

Segmented Labor Markets, Distributive Cycles and an Employer of Last Resort

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Abstract

The paper builds on the baseline Goodwin (1967) model which describes the reserve army mechanism of capitalist economies. We add to this model segmented labor markets as described in Marx's Capital, Vol.I. The model exhibits a unique steady state solution which depends on the speeds with which workers are pushed into or out of the labor market segments. We investigate the stability properties of this Goodwin model with segmented labor markets and find that, though there is a stabilizing inflation barrier term in our Phillips curve formulation, interaction with the latent and stagnant portions of the labor market generates potentially destabilizing forces. We then introduce an active labor market policy where government acts as employer-of-last-resort thereby eliminating the stagnant portion of the labor market, whilst erecting a benefit system that sustains the incomes of workers that leave the floating labor market into the latent one. We show that such policies guarantee the macro-stability of the economy's growth path.

Keywords: Reserve army, distributive cycles, segmented labor markets, (in-)stability, employer of last resort

JEL CLASSIFICATION SYSTEM: E32, E64, H11.

1 Segmented labor markets in Marx's and our times

'The folly is now patent of the economic wisdom that preaches to the labourers the accommodation of their number to the requirements of capital. The mechanism of capitalist production and accumulation constantly effects this adjustment. The first word of this adaptation is the creation of a relative surplus-population, or industrial reserve army. Its last word is the misery of constantly extending strata of the active army of labor, and the dead weight of pauperism.' Marx (1954, p.603).

In this paper we consider the Marxian trichotomy of floating, latent and stagnant segments of the labor market. This trichotomy is outlined in Capital I, ch. 25.4, where the last segment - the stagnant one - can also be described as a dead segment. The significance and relevance of this trichotomy can be easily observed in both advanced and developing countries. In relation to the German economy of the 21st century, we have normal occupations (the floating segment). The latent segment is made up of atypical, low-income employment, which is constituted by part-time-occupied workers, temporary work organized by special leasing firms and less paid, so-called mini-jobs with no minimal subsistence wage. The dead segment is made up of unemployed persons receiving unemployment benefits for one year, elderly people for 18-24 months, and long-time unemployed persons with only marginal chances of a return to proper work.

In 2009, there were 43 million persons in the active part of the German labor force, but nearly 5 million persons had only a minijob (Federal Statistical Office 2011). According to the Federal Institute for Employment, in 2010 there were 3 244 000 unemployed persons. Altogether, around 6 million employable persons received social security benefits in 2010, the so-called Hartz IV people. In addition to this, there were 1.5 million persons involved in a measure of labor market policy, mainly skill training. Of the long-term unemployed, more than 20 % have been unemployed for more than 2 years (Bundesagentur für Arbeit 2010), but a significant number of people who rely on social security have been in this system for more than 5 years. The persons who suffer from long-term unemployment often do not have sufficient qualifications. They are often linked to so-called "problem-groups", which include migrants without any professional or even educational qualifications, single parent households, unskilled persons in general, and early school leavers.

In the case of South Africa, with an estimated population of 50 million people, the Labor Force Survey for the fourth quarter of 2010 shows that people who have been unemployed for more than 1 year constitute 68% of the unemployed. The unemployment rate is estimated to be 25%, based on the narrow definition which excludes discouraged work-seekers. Including discouraged work-seekers the unemployment rate rose to 36% in 2010. It is estimated that at least 60% of the unemployed have less than secondary education. Of the unemployed, 43% are new entrants into the labor market. Between 2003 and 2008 an estimated 400 000 young people per annum could not proceed with their studies towards higher education. Recent statistics further shows that 30% of the South African population rely on social assistance, which mainly takes the form of child support grants and old age pensions. The headcount poverty rate is estimated to be 48% by the Presidential Report (2009) and atypical forms of employment have been on the rise. The UN Human Development Report (2010) notes that 44% of South African workers earn less than \$1.25 a day.

Therefore, it is admissible to compare Hartz IV with the stagnant (dead) segment of South Africa as described above. Of course, their situation is not comparable to the situation of the dead segment on the labor market of Marx's time, but in a cross-sectional comparison they are nevertheless in a comparable position concerning life-perspectives, neighborhood problems, tendencies to drug consumption and inclination towards violence. The same can be said about those discouraged work-seekers who are not covered by any form of social assistance in South Africa.

Moreover, there are certainly many difficult atypical working conditions in Germany and South Africa that can be related to the Marxian latent segment, though these workers have a chance to move up into the floating segment like skilled and well trained unemployed persons who may find a new job after some months. On the other hand, there are many persons in this segment who can also easily drop into the stagnant segment when, for example, part-time-occupied workers loose their job, or temporary work organized by special leasing firms is so little paid that the comparison with regular workers is discriminating. These types of workers are indeed also supported through the Hartz IV program to a significant degree. In the case of South Africa, there is no social security support for these low-paid workers, let alone the long-term unemployed.

In this paper we will - among other questions - also deal with proposed solutions to solve the unemployment and poverty problems such as Basic Income Guarantees (BIG) or an Employer of Last Resort (ELR). The importance of such programs cannot be underestimated, since the increase in child poverty that is accompanying mass unemployment is a ticking time bomb. In Germany, approximately 10 % of workers in the new classification system "Hartz IV families", are by and large chronically unemployed with no hope for improvement of their lot. This situation can prove fatal to a democracy. This is even more dangerous in the case of South Africa, where social security support is extremely limited and the distribution of income has worsened in a new democracy. Against such a background, this paper provides a model of Marx's segmented labor market analysis and a reform proposal that tries to cure this situation.

