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by

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Abstract

In this paper we present an endogenous growth model with productive government spending and pollution. As to pollution we suppose that it is a by-product of aggregate production and that it negatively affects utility of the household but not production possibilities directly. Analyzing our model it is first shown that for reasonable parameter values there exists a unique balanced growth path which is both locally and globally determinate. Second, growth effects of fiscal policy along the balanced growth path are analyzed and conditions for positive effects of policy measures are derived.

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1 Introduction

Recently, economists have begun to build models with an endogenously determined growth rate and pollution in order to understand the links between economic growth and pollution. Basically, two different ways of modelling can be distinguished. On the one hand, there are models in which economic activity gives rise to pollution. That line of research goes back to Forster (1973) and was extended by Gruver (1976).

On the other hand, there are models in which a stock of natural resources can be exploited which are used for production. An analogous formulation, from the technical point of view, is the assumption of economic activities which cause pollution which, for its part, negatively affects the environmental quality. Examples of that type of research within an endogenous growth model are the papers by Bovenberg and Smulders (1995) or Gradus and Smulders (1993)[1].

However, most of those models, when analyzing growth effects, assume that pollution or the use of resources influences prodution possibilities either through affecting the accumulation of human capital or by directly entering the production function. In this paper we intend to analyze a growth model where pollution only affects utility of a representative household but does not directly affect production possibilities. But there is an indirect effect of pollution by determining in a way the abatement activities which require resources. As to pollution we assume that it is an inevitable by-product of prodution and can be reduced by abatement activities but not completely. Concerning the growth rate we suppose that it is determined endogenously and that public investment in a productive public capital stock brings about sustained long-run per-capita growth. Thus, we adopt that type of endogenous growth models which was initiated by Barro (1990) and Futagami et.al. (1993).

The goal of our research then is to study how the balanced growth rate is affected by a shift of preferences towards a less polluted environment or by the use of a less polluting

[1]For a good survey of how to model pollution in growth models see Smulders (1995).

production technology. Further, we also intend to analyze growth effects of variations in tax rates, like a capital income tax rate or pollution tax rate.

The rest of the paper is organized as follows. In section 2 we present the basic model. In section 3 we solve the model and demonstrate that under a slight technical assumption there exists a unique balanced growth path (BGP) which is a saddle path. Section 4 gives growth effects of fiscal policy for our model on the BGP and section 5 concludes the paper.

2 The Model

We consider an economy which comprises three sectors: the household sector, a productive sector, and the government.

2.1 The Household

Our economy consists of many identical households which can be represented by one representative household or individual. The goal of this houshold is to maximize its discounted stream of utility arising from consumption $C(t)$ subject to its budget constraint:

$$J(\cdot) \equiv \max_{C(t)} \int_0^\infty e^{-\rho t} V(t)^{1-\sigma}/(1-\sigma) dt, \tag{1}$$

with $V(t)$ the instantaneous subutility function which depends positively on the level of consumption and negatively on effective pollution. $V(t)$ takes the Cobb-Douglas form $V(t) = (C(t) P_E(t)^{-\xi})$, where $\xi > 0$ gives the disutility arising from effective pollution[2]. $1/\sigma > 0$ gives the intertemporal elasticity of substitution of private consumption between two points in time for a given level of effective pollution and ρ in (1) is the subjective discount rate.

[2]For a survey of how to incorporate pollution in the utility function see Smulders (1995), p. 328-29.

The budget constraint is given by[3]

$$\dot{K} + C = w(1 - \tau_w) + rK(1 - \tau_K). \tag{2}$$

The budget constraint (2) states that the individual has, as usual, to decide how much to consume and how much to save, thus increasing consumption possibilities in the future. The depreciation of physical capital is assumed to equal zero.

w in the budget constraint is the wage rate and $\tau_w \in (0, 1)$ is the tax on labour income. The labour supply L is constant, supplied inelastically, and we normalize $L \equiv 1$. r is the return to per-capita capital K and $\tau_K \in (0, 1)$ stands for the capital income tax rate.

