could fix the weight on output stabilization equal to one and make the weight on inflation stabilization state dependent²¹ (dependent on the inflation rate).²² All of these options would not change our basic results .

3 Hysteresis Effects

Summarizing our discussions from section 2 and assuming an intertemporal perspective, the optimization problem of the monetary authority can be written as

$$\min_u \int_0^\infty e^{-\delta t} (h_1(\pi) + g(x)) dt, \quad (4)$$

subject to

$$\dot{x} = u, \quad (5)$$

with $\delta > 0$ the discount rate and $g(\cdot)$ a continuous function with continuous first and second derivatives. In particular, we assume that, in accordance with our considerations in section 2, $g(\cdot)$ is convex-concave-convex and satisfies in addition $\lim_{x \rightarrow -\infty} g(\cdot) = \lim_{x \rightarrow \infty} g(\cdot) = \infty$. As concerns the function $h_1(\pi)$ we take $h_1(\pi) = (\pi - \pi^*)^2$, with $\pi - \pi^*$ given by (2).

Candidates for optimal steady states and local solutions can be found either through the Hamiltonian or the HJB-equation. Here we derive some results using the Hamiltonian.

²¹Examples for such weighting functions are available upon request.

²²A nonlinear central bank interest rate feedback rule, with, however, the weight to output stabilization set to zero, can be found in Benhabib et al. (1998a,b). By adding the assumption of $\bar{\pi} > -r$, with $\bar{\pi}$ the steady state inflation rate and r the real interest rate, they also obtain multiple steady state equilibria.

Further details on the use of the Hamilton and the HJB-equation are given in appendix 1.

We apply the current-value Hamiltonian $H(\cdot)$

$$H(\cdot) = (g(x) + (\pi - \pi^*)^2) + \lambda u, \quad (6)$$

with λ the costate variable and $\pi - \pi^*$ determined by (2) respectively. The Maximum principle gives

$$u = -\frac{\lambda}{2\alpha^2} - \frac{\beta}{\alpha}(x - x_n) \quad (7)$$

and the costate variable evolves according to

$$\dot{\lambda} = \delta\lambda - g'(x) - 2\beta(\alpha u + \beta(x - x_n)). \quad (8)$$

Further, the limiting transversality condition

$$\lim_{t \rightarrow \infty} e^{-\delta t} \lambda x = 0 \quad (9)$$

must hold. Using the Maximum principle (7) the dynamics is completely described by the two-dimensional autonomous differential equation system

$$\dot{u} = \delta u + \frac{\beta}{\alpha} \left(\frac{\beta}{\alpha} + \delta \right) (x - x_n) + \frac{g'(x)}{2\alpha^2} \quad (10)$$

$$\dot{x} = u. \quad (11)$$

Rest points of this differential equation system yield equilibrium candidates for our economy. At equilibrium candidates, we have $\dot{x} = \dot{u} = 0$, implying $u = 0$ and x such that

$$\frac{\beta}{\alpha} \left(\frac{\beta}{\alpha} + \delta \right) (x - x_n) + \frac{g'(x)}{2\alpha^2} = 0 \quad (12)$$

holds. If $g(x)$ has a convex-concave-convex shape,²³ as argued in section 2, there may be three candidates for an equilibrium, which we denote as \hat{x}_1 , \hat{x}_2 and \hat{x}_3 , with $\hat{x}_1 < \hat{x}_2 < \hat{x}_3$.

²³The additional assumption $\lim_{x \rightarrow -\infty} g(\cdot) = \lim_{x \rightarrow \infty} g(\cdot) = \infty$ is sufficient for the existence of a solution to (12) with respect to x .

More concretely, since $g(x)$ is convex-concave-convex $g'(x)$ is concave-convex and the number of equilibria will depend on the linear term $(\beta/\alpha)((\beta/\alpha) + \delta)(x - x_n)$. If this term is not too large so that (12) is also concave-convex, multiple equilibria will exist. Below we will present a numerical example which illustrates this case. For now we will assume that this holds and derive results for our general model.