Goodwin's (1967) Marxian growth cycle model is one of the truly baseline models of macroeconomic theory, comparable to the orthodox Solow (1956) model in its simplicity, but totally different in its implications from the latter type of growth theory. This has indeed also been acknowledged by Solow (1990) himself and has led to numerous publications on modifications and extensions of this approach to the distributive cycle, as Taylor (2004, Ch.9) and Barbosa-Filho and Taylor (2006) have characterized this cycle mechanism. Recently, in 2006, there has been a special issue in the journal 'Structural Change and Economic Dynamics' on Goodwin's legacy and its continuation as well as an edited volume on this subject, see Flaschel and Landesmann (2008). There has also been recent empirical work on this distributive cycle by Harvie (2000), Mohun and Veneziani (2008), Franke et al. (2006) and others. This indicates that the model of the reserve army mechanism designed by Goodwin (1967, 1972) on the basis of Marx analysis of this mechanism is still attracting numerous studies of its further development and its empirical evaluation.

From this perspective, this paper extends the growth cycle of Goodwin (1967) to incorporate the various forms of unemployment first pointed out by Marx (1954) in Capital I, Chapter 25: the floating, latent and stagnant or dead segments of the labor market. In Goodwin's model, only the floating type of unemployment is considered. We extend Goodwin's model by postulating an

interaction between the three labor market segments and show that, in the presence of a benefit system that is undertaken by government as "employer-of-last-resort", the dead segment of the labor market can be assumed as eliminated, whilst preserving the macro-stability of the economy. To our knowledge, the first study of an unemployment benefit system in the context of the Goodwin growth cycle model was provided by Glombowski and Krüger (1984).

The paper shows that Goodwin's model, which is in a way characterized by the unrestricted operation of the "law of capitalist accumulation", can be reformulated in such a way as to produce socio-economic outcomes that are socially and politically acceptable. We should however mention that in our formulation of reformed capitalism that is presented here, the role of changes in the composition of capital remains to be explored. In his detailed exposition of the operation of capitalist accumulation, Marx places great emphasis on the rise in the organic composition of capital as a principal force behind the increase in structural unemployment. We do not pursue this matter in this paper and save it for further elaboration in subsequent research. Our main focus is to integrate the three segments of the labor market under the assumption of a given capital intensity (constant labor productivity) and show, on this basis, that active labor market policy can generate an outcome that allows us to eliminate the stagnant portion of unemployment.

The rest of the paper is structured as follows. Section 2 adds to the Goodwin model a segmented labor market structure, in which the different types of unemployment interact on the basis of rates of employment and unemployment. Section 3 provides a steady state analysis of the model, and shows that the steady state rates of employment in the latent and dead segments depend on the speeds with which workers are pushed into or out of these segments. Section 4 investigates the stability properties of the extended model. We find that adding the latent and dead portions of the labor market in this model generates potentially destabilizing forces, though there is a stabilizing inflation barrier term in our Phillips curve formulation. Section 5 introduces an active labor market policy where government acts as employer-of-last-resort thereby eliminating the stagnant portion of the labor market, whilst erecting a benefit system that sustains the incomes of workers that leave the floating labor market into the latent one. We show that this policy guarantees the macro-stability of the economy's growth path. Section 6 concludes.

2 Segmented labor and the distributive cycle

The model of the distributive cycle that is considered in Flaschel and Greiner (2009) by and large describes a viable situation for a capitalist economy. In that model, it was shown that the introduction of a benefit system, minimum and maximum real wage rules that are facilitated by a social consensus between labor and capital significantly improves economic performance. However, there is one serious neglect in such a scenario. Mass unemployment occurs without any social consequences for the household structure of the working class. In Marx's description of the reserve army mechanism this is taken note of and it is even claimed there that the distributive cycle will necessarily imply a hierarchy of three segments in the labor market which are unavoidable under an unrestricted evolution of the capitalist mode of production. In this paper, as distinct from Flaschel and Greiner (2009), we provide a model in which these unemployment hierarchies interact.

We consider the Marxian growth cycle model as it was formulated by Goodwin (1967) but add a consideration of the evolution of latent and stagnant portions of the labor markets. The growth cycle dynamics for the floating labor market can then be formulated (if the other segments of the labor market are still ignored):

$$\hat{\omega} = \beta_w \left(\frac{\bar{y}/\bar{z}}{l^s} - \bar{e} \right), \quad l^s = \frac{L^s}{K} \quad (1)$$

$$\hat{l}^s = n - \bar{y}(1 - \omega/\bar{z}), \quad \omega = w/p \quad (2)$$

where \bar{z} is a constant output-labor ratio and \bar{y} is a constant output capital ratio. Adding the other two segments of the labor market we assume now for the floating part of it (indexed by 1) as law of motion for their real wage:

$$\hat{\omega}_1 = \beta_{we_1} \left(\frac{\bar{y}/\bar{z}_1}{l_1^s} - \bar{e}_1 \right) + \beta_{we_2} e_2 - \beta_{wd} \frac{D}{L^s} + \beta_{w\omega_1} (\omega_1^o - \omega_1) \quad (3)$$

$$\hat{l}^s = n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2), \quad (4)$$

The real wage Phillips curve in the first labor market remains based on demand pressure term in the first labor market $\frac{\bar{y}/\bar{z}_1}{l_1^s} - \bar{e}_1$, and is now augmented by the positive influence of the second labor market through an increasing rate of employment $e_2 = \frac{\bar{y}/\bar{z}_2}{l_2^s}$ in the atypical (second) labor market and by a negative influence from the third (dead) segment of the labor market where there is no employment at all. The extent of this dead segment is measured by D/L^s and is related to what Marx's considers as pauperism in his chapter 25 section 4.

The law of motion for labor intensity is the same as before, but now refers to the whole of labor supply per unit of capital. This is driven by the rate of profit, where the given real wage per unit of capital has now to be deducted. We assume in this paper that the real wage in the latent segment of the labor market is a given subsistence wage, while there are no wages paid at all in the sphere of pauperism. Note that the given magnitudes \bar{z}_1, \bar{z}_2 of output per unit of employed labor have now to be interpreted in inverted form as employment coefficients since they are used here to calculate employment on the two active labor markets on the basis of a given output-capital ratio \bar{y} .