Assuming that a solution to (1) subject to (2) exists[4] we can use the current-value Hamiltonian to describe that solution. The Hamiltonian function is written as $\mathcal{H}(\cdot) = (CP_E^{-\xi})^{1-\sigma}/(1-\sigma) + \lambda(-C + w(1-\tau_w) + rK(1-\tau_K))$, with λ the costate variable. The necessary optimality conditions are given by

$$\lambda = C^{-\sigma} P_E^{-\xi(1-\sigma)}, \tag{3}$$

$$\dot{\lambda}/\lambda = \rho - r(1 - \tau_K), \tag{4}$$

$$\dot{K} = -C + w(1 - \tau_w) + rK(1 - \tau_K). \tag{5}$$

Since the Hamiltonian is concave in C and K jointly, the necessary conditions are also sufficient if in addition the transversality condition at infinity $\lim_{t\to\infty} e^{-\rho t}\lambda(t)(K(t) - K^{\star}(t)) \geq 0$ is fulfilled with $K^{\star}(t)$ denoting the optimal value. Moreover, strict concavity in C also guarantees that the solution is unique (cf. Seierstad and Sydsaeter (1987), pp. 234-235).

[3] In what follows we will suppress the time argument if no ambiguity arises.

[4] A formal proof can be obtained by adopting the proof of proposition 1 in Greiner and Semmler (1996).

2.2 The Productive Sector

The productive sector in our economy can be represented by one firm which chooses inputs in order to maximize profits and which behaves competitively. As to pollution, we suppose that it is the result of aggregate production. In particular, we assume that pollution $P(t)$ is a by-product of per-capita output $Y(t)$, i.e. $P(t) = \varphi Y(t)$, with $\varphi = const. > 0$. Thus, we follow the line invited by Forster (1973) and worked out in more details by Luptacik and Schubert (1982).

Pollution is taxed at the rate $\tau_p \in (0,1)$ and the firms take into account that one unit of output causes φ units of pollution for which they have to pay $\tau_p \varphi$ per unit of output. The per-capita production function is given by,

$$Y = K^{\alpha} H^{1-\alpha}, \tag{6}$$

with H denoting the stock of productive public capital and $\alpha \in (0,1)$ gives the per-capita capital share. Recall that K denotes per-capita capital and that L is normalized to one.

Assuming competitive markets and taking public capital as given the first-order conditions for a profit maximum are obtained as

$$w = (1-\tau_p\varphi)(1-\alpha)K^{\alpha}H^{1-\alpha}, \tag{7}$$

$$r = (1-\tau_p\varphi)\alpha K^{\alpha-1}H^{1-\alpha}. \tag{8}$$

2.3 The Government

The government in our economy uses resources for abatement activities $A(t)$ which reduce total pollution. Abatement activities $A \geq 0$ are financed by the tax revenue coming from the tax on pollution, i.e. $A(t) = \eta \tau_p P(t)$, with $\eta > 0$. If $\eta < 1$ not all of the pollution tax revenue is used for abatement activities and the remaining part is used for public investment in the public capital stock I_p, $I_p \geq 0$, in addition to the tax revenue resulting from the taxation of labour and capital income. For $\eta > 1$ a certain part of the tax revenue resulting from the taxation of labour and capital income is used for abatement activities in addition to the tax revenue which is gained by taxing pollution.

However, pollution cannot be eliminated completely. We call that part of pollution which remains in spite of abatement activites the effective pollution $P_E(t)$. In particular, we follow Gradus and Smulders (1993) and Lighthart and van der Ploeg (1994) and take the following specification

$$P_E = \frac{P}{A^\beta} = P^{1-\beta}\eta^{-\beta}\tau_p^{-\beta},\ 0 < \beta \leq 1. \tag{9}$$

The limitation $\beta \leq 1$ assures that a positive growth rate of aggregate production goes along with an increase in effective pollution, $\beta < 1$, or leaves effective pollution unchanged, $\beta = 1$. We make that assumption because we think that it is realistic to assume that higher production also leads to an increase in pollution, although at a lower rate because of abatement. Looking at the world economy that assumption is certainly justified.