The local dynamics is described by the eigenvalues of the Jacobian matrix. The Jacobian matrix corresponding to this system is given by

$$J = \begin{pmatrix} \delta & (\beta/\alpha)(\delta + \beta/\alpha) + g''(x)/2\alpha^2 \\ 1 & 0 \end{pmatrix}.$$

The eigenvalues are obtained as

$$\mu_{1,2} = \frac{\delta}{2} \pm \sqrt{\left(\frac{\delta}{2}\right)^2 - \det J}. \quad (13)$$

The eigenvalues are symmetric around $\delta/2$ implying that the system always has at least one eigenvalue with a positive real part. For $\det J < 0$ the eigenvalues are real with one being positive and one negative. In case of $\det J > 0$ the eigenvalues are either real and both positive or complex conjugate with positive real parts. That is in the latter case the system is unstable.

Since (12) is concave-convex, the $\dot{u} = 0$ isocline, given by $u = -[(\beta/\alpha)(\delta + \beta/\alpha)(x - x_n) + g'(x)]/2\alpha^2/\delta$, is convex-concave in x . Consequently, $(\beta/\alpha)(\delta + \beta/\alpha) + g''(x)/2\alpha^2$, which is equal to $-\delta \times$ the slope of the $\dot{u} = 0$ isocline, is positive for $x = \hat{x}_1$ and $x = \hat{x}_3$ while it is negative for $x = \hat{x}_2$. This implies that \hat{x}_1 and \hat{x}_3 are saddle point stable while \hat{x}_2 is unstable. This outcome shows that there exists, in between \hat{x}_1 and \hat{x}_3 , a so-called Skiba point x_s (see Brock and Malliaris, 1989, or Dechert, 1984).

From an economic point of view the existence of a Skiba point has the following implication. If the initial level of production $x(0)$ is smaller than the Skiba point x_s , the monetary authority has to choose $u(0)$ such that the economy converges to \hat{x}_1 in order to minimize (4). If $x(0)$ is larger than x_s the optimal $u(0)$ is the one which makes the

economy converging to \hat{x}_3 . If x_0 is equal to x_s the optimal long-run aggregate output level is indeterminate, that is convergence to \hat{x}_1 yields the same value for (4) as convergence to \hat{x}_3 .

Given this property of our model, history dependence and hysteresis effects on the labor market can arise in the following way. Assume that the economy originally is in the high output equilibrium \hat{x}_3 . If the economy is struck by a shock reducing output below the Skiba point, it is optimal for the central bank to steer the economy towards the low output equilibrium \hat{x}_1 , which goes along with a lower inflation rate. It should be noted that this monetary policy is optimal and the hysteresis effect arises given complete information and the central bank's knowledge of the Skiba point. So, it must also be pointed out that in reality the central bank does probably not dispose of the necessary information to achieve a minimum. In this case, however, the emergence of hysteresis is not less likely. For example, the central bank could conduct a sub-optimal monetary policy and steer the economy to the low output equilibrium although convergence to the high output equilibrium would be optimal. The emergence of hysteresis effects is independent of the assumption that the central bank conducts an optimal policy. What is crucial for hysteresis is the shape of the function $g(x)$.

To illustrate these theoretical considerations and in order to gain additional insight, we resort to the function of our numerical example in section 2, that is $g(\cdot) = -10 - (x - 50) - 0.3(x - 50)^2 + 0.33(x - 50)^3 + 0.1(x - 50)^4$. α, β and δ are set to $\alpha = 0.09$, $\beta = 0.01$ and $\delta = 0.05$. x_n is assumed to be given by $x_n = 50$. With these parameters, candidates for optimal equilibria are $\hat{x}_1 = 47.3133$, $\hat{x}_2 = 49.1354$ and $\hat{x}_3 = 51.0763$. The eigenvalues are $\mu_1 = 1.687$ and $\mu_2 = -1.637$ corresponding to \hat{x}_1 , $\mu_{1,2} = 0.025 \pm 1.182\sqrt{-1}$ for \hat{x}_2 and $\mu_1 = 1.74$ and $\mu_2 = -1.69$ for \hat{x}_3 . Thus, \hat{x}_1 and \hat{x}_3 are saddle point stable while \hat{x}_2 is an unstable focus.

Figure 3 shows a qualitative picture of the phase diagram in the $x - u$ phase diagram where saddle points and the unstable focus are drawn.