Note also that we have, by definition, the identity $L^s = L_1^s + L_2^s + D$ where total labor supply grows at the natural rate n . The split of this labor supply into a floating, a latent and a dead segment must now be formulated in order to complete the model. We further assume that there are upward and downward movements between the floating and the latent segments of the labor market. We denote the floating and latent segments as type 1 and type 2 employment respectively, as indexed above. The unemployment rate in the floating segment is an indicator of the percentage of type 1 workers that are compelled to move into the latent segment and the employment rate of the latent segment is an indicator of the percentage of people who get the chance to move back into the first labor market. This gives rise to the following laws of motion:

$$\hat{L}_2^s = \gamma_1^d(1 - e_1) - \gamma_1^u e_1 + n \quad (5)$$

$$\hat{D} = \gamma_2^d(1 - e_2) - \gamma_2^u e_2 + n \quad (6)$$

We have already added here a similar law of motion for the movement in and out of the dead segment of the labor market which therefore also assumes that there are ways to leave the sphere of pauperism.

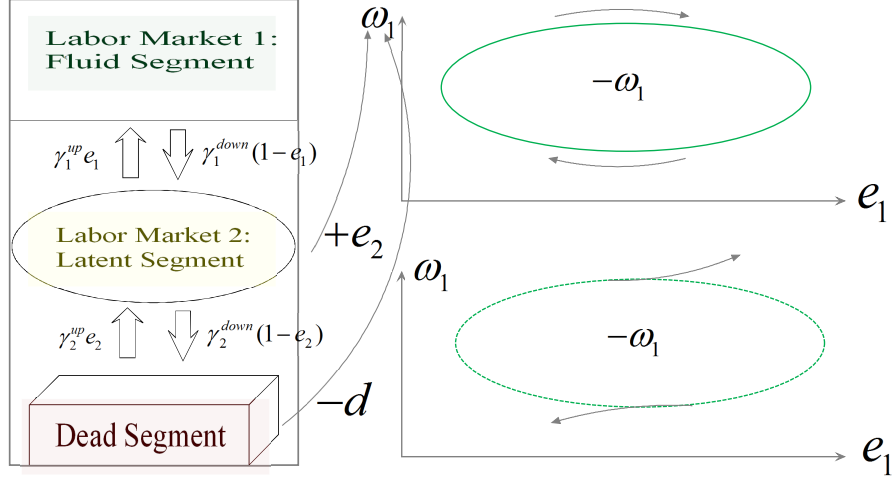


Figure 1: The flow of workers between the segments of the labor market.

Yet the downward leading coefficients γ_1^d, γ_2^d will be significantly larger than the upward leading ones γ_1^u, γ_2^u . This is briefly exemplified for the first labor market as follows. In the steady state we will have

$$\gamma_1^d(1 - e_1) - \gamma_1^u e_1 = 0, \quad i.e., \quad e_1 = \frac{\gamma_1^d}{\gamma_1^d + \gamma_1^u} = \frac{1}{1 + \gamma_1^u/\gamma_1^d}$$

This suggests that for plausible values of e_1 , say values greater than 50%, the parameter γ_1^u must be significantly less than γ_2^d . Taken together we have as laws of motion for this economy with three labor market segments the differential equations (where everything is expressed per unit of capital and denoted in lowercase letters):

$$\hat{\omega}_1 = \beta_{we} \left(\frac{\bar{y}/\bar{z}_1}{l_1^s} - \bar{e}_1 \right) + \beta_{we2} \frac{\bar{y}/\bar{z}_2}{l_2^s} - \beta_{wd} d/l^s + \beta_{w\omega_1} (\omega_1^o - \omega_1) \quad (7)$$

$$\hat{l}^s = n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (8)$$

$$\hat{l}_2^s = \gamma_1^d - (\gamma_1^d + \gamma_1^u) \frac{\bar{y}/\bar{z}_1}{l_1^s} + n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (9)$$

$$\hat{d} = \gamma_2^d - (\gamma_2^d + \gamma_2^u) \frac{\bar{y}/\bar{z}_2}{l_2^s} + n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (10)$$

where the statically endogenous variable l_1^s is given by $l_1^s = l^s - l_2^s - d$. This represents a description of the Marxian reserve army mechanism with the three segments of the labor market he assumed as typical for its working under the capitalism of his time.

We summarize the structure of the considered economy by way of figure 1. The figure 1 shows on its left hand side the flows occurring between the segments of the labor market which are therefore not completely separated from each other, but segmented to a degree that is mirrored through the size γ parameters. The figure top right shows an example of a Goodwin type distributive cycle which will be modified to a convergent dynamics if the real wage barrier term is added to it, here simply visualized by the $-\omega_1$ -expression in the center of it, the real wage of the workers in the fluid part of the labor market. The arrows in the middle indicate the forces that impact the fluid labor market

because of the presence of the other two labor markets, namely the state of employment in the latent part of the labor market as measures by the employment rate e_1 and the size of the dead segment of the labor market, d , here measured relative to the size of the capital stock. The size of the first variable has a positive impact on the wage claims made in the fluid labor market while the size of the second one has a negative effect on the wage negotiations. The analysis of the model in subsequent section will show that these feedbacks arising from the lower labor markets onto the dynamics in the first one will add destabilizing forces to the distributive cycle generated in the fluid part of the labor market, as indicated in the figure bottom right.