Moreover, the government in our economy runs a balanced budget at any moment in time. Thus, the budget constraint of the government is written as

$$I_p = \tau_p P(1-\eta) + w\tau_w + rK\tau_K. \tag{10}$$

The evolution of public capital is described by

$$\dot{H} = I_p, \tag{11}$$

where for simplicity we again assume that the depreciation of public capital is zero. As to the governmental decision rules, we do not try to find out the second best optimal level for the tax rates or the amount of abatement activities nor the socially optimal decisions for consumption and the fiscal parameters. Instead, we only consider how the growth rate reacts to changes in fiscal policy. This seems to be of more relevance for real world economies with a democratic government because government behaviour may be hampered by bureaucracy and by political or institutional constraints (as to this argumentation see also van Ewijk and van de Klundert (1993)).

3 Equilibrium Conditions

Combining the budget constraint of the government and the equation describing the evolution of public capital over time, the accumulation of public capital can be written as $\dot{H} = -\eta\varphi\tau_p K^\alpha H^{1-\alpha} + \tau_p\varphi K^\alpha H^{1-\alpha} + \tau_w w + rK\tau_K = K^\alpha H^{1-\alpha}(\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)\tau_w(1-\alpha) + (1-\varphi\tau_p)\tau_K\alpha)$, where we have used (7) and (8). To obtain the other differential equations describing our economy we note that the growth rate of private consumption is obtained from (3) and (4) as $\dot{C}/C = -\rho/\sigma + \alpha(1-\tau_K)(1-\varphi\tau_p)(H/K)^{1-\alpha}/\sigma - \xi(\dot{Y}/Y)(1-\sigma)(1-\beta)/\sigma$, with r taken from (8) and where we have used $\dot{P}_E/P_E = (1-\beta)\dot{Y}/Y$. Using (7) and (8) $\dot{K}/K$ is obtained from (5) as $\dot{K}/K = -C/K + (H/K)^{1-\alpha}(1-\varphi\tau_p)((1-\tau_w)(1-\alpha) + \alpha(1-\tau_K))$. It should be noted that the accumulation of public capital which is positive for $I_p > 0$ is the source of sustained economic growth in our model and makes the growth rate an endogenous variable.

Thus, the dynamics of our model are completely described by the following differential equation system:

$$\frac{\dot{C}}{C} = -\frac{\rho}{\sigma} + \sigma^{-1}(1-\tau_K)(1-\varphi\tau_p)\alpha\left(\frac{H}{K}\right)^{1-\alpha} - \xi(1-\beta)\frac{1-\sigma}{\sigma}\left(\frac{\dot{Y}}{Y}\right), \tag{12}$$

$$\frac{\dot{K}}{K} = -\frac{C}{K} + \left(\frac{H}{K}\right)^{1-\alpha}(1-\varphi\tau_p)\left((1-\tau_w)(1-\alpha) + \alpha(1-\tau_K)\right), \tag{13}$$

$$\frac{\dot{H}}{H} = \left(\frac{H}{K}\right)^{-\alpha}\left(\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)(\tau_w(1-\alpha) + \tau_K\alpha)\right). \tag{14}$$

The initial conditions $K(0)$ and $H(0)$ are given and fixed and $C(0)$ can be chosen freely by the economy. Further, the transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda(t)(K(t) - K^\star(t)) \geq 0$ must be fulfilled, with $K^\star(t)$ denoting the optimal value and λ determined by (3).

In the following we will first examine our model as to the existence and stability of a balanced growth growth (BGP) on which all variables grow at the same constant rate.