To show that a Skiba-point exists we need two additional results. First, the minimum of (4) is given by $H^0(x(0), u(0))/\delta$, with $H^0(\cdot)$ denoting the minimized Hamiltonian. Second, the minimized Hamiltonian, $H^0(\cdot)$, is strictly concave in u and reaches its maximum along the $\dot{x} = 0 = u$ isocline. These results imply that for any $x(0)$ the minimizing $u(0)$ must lie on either the highest or lowest branch of the spirals converging to $\hat{x}_1 = 47.3133$ or to $\hat{x}_3 = 51.0763$ (because of the strict concavity of H^0 in u). For $x(0) = x_1$, we may set either $u(0) = 0$, leading to $\hat{x}_3 = 51.0763$, or $u(0) = u_1$, which implies convergence to $\hat{x}_1 = 47.3133$. Since the minimized Hamiltonian takes its maximum along the $\dot{x} = 0$ isocline, $u(0) = 0$ yields a maximum and cannot be optimal. Instead, $u(0) = u_1$ yields the minimum for (4).

Figure 3: Local Dynamics of the Equilibria

If $x(0) = x_2$ the same argument shows that setting $u(0) = 0$, leading to $\hat{x}_1 = 47.3133$, is not optimal. In this case, $u(0) = u_2$ yields the minimum. Therefore, if $x(0) \leq (\geq) x_1$ (x_2) convergence to $\hat{x}_1 = 47.3133$ ($\hat{x}_3 = 51.0763$) yields the minimum for (4). Consequently, the Skiba-point lies between x_1 and x_2 . To find the exact location for the Skiba-point we would have to calculate the value function on the stable branch of the saddle point, what, however, we will not undertake here.²⁴

It should be mentioned that the high equilibrium point \hat{x}_3 is expected to be lower than potential output which is about 51.7 in our example. This is due to the inclusion of the term $(\pi - \pi^*)^2$ in the central bank's objective function. Since the central bank wants to control inflation besides output it will not steer the economy towards potential aggregate output. Note, however, that in our example \hat{x}_3 is almost equal to potential output since we have chosen small values for α and β , implying that deviations from the potential output do not bring about strong inflationary pressure. Supposing that the core inflation

²⁴For a short description of the numerical method to find the Skiba point, see appendix 1.

π^* equals 3 percent, the actual inflation rate is about 4.1 percent for the high output equilibrium \hat{x}_3 . The low equilibrium output \hat{x}_1 goes along with an inflation rate of about 0.3 percent. The capacity utilization in this equilibrium is about 92.6 percent.²⁵

We should also point out that it is not optimal to steer the economy to the NAIRU corresponding to x_n . This outcome results from our assumption that deviations from the NAIRU are explicitly considered in the objective function. Therefore, a higher level of aggregate output, which gives an inflation rate above the desired level π^* , may be optimal. This holds because the negative effect of a higher inflation rate is compensated by the benefits of a higher level of aggregate output. A lower level of aggregate output, in our example \hat{x}_1 , is an optimal solution because it implies a lower inflation rate. If the NAIRU corresponding to the natural rate of output, x_n , coincides with a solution of $g(\cdot) = 0$, a steady state level of aggregate output can be achieved for which $\pi = \pi^*$ is optimal.

4 Conclusion

In this paper we have attempted to show that monetary policy may contribute to hysteresis effects on the labor market. Yet, we do not want to neglect the hysteresis effects that stem from the labor market itself. The studies of hysteresis effects in labor markets originates in the work by Blanchard and Summers (1986, 1988) and has recently revived in Blanchard and Katz (1997) and Stiglitz (1997). This research agenda attempts to explain the time variation of the natural rate due to large shocks.²⁶ The hysteresis hypothesis has been

²⁵The average capacity utilization rate of the West German economy was 96.75 percent on average from 1974-1996.