3 Steady state analysis

In the steady state, where time derivatives are zero, the system can be expressed as follows:

$$0 = \beta_{we_1} \left(\frac{\bar{y}/\bar{z}_1}{l_1^s} - \bar{e}_1 \right) + \beta_{we_2} \frac{\bar{y}/\bar{z}_2}{l_2^s} - \beta_{wd} d/l^s + \beta_{w\omega_1} (\omega_1^o - \omega_1) \quad (11)$$

$$0 = n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (12)$$

$$0 = \gamma_1^d - (\gamma_1^d + \gamma_1^u) \frac{\bar{y}/\bar{z}_1}{l_1^s} + n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (13)$$

$$0 = \gamma_2^d - (\gamma_2^d + \gamma_2^u) \frac{\bar{y}/\bar{z}_2}{l_2^s} + n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (14)$$

which gives for the determination of the steady state the equations

$$0 = \beta_{we} \left(\frac{\bar{y}/\bar{z}_1}{l_1^s} - \bar{e}_1 \right) + \beta_{we_2} \frac{\bar{y}/\bar{z}_2}{l_2^s} - \beta_{wd} d/l^s \quad (15)$$

$$0 = n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \rightarrow \omega_1^o = \frac{\bar{y}(1 - \bar{\omega}_2/\bar{z}_2) - n}{\bar{y}\bar{z}_1} > 0 \quad (16)$$

$$0 = \gamma_1^d - (\gamma_1^d + \gamma_1^u) \frac{\bar{y}/\bar{z}_1}{l_1^s} \rightarrow l_1^{so} \quad (17)$$

$$0 = \gamma_2^d - (\gamma_2^d + \gamma_2^u) \frac{\bar{y}/\bar{z}_2}{l_2^s} \rightarrow l_2^{so} \quad (18)$$

Substituting the steady state expressions for e_1 and e_2 into these equations gives the following system:

$$0 = \beta_{we_1} \left(\frac{\gamma_1^d}{\gamma_1^d + \gamma_1^u} - \bar{e}_1 \right) + \beta_{we_2} \frac{\gamma_2^d}{\gamma_2^d + \gamma_2^u} - \beta_{wd} d^o/l^{so} \quad (19)$$

$$0 = n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \rightarrow \omega_1^o > 0 \quad (20)$$

$$0 = \gamma_1^d - (\gamma_1^d + \gamma_1^u) \frac{\bar{y}/\bar{z}_1}{l_1^s} \rightarrow l_1^{so} = \frac{\gamma_1^d + \gamma_1^u}{\gamma_1^d} \frac{\bar{y}}{\bar{z}_1} = l^{so} - l_2^{so} - d^o > 0 \quad (21)$$

$$0 = \gamma_2^d - (\gamma_2^d + \gamma_2^u) \frac{\bar{y}/\bar{z}_2}{l_2^s} \rightarrow l_2^{so} = \frac{\gamma_2^d + \gamma_2^u}{\gamma_2^d} \frac{\bar{y}}{\bar{z}_2} > 0 \quad (22)$$

This further gives, for the determination of the steady state values of d, l^s , the two equations:

$$d^o = [\beta_{we1}(\frac{\gamma_1^d}{\gamma_1^d + \gamma_1^u} - \bar{e}_1) + \beta_{we2} \frac{\gamma_2^d}{\gamma_2^d + \gamma_2^u}] l^{so} / \beta_{wd} \quad (23)$$

$$d^o = l^{so} - [\frac{\gamma_1^d + \gamma_1^u}{\gamma_1^d} \frac{\bar{y}}{\bar{z}_1} + \frac{\gamma_2^d + \gamma_2^u}{\gamma_2^d} \frac{\bar{y}}{\bar{z}_2}] \quad (24)$$

We assume that the term in the square bracket in eq. (23) provides a value between zero and one so that this linear curve is positively sloped and flatter than the other straight line, given by eq. (24). It is then obvious that these two equations have a positive intersection and this solution automatically fulfills $d^o/l^{so} < 1$, i.e., $D/L^s < 1$ as it should be the case. We note that the slope of this curve depends on the speeds of adjustment in the first labor market, in the form of ratios with β_{wd} in the denominator. The adjustment parameters β_{we1} and β_{we2} must therefore be sufficiently small relative to β_{wd} in order to allow for a positive steady state solution. In other words, a system where the influence of β_{wd} onto the fluid labor market is too weak is therefore problematic even from the steady state point of view, independently of the stability features of this steady state.

Note also that the steady state rates of employment in the first and the second labor market are given by

$$e_1^o = \frac{\gamma_1^d}{\gamma_1^d + \gamma_1^u}, \quad e_2^o = \frac{\gamma_2^d}{\gamma_2^d + \gamma_2^u}$$

The rate e_x^o is thus different from what is usually considered as the NAIRU employment rate \bar{e}_x and it increases with γ_x^d , the speed by which workers are pushed into a lower segment of the labor market, and it decreases with γ_x^u , the speed by which workers can climb up again into the higher segment of the labor market. Though there is social segmentation through this labor market structure, it is nevertheless possible to overcome the barriers created by the workings of unrestricted capitalism to a certain degree. We summarize the steady state analysis by way of what is shown in figure 2. Top left we show the feedback structure of the dynamics as it is represented by the signs of the entries in the Jacobian matrix of the model at the steady state. The slopes of the two isoclines that determine the steady state values l^{so}, d^o are such that indeed positive values for these two ratios exist and are uniquely determined. We, for example, get from the figure 2 that increasing output to factor employment ratios z_i^1 will reduce the dead segment of the labor market on average – or if the model is convergent – in the long-run. This is a very astonishing result, since the impact effect of an increase in the z_i^1 s is to lower labor demand in the corresponding markets.

4 Stability of balanced growth?

In this section we investigate the stability properties of the Marxian growth cycle model with the segmented labor markets that we have formulated above. We consider again the 4D autonomous

¹here no longer really measures of labor productivity in this partial form, since the model does not allow for isolated changes in the employment of the factors of production.