In case of sustained growth the growth rates of C, K, and H are always positive and we first have to perform a change of the variables. Defining $c = C/K$ and $h = H/K$ and differentiating these variables with respect to time we get $\dot{c}/c = \dot{C}/C - \dot{K}/K$ and

$\dot{h}/h = \dot{H}/H - \dot{K}/K$. A rest point of this new system then corresponds to a BGP of our original economy where all variables grow at the same constant rate. Using $\dot{Y}/Y = \alpha(\dot{K}/K) + (1-\alpha)(\dot{H}/H)$ the system describing the dynamics around a BGP is given by

$$\begin{aligned}
\frac{\dot{c}}{c} &= -\frac{\rho}{\sigma} + \frac{\alpha(1-\tau_K)h^{1-\alpha}(1-\varphi\tau_p)}{\sigma} - (1-\alpha)\xi(1-\beta)\frac{1-\sigma}{\sigma}h^{-\alpha}\cdot \\
&\quad (\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)(\tau_w(1-\alpha)+\tau_K\alpha)) + \left(1+\alpha\xi(1-\beta)\frac{1-\sigma}{\sigma}\right)\cdot \\
&\quad (c - h^{1-\alpha}(1-\tau_p\varphi)((1-\tau_w)(1-\alpha)+\alpha(1-\tau_K))) \qquad (15)
\end{aligned}$$

$$\begin{aligned}
\frac{\dot{h}}{h} &= c - h^{1-\alpha}(1-\varphi\tau_p)((1-\tau_w)(1-\alpha)+\alpha(1-\tau_K)) + \\
&\quad h^{-\alpha}(\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)(\tau_w(1-\alpha)+\tau_K\alpha)). \qquad (16)
\end{aligned}$$

Concerning a rest point of system (15) and (16) it should be noted that we only consider interior solution. That means that we exclude the economically meaningless stationary point $c = h = 0$ such that we can consider our system in the rates of growth. As to the existence and stability of a BGP we can state proposition 1.

Proposition 1 *If* $1+\xi(1-\beta)(1-\sigma)/\sigma \geq 0$ *there exists a unique BGP which is a saddle path.*

Proof: To prove that proposition we first calculate c^∞ on a BGP which is obtained from $\dot{h}/h = 0$ as $c^\infty = h^{1-\alpha}(1-\varphi\tau_p)((1-\tau_w)(1-\alpha)+\alpha(1-\tau_K)) - h^{-\alpha}(\varphi\tau_p(1-\eta)+(1-\varphi\tau_p)(\tau_w(1-\alpha)+\tau_K\alpha))$. Inserting c^∞ in (15) gives after some modifications $f(\cdot) \equiv \dot{c}/c = -\rho/\sigma + (1-\tau_K)(1-\varphi\tau_p)\alpha h^{1-\alpha}/\sigma - h^{-\alpha}(\varphi\tau_p(1-\eta)+(1-\varphi\tau_p)(\tau_w(1-\alpha)+\tau_K\alpha))(1+\xi(1-\beta)(1-\sigma)/\sigma)$, with $\lim_{h\to 0} f(\cdot) = -\infty$ (for $I_p > 0$) and $\lim_{h\to\infty} f(\cdot) = \infty$. A rest point for $f(\cdot)$, i.e. a value for h such that $f(\cdot) = 0$ holds, then gives a BGP for our economy. Further, we have $\partial f(\cdot)/\partial h = (1-\tau_K)(1-\varphi\tau_p)(1-\alpha)\alpha h^{-\alpha}/\sigma + \alpha h^{-\alpha-1}(\varphi\tau_p(1-\eta)+(1-\varphi\tau_p)(\tau_w(1-\alpha)+\tau_K\alpha))(1+\xi(1-\beta)(1-\sigma)/\sigma) > 0$, for $I_p > 0$ and $1+\xi(1-\beta)(1-\sigma)/\sigma \geq 0$. Note that on a BGP $\dot{H}/H > 0$ must hold implying $I_p > 0$ and, thus, $\varphi\tau_p(1-\eta)+(1-\varphi\tau_p)(\tau_w(1-\alpha)+\tau_K\alpha) > 0$. $\partial f/\partial h > 0$ for h such that $f(\cdot) = 0$ means that $f(\cdot)$ cannot intersect the horizontal axis from above. Consequently, there exists a unique h^∞ such that $f(\cdot) = 0$ and, therefore, a unique BGP.