²⁶The hysteresis hypothesis has been applied to compare the time variation of the natural rate in the U.S. and Europe. Empirically it has been shown (Stiglitz 1997 and Gordon 1997) that in the U.S. the natural rate has moved from a high to a low level and, on the other hand, in Europe it has moved from a low to a high level. Each of those economies have experienced different levels of the natural rate over the last forty years. Yet, whether econometrically the persistence of unemployment is described best by a unit root process (hysteresis process) or by a mean reverting process, with changing mean, is still controversial, see Phelps and Zoega (1998).

given some further foundation by labor market search theory (Mortensen 1989, Howitt and McAfee 1992). The hysteresis theory states that with a large negative economic shock the unemployment rate becomes history dependent. With large unemployment the improvement in unemployment benefits may generate less competition on the supply side for labor, a wage aspiration effect (Stiglitz, 1997) from previous periods of higher employment keeps real wage increasing (possibly higher than productivity), long term unemployment may arise without pressure on the labor market and there may be loss of human capital, shortage of physical capital and a bias toward labor saving technologies (Blanchard 1998). Thus, the natural rate of unemployment may tend to move up or the natural rate of employment may tend to move down with large shocks.²⁷ This process has been assumed to have occurred in Europe.²⁸

In the paper we have demonstrated that under reasonable assumptions central banks may add to the hysteresis effect on the labor market if the central bank's objective function is non-quadratic. The objective function of the central bank will be non-quadratic if it exhibits state dependent weights for output or inflation stabilization. This may give rise to a convex-concave-convex shape of the function resulting in multiple optimal steady-state equilibria. In such a model there can be three candidates for steady-state equilibria but only two are optimal. There is history dependence since, the initial conditions crucially determine which equilibrium should be selected. 'Optimal' hysteresis effects on the labor market arise if an exogenous shock leads to a decrease in production such that convergence to the low-level equilibrium becomes optimal whereas convergence to the high-level

²⁷For the opposite view, namely that the currently higher European unemployment is a result of a moving natural rate, see Phelps and Zoega (1998). They associate the rise of the European natural rate with rising and high interest rates in Europe. Moreover, they argue that the hysteresis theory still lacks state variables such as wealth, capital stock or customer stock.

²⁸On the other hand, with a positive shock to employment and rising employment the hysteresis effect may work in the opposite direction. The long term unemployed come back to the labor market, increase competition, wage aspiration is low (compared to productivity), there is reskilling of human capital due to higher employment and product market competition keeps prices down (Stiglitz 1997, Rotemberg and Woodford 1996). This case, is usually associated with the recent U.S. experience, where the natural rate of unemployment has moved down or the natural rate of employment has moved up.

equilibrium may have been optimal before the shock. It must be underlined that, in this case, it is indeed optimal to realize the low-level equilibrium so that we may speak of ‘optimal’ hysteresis effects.²⁹

However, we should also point out that the central bank must be able to find out which equilibrium yields the optimum, a task which is definitely non-trivial in practice. Therefore, it is conceivable that the central bank chooses a non-optimal equilibrium and hysteresis effects may occur which turn out to be non-optimal. Again, imagine that an exogenous shock reduces output. If it is optimal to return to the high level equilibrium after the shock, but the central bank conducts a monetary policy implying convergence to the low-level equilibrium, welfare losses will result. In this situation, the higher output equilibrium, if achievable, would yield an increase in welfare.

Finally, we want to note that the pursuit of a proper policy will be made feasible by computing, as we have suggested here, the welfare function (the value of the objective function) outside the steady-state equilibria which will reveal the thresholds at which a change of the policy should occur.

²⁹This may be a scenario describing the situation in Europe with protracted period of high unemployment rate, see for example, Phelps and Zoega (1998) who have pointed to the high interest rate policy in Europe in the 1980’s and 1990’s as cause for protracted period of unemployment. Of course, as above discussed, labor market conditions presumably also have contributed to hysteresis effects.

4.1 Appendix 1: Methods to Solve Models with Non-Quadratic Welfare Function

Our central bank's welfare function can be described by a convex-concave-convex function which should have the same global properties as the quadratic loss function. We have presumed that the function $g(x)$ is a smooth function for which globally the following holds:

$$\lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow \infty} g(x) = \infty.$$

For large x the function $g(x)$ behaves the same way as the original quadratic penalty functions, yet locally there are two minima.