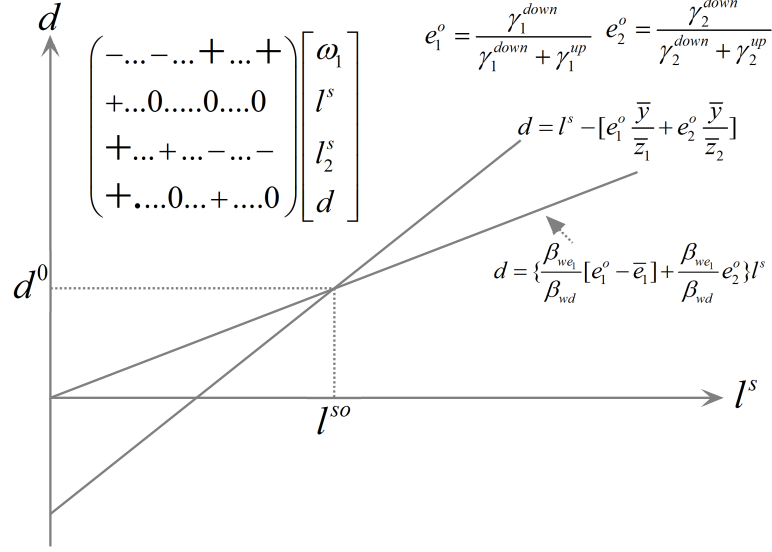


Figure 2: Steady state employment rates and the determination of l^{so} , d^o .

system given by:

$$\hat{\omega}_1 = \beta_{we1} \left(\frac{\bar{y}/\bar{z}_1}{l^s - l_2^s - d} - \bar{e}_1 \right) + \beta_{we2} \frac{\bar{y}/\bar{z}_2}{l_2^s} - \beta_{wd} d/l^s + \beta_{w\omega_1} (\omega_1^o - \omega_1) \quad (25)$$

$$\hat{l}^s = n - \bar{y} (1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (26)$$

$$\hat{l}_2^s = \gamma_1^d - (\gamma_1^d + \gamma_1^u) \frac{\bar{y}/\bar{z}_1}{l^s - l_2^s - d} + n - \bar{y} (1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (27)$$

$$\hat{d} = \gamma_2^d - (\gamma_2^d + \gamma_2^u) \frac{\bar{y}/\bar{z}_2}{l_2^s} + n - \bar{y} (1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (28)$$

Since linear dependent expressions cancel in the calculation of the determinant of the Jacobian of this system at the steady position, we can simplify the right hand side of these equations as follows, without changing the sign of the determinant of this Jacobian:

$$\hat{\omega}_1 = \beta_{we1} \left(\frac{\bar{y}/\bar{z}_1}{l^s - l_2^s - d} - \bar{e}_1 \right) + \beta_{we2} \frac{\bar{y}/\bar{z}_2}{l_2^s} - \beta_{wd} d/l^s \quad (29)$$

$$\hat{l}^s = \omega_1 \quad (30)$$

$$\hat{l}_2^s = -(\gamma_1^d + \gamma_1^u) \frac{\bar{y}/\bar{z}_1}{l^s - l_2^s - d} \quad (31)$$

$$\hat{d} = -(\gamma_2^d + \gamma_2^u) \frac{\bar{y}/\bar{z}_2}{l_2^s} \quad (32)$$

After further reductions we arrive at the Jacobian of this system, which reads:

$$J_o = \begin{pmatrix} 0 & d/(l^s)^2 & 0 & -1/l^s \\ 1 & 0 & 0 & 0 \\ 0 & 1/(l^s - d)^2 & 0 & -1/(l^s - d)^2 \\ 0 & 0 & 1/(l^s)^2 & 0 \end{pmatrix}$$

This gives for the determinant of this Jacobian the expression:

$$J_o = (-d/l^s + 1)/[(l^s)^3(l^s - d)^2] = (1 - d/l^s)/[(l^s)^3(l^s - d)^2] > 0$$

since $d < l^s$ holds at the steady state.

With respect to the other stability conditions we have to consider the sign distribution within the Jacobian of the dynamical system:

$$\hat{\omega}_1 = \beta_{we1} \left(\frac{\bar{y}/\bar{z}_1}{l^s - l_2^s - d} - \bar{e}_1 \right) + \beta_{we2} \frac{\bar{y}/\bar{z}_2}{l_2^s} - \beta_{wd} d/l^s + \beta_{w\omega_1} (\omega_1^o - \omega_1) \quad (33)$$

$$\hat{l}^s = n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (34)$$

$$\hat{l}_2^s = \gamma_1^d - (\gamma_1^d + \gamma_1^u) \frac{\bar{y}/\bar{z}_1}{l^s - l_2^s - d} + n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (35)$$

$$\hat{d} = \gamma_2^d - (\gamma_2^d + \gamma_2^u) \frac{\bar{y}/\bar{z}_2}{l_2^s} + n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (36)$$

which is given by

$$J_o = \begin{pmatrix} - & ? & ? & ? \\ + & 0 & 0 & 0 \\ + & + & - & - \\ + & 0 & + & 0 \end{pmatrix}$$

If the first term in the Phillips curve is dominating the second and the third one with respect to the state variable l^s, l_2^s, d^2 so that the floating part dynamics of the model is in particular of the type of a Goodwin cycle $J_{12} < 0$ (with damped oscillations however) we in particular get³

$$J_o = \begin{pmatrix} - & - & + & + \\ + & 0 & 0 & 0 \\ + & + & - & - \\ + & 0 & + & 0 \end{pmatrix}$$

In addition to the stability result obtained above we see further stabilizing feedback channels at work. In the Goodwin subdynamics, i.e., the interaction of the state variables ω_1, l^s , we have again the result of section 2.3 of the distributive cycle without segmented labor markets. And in the lower two segments we have also a stable, partial interaction of the state variables l_2^s, d (see the 2D submatrix at the bottom to the right).

There are however also destabilizing feedback chains at work now. There is first the cumulative interaction of d, ω_1 in the laws of motion of real wages and of the dead segment of the labor market.

²such that the slope of the $\dot{\omega}_1$ -isocline remains smaller than 1 (otherwise the model is per se not a viable one, see the discussion of its balanced growth path).

³In order to get this result the conditions

$$\beta_{we1} \frac{\bar{z}_2}{\bar{z}_1} \left(\frac{l_2^s}{l_1^s} \right)^2 > \beta_{we2}, \quad \beta_{we1} \frac{\bar{y}}{\bar{z}_1} \left(\frac{l^s}{l_1^s} \right)^2, \beta_{we1} \frac{\bar{y}}{\bar{z}_1 l^s} \left(\frac{l^s}{l_1^s} \right)^2 > \beta_{wd}$$

must hold. However, other cases with other stability properties are possible and thus make the outcome of this model of unrestricted capitalism somewhat ambiguous.