The saddle path property is shown as follows. Denoting with J the Jacobian of (15) and (16) evaluated at the rest point we first note that det $J < 0$ is a necessary and sufficient condition for saddle path stability, i.e. for one negative and one positive eigenvalue. The Jacobian in our model can be written as

$$J = \begin{bmatrix} 1 + \alpha\xi(1-\beta)(1-\sigma)/\sigma & \phi \\ 1 & \upsilon \end{bmatrix},$$

with ϕ given by $\phi = (1-\varphi\tau_p)(1-\alpha)h^{-\alpha}(-(1-\alpha)(1-\tau_w) - \alpha(1-\tau_K) + \alpha(1-\tau_K)/\sigma) - (\xi(1-\beta)(1-\sigma)/\sigma)(1-\alpha)\alpha h^{-\alpha-1}[h(1-\varphi\tau_p)((1-\alpha)(1-\tau_w) + \alpha(1-\tau_K)) - (\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)(\tau_w(1-\alpha) + \tau_K\alpha))]$ and $\upsilon = -\alpha h^{-\alpha-1}(\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)(\tau_w(1-\alpha) + \tau_K\alpha)) - (1-\alpha)h^{-\alpha}(1-\varphi\tau_p)((1-\alpha)(1-\tau_w) + \alpha(1-\tau_K))$. The determinant can be calculated as det $J = -(1-\tau_K)\alpha(1-\alpha)h^{-\alpha}/\sigma - \alpha h^{-\alpha-1}(\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)(\tau_w(1-\alpha) + \tau_K\alpha))(1 + \xi(1-\beta)(1-\sigma)/\sigma) < 0$, for $(1 + \xi(1-\beta)(1-\sigma)/\sigma) \geq 0$. □

That proposition states that our model is both locally and globally determinate, i.e. there exists a unique value for $c(0)$ such that the economy converges to the BGP in the long run[5]. A prerequisite for that outcome is $1 + \xi(1-\beta)(1-\sigma)/\sigma \geq 0$. If that inequality is not fulfilled we probably can observe multiple steady states. An explicit study of that phenomenon, however, is beyond the scope of this paper and in the following we will confine our investigations to the case $1 + \xi(1-\beta)(1-\sigma)/\sigma \geq 0$ throughout the paper which seems to be more likely. For $\beta = 1$, for example, that inequality is always fulfilled.

For $\beta < 1$ it is also fulfilled provided $\sigma < 1$ holds. For $\sigma > 1$, however, the expression depends on the values of ξ, β and σ. In order to show that our assumption $1 + \xi(1-\beta)(1-\sigma)/\sigma \geq 0$ is only slight we consider the range of ξ which is compatible with this inequality. For that inequality to be fulfilled we see that $\xi \leq \sigma/(\sigma-1)(1-\beta)$ must hold implying that the higher σ the smaller ξ has to be. As to the values for σ we know that most empirical estimates for σ reach the conclusion that σ takes on values at or above

[5] For a definition of local and global determinacy see e.g. Benhabib and Perli (1994) or Benhabib, Perli, and Xie (1994).

unity (cf. Blanchard and Fischer (1989), p. 44). Lucas (1990) in his paper takes $\sigma = 2.0$ but asserts that even $\sigma = 2.0$ seems high. So taking $\sigma = 2.0$ as a plausible upper bound seems to be reasonable. Then, we see that ξ can be out of the range $[0, 2(1-\beta)^{-1}]$. That means that in case of very ineffective abatement activities (β only slightly larger zero) one additional unit of pollution raises disutility more than proportional in relation to one additional unit of the consumer good. Even if pollution is relatively high such a value for ξ will reflect consumers' preferences sufficiently well. Therefore, our assumption $1 + \xi(1-\beta)(1-\sigma)/\sigma \geq 0$ does not impose too strong a limitation from the economic point of view.

It should also be mentioned that proposition 1 implies that σ must be larger one for multiple BGPs to be possible. In other papers a small value for σ is a necessary condition for multiple steady states (see for example Greiner and Semmler (1996)). As to that point it must be stated that the result in our paper changes if we allow for $\beta > 1$ which would be feasible from the technical point of view but which seems to be less realistic. Then, multiple BGPs are the more likely the smaller σ. It is our assumption $\beta \leq 1$ which determines the conditions necessary for multiple BGPs.