For the computation of the global value function and thus the thresholds, or Skiba points, we can employ two methods. For the case of our nonlinear welfare function, as studied in section 2, one can use, as shown above, Pontryagin's maximum principle and the associated Hamiltonian function and numerically compute exactly the location of the Skiba point, the threshold where the global dynamics converge to different domains of attraction. If, as in the example of section 3, two saddle points, \hat{x}_1, \hat{x}_3 and one unstable point \hat{x}_2 for the state and co-state variables arise, one can compute the threshold (Skiba point) by approximating the stable manifolds leading into the candidates for equilibria \hat{x}_1, \hat{x}_3 . This is done by solving boundary value problems for differential equations (for details, see Beyn, Pampel and Semmler (2000)). In addition for each initial condition on the line x one computes - by staying approximately on the stable manifold - the integral which represents the value of the objective function corresponding to the initial conditions.

Figure 4: Computation of the Value Function by the Hamiltonian Function

The connection of the integral points gives the two lines V_I and V_{II} . The intersection of the two value functions represents the threshold where the dynamics separate to the

low and high level equilibria (\hat{x}_1, \hat{x}_3) . For the details of such an procedure, see Beyn, Pampel and Semmler and Semmler and Greiner (1999).

Another method to compute the global dynamics and the thresholds can be derived from the HJB-equation³⁰

$$\delta V = \min_u [h(x, u) + V'(x)f(x, u)] \quad (\text{A1})$$

$$s.t. \dot{x} = f(x, u)$$

where $h(x, u)$ is the pay-off function. For our system (4)-(5) using (2) we can follow three steps to compute the solution.

First, we compute the candidates for equilibria. By using for $h(x, u) = h_1(\pi) + g(x)$

$$h_u(x, u) + h_x(x, u) = 0 \quad (\text{A2})$$

For our system (4) and (5) we obtain

$$- [2\alpha^2 u + 2\alpha\beta(x - x_n)] \cdot \delta + [g'(x) - 2\alpha\beta u - \beta^2 2(x - x_n)]$$

For $u = 0$ we get

$$-2\alpha\beta(x - x_n)\delta + g'(x) - \beta^2 2(x - x_n) = 0$$

$$- (2\alpha\beta\delta + \beta^2 2)(x - x_n) + g'(x) = 0 \quad (\text{A3})$$

Note that the equ. (A3) is the same as obtained from the Hamiltonian, equ. (12).

Thus, the candidates for equilibria will be the same.

³⁰For details, see Semmler and Sieveking (1999). For a growth model with resources exhibiting multiple equilibria, see Semmler and Sieveking (2000).

Second, we compute from (A1) the differential equation $V'(x)$ which gives us local solutions.

We obtain

$$u^* = \frac{V'(x) - 2\alpha\beta y}{2\alpha^2} \quad (\text{A4})$$

substituting (A4) into (A1) and solving for $V'(x)$ we obtain:

$$\begin{aligned} V'(\cdot) &= 2(\alpha\beta(x - x_n) \pm \sqrt{-\alpha^2 b_0 + \alpha^2 \delta V(\cdot) - \alpha^2 b_1 x + \alpha^2 \beta^2 x^2 - \alpha^2 b_2 x^2 - \alpha^2 b_3 x^3 - \alpha^2 b_4 x^4 - 2\alpha^2 \beta^2 x_n (x - x_n)}) \\ &= 2(\alpha\beta(x - x_n) \pm \sqrt{\alpha^2 \delta V(x) - g'(x) \cdot \alpha^2 + \alpha^2 \beta^2 x^2 - 2\alpha^2 \beta^2 (x - x_n)x_n}) \end{aligned} \quad (\text{A5})$$

Furthermore, we solve the differential equation (A5) forward and backward with the initial condition

$$V(\hat{x}_i) = \frac{1}{\delta} h(\hat{x}_i) \quad (\text{A6})$$

Finally, we compute

$$V(x) = \underset{i}{Max} V_i$$

where $V(x)$, the value function, is the outer envelop of all our piecewise solutions generated by (A5) with initial conditions (A6). Figure 5 illustrates the procedure for a numerical example.

Figure 5: Computation of the value function by the HJB-equation

Details of the numerical example can be found in Semmler and Sieveking (1999). The broken line with the arrows represent the piecewise solutions starting with the initial conditions \hat{x}_1, \hat{x}_3 . The point of intersection, s , is the Skiba-point which represents a threshold for the global dynamics. In those models large shocks produce history dependence. For details of the employed functions and parameters, see Semmler and Sieveking (1999).