And secondly, there is the cumulative interaction between the state variables l_2^s, ω_1 in the laws of motion for real wages and the latent segment of the labor market. These positive feedback loops are represented in the Jacobian matrix shown in figure 2.2 by the enlarged plus signs.

Of course all these statements are made from a partial perspective concerning the principal minors of order 2 of the Jacobian J solely. There are no destabilizing adjustment processes in the trace of J . We however conclude from these observations that the stable Goodwin growth cycle within the floating element of the labor market is plagued by some positive feedback chains caused by the existence of the latent and the dead part of the labor market and their interaction with the real wage dynamics in particular. Should these feedback chains make the overall dynamics unstable this will occur by way of a Hopf bifurcation, through the death or birth of unstable or stable limit cycles, respectively, since the determinant of the system cannot change its sign, i.e., the roots of the Jacobian can only enter the positive part of the complex plane of complex numbers by becoming complex at the bifurcation point. The loss of stability therefore necessarily occurs in the presence of business fluctuations.

5 Active labor market policy and an employer of last resort

We have so far considered an economic system with three labor markets, a fluid one, a latent one and a dead one, where people are only casually doing some work, but primarily live on charity of the other members of the workforce.⁴ We have assumed for simplicity that people in the ‘dead’ segment of the labor market live on charity received from the other members of the working class, redistributing part of the latter’s consumption demand. Moreover, this segment is not totally a ‘dead’ one, since the parameter γ_2^u provides the extent that they can return to the latent segment of the labor market, depending on the rate of employment in this market. Workers employed in this second, latent portion of the labor market receive a given (minimum) real wage, while the fluid labor market works in the way it is assumed to work in the context of models of the distributive cycle. We have abstracted in the extension of the Goodwin model to the present situation from the social legislation, and thus assume again that the unemployed in the two active labor markets are living on the basis of income of larger families they are belonging to as it is for example often the case in Spain and its larger family structures.

Yet, in the present section we return to an unemployment benefit system in the two active labor markets of the model and assume that this addition makes the secure in the downward direction, i.e., the massive generations of a totally degraded workforce is thereby no longer possible. Of course, there may exist disabled people of various kinds, but this is not a problem a macro-model has to deal with so that we now can simply assume that $\gamma_2^d = 0$ is established through a social network for the unemployed. Moreover we are assuming an active labor market policy, attempting to moderate the operation of the Marxian growth cycle within the context of a dual structure of segmented labor

⁴In the extreme such people have been characterized in Marx ‘The Eighteenth Brumaire of Louis Napoleon (1852)’ as “Lumpenproletariat”, the “refuse of all classes”, including “swindlers, confidence tricksters, brothel-keepers, rag-and-bone merchants, beggars, and other flotsam of society”. We view the third segment of the labor market – when increasing – as being on the way to such a social structure, at Marx’s times and today related to alcoholism, drug dealing and consuming, youth gang formation and racism.

markets, here in particular through an activation program that tries to counteract the flow of workers dismissed from the first labor market into the latent one, which now consider more of the type of a low income sector, representing ‘atypical employment’. Last, but not least, we postulate that the public sector can demand services from the unemployed (who are all receiving unemployment benefits). The organization of these (social) services of course demands microeconomic coordination and incentives.

We thus assume now the existence of a public Employer of Last Resort who organizes the sector of those workers who are not employed by capitalist firms. The classic, but not the best example for such a task are military services, for instance in the USA, which may be supplemented by the alternative of civilian service and the like. The public sector thus now takes care of a better working of the fluid labor market, tries to reduce the extent of second one, the low income sector, as much as possible and administers the funds (received from the employed) for the unemployed for in combination of which the government also conducts a system of social services supplied by the ‘unemployed’ (including skill preservation processes).

In order to do this, we start again from the 4D model of unrestricted capitalism:

$$\hat{\omega}_1 = \beta_{we_1}(e_1 - \bar{e}_1) + \beta_{we_2}(e_2 - \bar{e}_2) - \beta_{wd}d/l^s + \beta_{w\omega_1}(\omega_1^o - \omega_1) \quad (37)$$

$$\hat{l}^s = n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (38)$$

$$\hat{l}_2^s = \gamma_1^d - (\gamma_1^d + \gamma_1^u)e_1 + n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (39)$$

$$\hat{d} = \gamma_2^d - (\gamma_2^d + \gamma_2^u)e_2 + n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (40)$$

With the introduction of the Employer of Last Resort the dead segment of the labor market will be eliminated by assumption, and the latent segment is augmented by a social security supplemented by an activating labor market policy. This implies that $d/l^s = 0$ and by definition $\hat{D} = 0$. Based on this, we now modify the laws of motion for real wages and the labor markets of the economy as follows. Note that we have added an active labor market policy term to the third law of motion, the role of which is investigated later on.

$$\hat{\omega}_1 = \beta_{we_1}(e_1 - \bar{e}_1) + \beta_{we_2}(e_2 - \bar{e}_2) + \beta_{w\omega_1}(\omega_1^o - \omega_1) \quad (41)$$

$$\hat{l}^s = n - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) \quad (42)$$

$$\hat{l}_2^s = \gamma_1^d - (\gamma_1^d + \gamma_1^u)e_1 - \bar{y}(1 - \omega_1/\bar{z}_1 - \bar{\omega}_2/\bar{z}_2) - \phi(\bar{e}_2 - e_2) \quad (43)$$

with

$$e_1 = \frac{\bar{y}/\bar{z}_1}{l^s - l_2^s}, \quad e_2 = \frac{\bar{y}/\bar{z}_2}{l_2^s}, \quad \bar{e}_1 = \frac{\gamma_1^d}{\gamma_1^d + \gamma_1^u}, \quad \bar{e}_2 = \frac{\gamma_2^d}{\gamma_2^d + \gamma_2^u}$$

$$\phi = \gamma_2^d + \gamma_2^u.$$

By the choice of the benchmark levels of the employment rates we have adjusted the steady state of the model to the one of the model of unrestricted capitalism. Moreover we assume that the size of the γ value is in an adequate range, i.e. they are assumed to be manipulated by policy such that the normal rates of employment are around 95 percent as in say, advanced capitalist economies.

The elimination of the dead segment is achieved by a policy that erects an unemployment benefit system with reserves B as follows:

$$\dot{B} = \tau_b \omega_1 L_1^d - \bar{\omega}_2 (L_1^s - L_1^d) - \alpha \bar{\omega}_2 (L_2^s - L_2^d) \quad (44)$$

$$= \tau_b \omega_1 e_1 L_1^s - \bar{\omega}_2 (1 - e_1) L_1^s - \alpha \bar{\omega}_2 (1 - e_2) L_2^s \quad (45)$$

Workers employed in the first labor market are now taxed with rate τ_b in order to create inflows into the reserves of an unemployment benefit system. The outflow goes to unemployed workers of type 1 who receive the wage of the employed workers of type 2 as unemployment benefits. Unemployed workers of type 2 receive only a portion α of this wage however, yet they have income now and any decline into a thereby reopened dead segment of the labor market is prevented by an activating labor market policy. This policy at least guarantees skill preservation for workers of type 1 and 2, by employing the $(1 - e_1)L_1^s + (1 - e_2)L_2^s$ workers as an Employer of Last Resort in public institutions, which provide social services and more, yet work that is not in competition with the activities occurring in the private sector of the economy. This amount of unemployment therefore is kept intact as suppliers of work of at least type 2. Moreover, active labor market policy also demands that government or private institutions attempt to channel back workers of type 2 into the first labor market. This gives rise to the $-\phi(\bar{e}_2 - e_2)$ expression in the law of motion that increases or decreases the extent of the second labor market. A labor market reform along these lines thus eliminates the existence of a dead segment on the labor market and increases the flow back into the first labor market. Nevertheless, we still consider this as very basic reforms of the labor market institutions. Note that all workers still consume all of their income so that the redistribution of income between the employed and the unemployed does not question the validity of Say's law in such a Goodwin-type economy. Moreover we have for the variable $b = B/K$, the benefit funds per unit of capital, the law of motion

$$\dot{b} = \frac{\dot{B}}{K} - b\hat{K} \quad (46)$$

$$= (\tau_b \omega_1 e_1 l_1^s - \bar{\omega}_2 (1 - e_1) l_1^s - \alpha \bar{\omega}_2 (1 - e_2) l_2^s) - \bar{y} (1 - \omega_1 / \bar{z}_1 - \bar{\omega}_2 / \bar{z}_2) b \quad (47)$$

with $e_1 = \frac{\bar{y}/\bar{z}_1}{l_1^s}$, $e_2 = \frac{\bar{y}/\bar{z}_2}{l_2^s}$. In the steady state this gives

$$\dot{b} = \tau_b \omega_1^o \frac{\gamma_1^d}{\gamma_1^d + \gamma_1^u} l_1^{so} - \bar{\omega}_2 \frac{\gamma_1^u}{\gamma_1^d + \gamma_1^u} l_1^{so} - \alpha \bar{\omega}_2 \frac{\gamma_2^u}{\gamma_2^d + \gamma_2^u} l_2^{so} - nb_o = 0$$

which when solved for the tax rate gives

$$\tau_b = \frac{\bar{\omega}_2 \gamma_1^u}{\omega_1^o \gamma_1^d} + \frac{\alpha \bar{\omega}_2 \frac{\gamma_2^u}{\gamma_2^d + \gamma_2^u} l_2^{so} + nb_o}{\frac{\gamma_1^d}{\gamma_1^d + \gamma_1^u} l_1^{so}} = \frac{\bar{\omega}_2 \gamma_1^u}{\omega_1^o \gamma_1^d} + \frac{\alpha \bar{\omega}_2 \frac{\gamma_2^u}{\gamma_2^d} \frac{\bar{y}}{\bar{z}_2} + nb_o}{\frac{\bar{y}}{\bar{z}_1}} \quad (48)$$

$$= \frac{\bar{\omega}_2 \gamma_1^u}{\omega_1^o \gamma_1^d} + \alpha \bar{\omega}_2 \frac{\gamma_2^u}{\gamma_2^d} \frac{\bar{z}_1}{\bar{z}_2} + \frac{nb_o \bar{z}_1}{\bar{y}} \quad (49)$$

This expression shows that the parameters of the model have to be determined with care such that the wage income after taxes of households of type 1 is of an appropriate size. Decreasing movements from the second into the first labor market decreases the tax rate of the employed workforce of type 1 as does an increase in the downward direction. There is therefore a variety of ways (indeed still

more) by which the tax rate on the income of those working in the first segment of the labor market can be reduced.

It is a bit surprising to get the result that an increasing value of α improves the income situation of the employed households of type 2, as does the real wage of workers employed in the second labor market as far as the tax rate is concerned. It lowers the value of ω_1^o however. Decreasing the downward movement of unemployed workers in the second labor market has also the side effect that workers of type 2 get more jobs in this market, since it is plausible to assume that workers of type 1 who enter the second labor market will have better chances to get a job there and will crowd out the type 2 workers who are looking for a job in their labor market.