4.2 Appendix 2: Deriving the Optimal Taylor Rule

Here we will discuss through what instruments the central bank may be able to steer $u = \dot{x}$. We follow Svensson (1997) and derive an interest rate reaction function of the central bank by using an IS-equation as in Woodford (1999). The interest rate reaction function will resemble the Taylor rule but is not equivalent to it. Note that in our case there is an optimal Taylor rule which is implicitly given through an IS equation. Our procedure is different from Christiano and Gust (1999) and Rotemberg and Woodford (1999) who postulate different variants of the Taylor rule and explore their effects on the household's welfare at a steady state.

Note that our model allows for inflation and output stabilization but the central bank, in the presence of multiplicity of equilibria, may settle at any steady state for employment which might, however, be different from the natural rate. Yet not all of the steady states may be optimal.

We want to show that the implicit Taylor rule is consistent with our proposed monetary control problem (4)-(5). If we call $u^* = \varphi(x)$, either given by equ. (7) or by (A4) of appendix 2, and assume additionally to the Phillips-curve (2) an IS-equation, such as employed in Woodford (1999), then we can write³¹

$$\varphi(x) = \gamma_1 \left(r_0 - (i - \bar{\pi}) \right) + \gamma_2 y \quad (\text{A7})$$

where r_0 is the real rate of return on capital and $\bar{\pi}$ is an expected inflation rate, for example given by the core inflation π^* as discussed in section 2 (or given by $\bar{\pi}$ as either a forward looking variable or a weighted average of lagged inflation rates, as in Rudebush and Svensson 1999). The monetary authority controls u through $\varphi(x)$ but uses the interest rate, i , as monetary instrument. Note that if the central bank targets the core inflation rate (assumed to be given in the vicinity of the steady state) it targets the nominal

³¹We leave aside the time index, since all variables refer to the same time period.

interest rate, since the nominal interest rate minus the inflation rate should be equal to the real return on capital at a steady state. This should hold, at least, if we assume no capital market imperfections.

We then can write

$$(i - \bar{\pi}) = r_0 + \beta_1 y - \beta_2 \varphi(x) \quad (\text{A8})$$

where $\beta_1 = \gamma_2/\gamma_1$ and $\beta_2 = 1/\gamma_1$. Moreover, we know that the optimal control gives us $u^* = \varphi(x)$.

From (A4) we obtain for the optimal u :

$$u^* = \frac{g'(x) - 2\beta^2(x - x_n) - 2\delta\alpha^2\beta(x - x_n)}{2\delta\alpha^2 + 2\alpha\beta} \quad (\text{A9})$$

Substituting (A9) into (A8) we get

$$i_t = r_0 + \bar{\pi} + \beta_3(x - x_n) - \frac{1 - \gamma_1\alpha}{\gamma_1} \frac{g'(x)}{2\alpha(\beta + \delta\alpha)} \quad (\text{A10})$$

with³²

$$\beta_3 = \beta + \beta_1 + \frac{\beta(\beta + \delta\alpha^2)\beta_2(1 - \gamma_1\alpha)}{\alpha(\beta + \delta\alpha)}.$$

Equ. (A10) gives the implicit (optimal) interest rate reaction function of the central bank for the monetary control problem (2) - (5) given the optimal policy $\varphi(x)$ in feedback form from the state equation and the IS-equation (A7). Note that (A10) implies that at the steady state the nominal interest rate responds to the long-run real return on capital and the core inflation rate, both determining, in equilibrium, the long-run nominal interest rate.

³²Note that close to a steady state $\bar{\pi}$ in (A10) corresponds to the steady state value of inflation rate implied by the policy rule. In the steady state (or either of the steady state) the nominal interest rate must equal the sum of the equilibrium steady state real interest rate, r_0 and steady state inflation rate. Thus, if r_0 is independent of the monetary policy rule the monetary authority's choice of $\bar{\pi}$ implies a value of the nominal interest rate.

It should be noted that, given that $g(x) = w(x)h_2(x)$, an exogenous increase in the weighting function, i.e. $w(x)$ rises for all x while $w'(x)$ does not change, also implies a lower interest rate provided that $1 - \gamma_1\alpha > 0$ holds which, however, is not too strict an assumption. This holds because then the r.h.s. in (A10) becomes smaller implying that i decreases. The optimal interest rate in (A10) is also affected by the deviation of actual employment from the NAIRU. The optimal interest rate is the higher the higher the deviation of actual output from the NAIRU provided $\beta_3 > 0$ holds which is automatically fulfilled for $1 - \gamma_1\alpha > 0$.

Figures

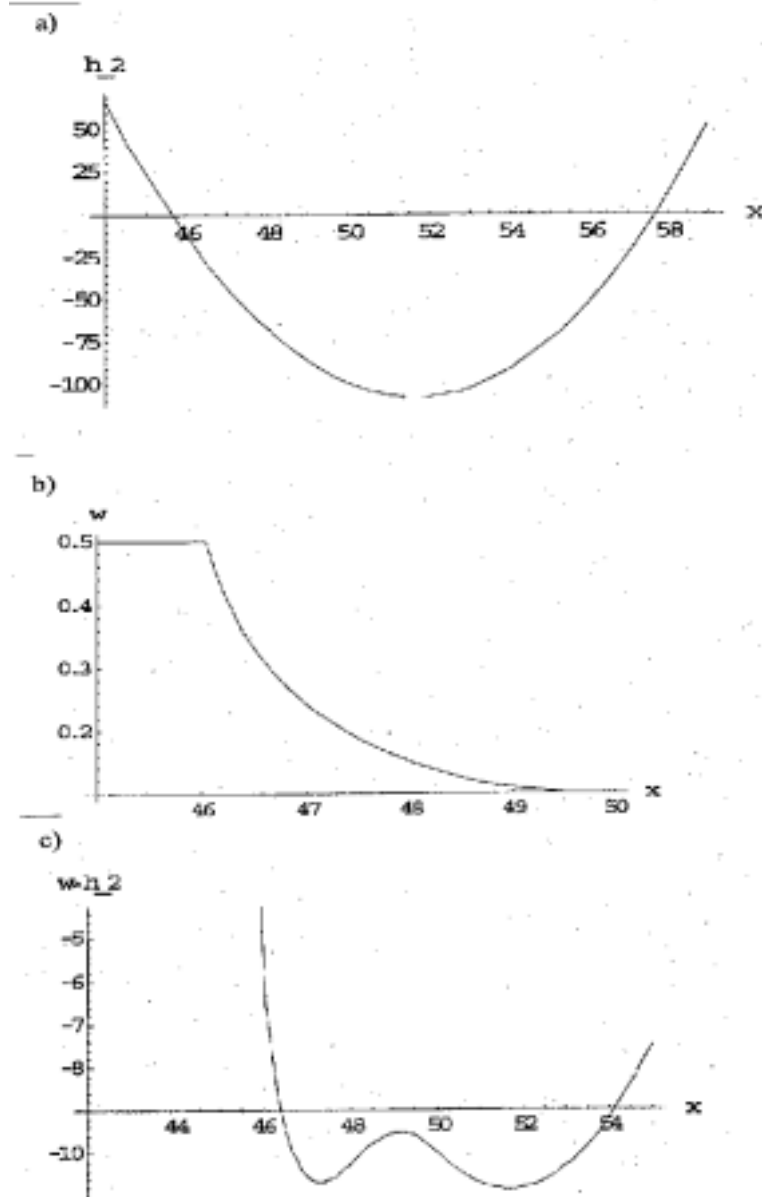


Figure 1

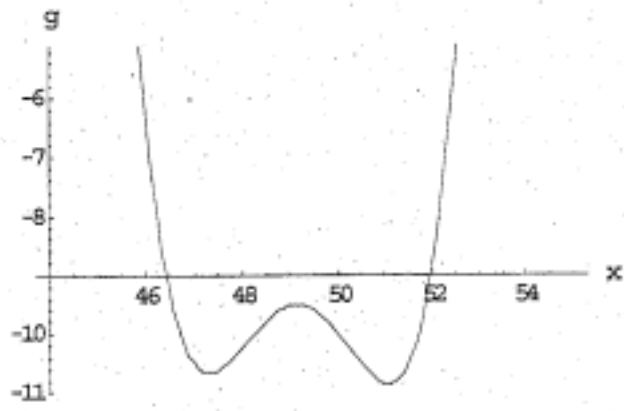


Figure 2

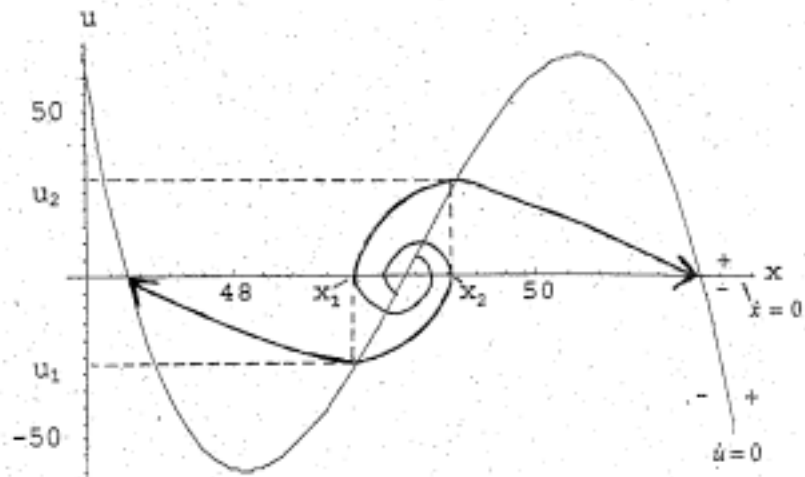


Figure 3

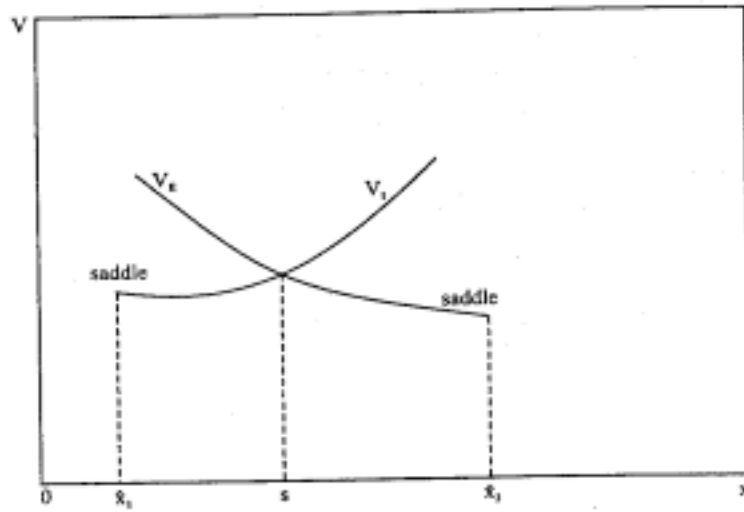


Figure 4

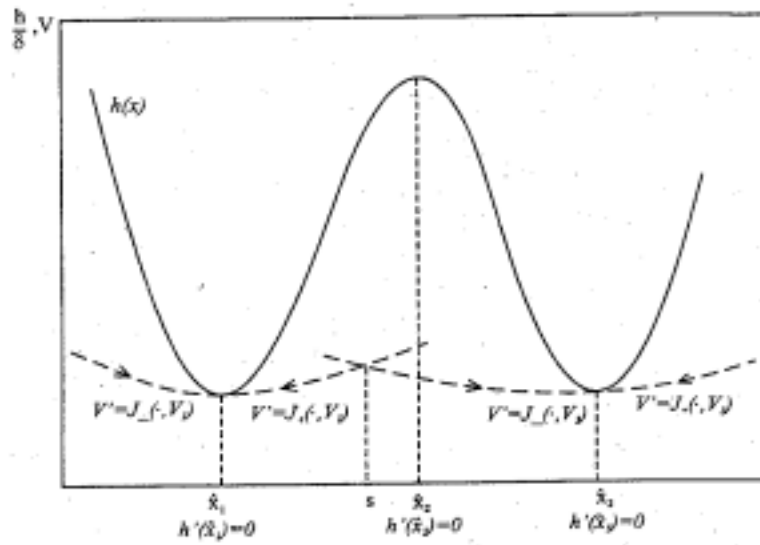


Figure 5

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