For the Jacobian of the dynamics of our reform of capitalism towards one with an active and social labor market policy we get the sign distribution

$$J_o = \begin{pmatrix} - & - & - \\ + & 0 & 0 \\ + & + & - \end{pmatrix}$$

if we assume that adjustment term $\beta_{we_2}(e_2 - \bar{e}_2)$ is sufficiently strong. We then have that the coefficients $a_i, i = 1, \dots, 3$ in the Routh Hurwitz polynomial are all positive (a positive J_{13} that is sufficiently small is already sufficient for this). In fact the determinant of J can be shown to be negative under mild conditions, so that only one minor of order two is creating a positive feedback loop which can destabilize the economy. The final Routh-Hurwitz stability condition is $a_1 a_2 - a_3 > 0$ and this condition can be fulfilled if the parameter ϕ is chosen sufficiently large, since its appearance in the determinant cancels against one of its appearances in the term $a_1 a_2$.

The considered system can therefore be stabilized in an obvious way. Yet, these sufficient conditions are not exploiting all the parameter combinations where the balanced growth path of the model with an active labor market policy is attracting.

Figure 3 summarizes what we have discussed above and shows in particular the more robust and stable feedback structure of the ELR variant of the model of this paper. The case of Germany after WWII provides an example where things went wrong due to a lack of cooperating corporatism between capital and labor primarily in the question of income distribution, though the concept of a social market economy was a success story in the prosperity phase after WWII. It was shown in Flaschel and Greiner (2009) that adding minimum real wages as well as maximum ones to the distributive cycle can reduce its amplitude significantly both in the prosperity phase and the depressed phase. The reserve army of unemployed thus thereby is reduced in the depression, and the social degradation of part of the workforce avoided. But in the prosperity phase, unions in Germany did not think in terms of Marx's reserve army mechanism and did not consider ceilings to their real wage claims. Chancellor Willy Brandt supported indirectly this behavior when he proclaimed that full employment would now be maintained forever. But Marx's overshooting income claims mechanism worked in this context (leading first to stagflation and later on to stagnation without inflation). By contrast, minimum real wage legislation was not taken seriously after the Iron Curtain came down, neither by the social democrats who under Chancellor Schröder implemented the Hartz I – IV reforms, nor under Chancellor Merkel where the discussion about minimum wages was only conducted from a

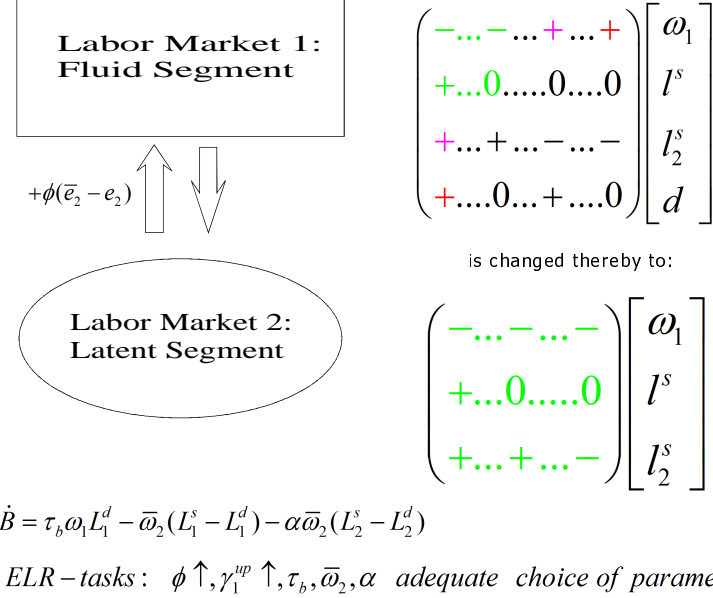


Figure 3: The Marxian distributive cycle with an Employer of Last Resort.

very microeconomic perspective. The result of such policies was the establishment of a progressively increasing low-income or part-time labor market segment and from there the flow of workers into Hartz VI which can by and large be considered a dead segment from the perspective of the social standards of the fluid segment of the labor markets in Germany.

These policies opened watergates on the labor market into a downward direction and contributed significantly to a return of a labor market structure as investigated already by Marx (1954) in *Capital*, Vol.I and modeled in this paper. Lacking insights into the Marxian reserve army mechanism on both sides of the conflict about income distribution (concerning agreements on both maximum and minimum real wages) as well as on both sides of the political spectrum in Germany (concerning resistance to processes of social degradation within the workforce) have now led in Germany to a situation where processes of social segmentation are difficult to overcome (even if policy would be willing to act accordingly).

The 1960's and early 1970's (where the Marxian insight into the working of capitalism and the reserve army mechanism was totally neglected) can thus be considered as a time of lost chances, since maximum and minimum real wages are easier to negotiate and implement by law in prosperity phases. To a certain degree the consequences of this failure was the disintegration of the concepts that constituted the German way to a 'Social Market Economy' into the direction of low income work and widespread poverty and its social consequences.

6 Conclusion

In this paper, we started from a baseline version of the Goodwin (1967) model of the distributive cycle which describes the implications of the reserve army mechanism of capitalist economies. We have added to this model segmented labor markets as described in Marx's Capital, Vol.I. The models exhibited a unique steady state solution which depends on the speeds with which workers are pushed into or out of the labor market segments. We investigated the stability properties of this model and found that, though there was a stabilizing inflation barrier term in our wage Phillips curve, the interaction between the latent and the dead portions of the labor market generated potentially destabilizing forces. We then introduced an active labor market policy where government acts as employer-of-last-resort thereby eliminating the stagnant portion of the labor market, whilst erecting an unemployment benefit system that sustains the incomes of workers that leave the floating labor market into the latent one. We showed that this policy guarantees the macro-stability of the economy's growth path. However, the affordability of such a structure needs a certain level of real wages and thus should be embedded into a model where there is growth of labor productivity. In such a system, where full employment is guaranteed, concerns about inflationary pressures should be taken into account. We have therefore proposed a wage Phillips curve where elements of cautiousness are incorporated in a simple way, suggesting that more reflection is needed in order to design a wage management system that avoids strong inflationary pressure in the boom and that eschews the danger of deflation during recessions.

We conclude from the above that a reformed type of capitalism is working much better compared to the unrestricted one, where labor market segmentation can present big economic and social problems. These problems can range from loss of social cohesion to social conflict and political instability.

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