That also makes sense from an economic point of view: Global indeterminacy means that the economy may either converge to the BGP with the high balanced growth rate or to the BGP with the low balanced growth rate in the long run. So it may either choose a path with a higher initial consumption level (but lower initial investment) or a path with a lower level of initial consumption (but higher initial investment). In the latter case, the household must be willing to forgo current consumption and shift it into the future. If production and, thus, consumption do not have negative effects in form of pollution then the household will do that only if he has a high intertemporal elasticity of substitution of consumption (cf. Benhabib and Perli (1994, p. 114)). However, if production and, thus, consumption do have negative repercussions because they lead to a rise in effective pollution (if β is small) then the household is willing to forgo current consumption even with a low intertemporal elasticity of substitution because renouncing to consumption also has a positive effect since effective pollution is then lower, too, which raises current utility.

Proposition 1 shows us that our economy generates long-run per capita growth. As concerns sustainable growth we adopt the definition by Byrne (1997). According to that concept sustainable growth is given if the growth rate of instantaneous utility is positive, that is if $\dot{V}/V = \dot{C}/C - \xi \dot{P}_E/P_E = \dot{C}/C - \xi(1-\beta)\dot{Y}/Y > 0$ holds

4 Growth Effects of Fiscal Policy

In the last section we have demonstrated that under a slight additional assumption our model has one unique BGP which is a saddle path so that our model, including the transitional dynamics, is completely characterized. In this section we will analyze how the long-run balanced growth rate of our economy reacts to fiscal policy. But before doing that we analyze the impact of varying ξ, the parameter determining the disutility arising form pollution, and the impact of introducing a less polluting production technology, i.e. the impact of a decline in φ.

The balanced growth rate which we denote with g is given by (14) as $g = \dot{H}/H = (H/K)^{-\alpha} (\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)(\tau_w(1-\alpha) + \tau_K\alpha))$. Differentiating g with respect to ξ leads to $\partial g/\partial\xi = -\alpha h^{-\alpha-1} (\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)(\tau_w(1-\alpha) + \tau_K\alpha)) \, \partial h/\partial\xi$. The sign of $\partial h/\partial\xi$ can be obtained by implicitly differentiating h with respect to ξ from $f(\cdot) = 0$. This gives $\partial h/\partial\xi = g(1-\beta)(1-\sigma)/(\sigma\partial f/\partial h)$. From the proof of proposition 1 we know that $\partial f/\partial h > 0$ such that we can state the following result.

Proposition 2 *(i) For $\beta < 1$ a shift of preferences towards less pollution, i.e. an increase in ξ, lowers (raises) the balanced growth rate if $\sigma < (>)1$.*

(ii) If $\beta = 1$ a shift of preferences towards less pollution does not affect the balanced growth rate.

That proposition states that a shift towards greener preferences may either raise or lower the long-run balanced growth rate, depending on the intertemporal elasticity of substitution as long as a higher growth rate of output raises effective pollution (if $\beta < 1$). To interpret this outcome we have to look at equation (3), i.e. the maximum principle stating that marginal utility of consumption (r.h.s.) equals the shadow price of the savings of the household (l.h.s.). If $\sigma > 1$ a shift towards greener preferences, i.e. an increase in ξ, reduces the marginal product of consumption. That is the r.h.s. in (3) becomes smaller. Since the shadow price of capital is not changed by an increase in ξ the l.h.s. remains the same. Consequently, consumption has to rise such that the l.h.s. declines in order for (3) to hold. This implies that the household raises the level of consumption and reduces savings leading to a lower balanced growth rate. It should be noted that this is just opposite to the result derived by Smulders and Gradus (1996). The reason for this result is to be seen in the fact that in Smulders and Gradus (1996) pollution has a direct effect on production possibilities, in contrast to our model, thus affecting the shadow price of savings.

If abatement activities are very effective ($\beta = 1$) implying that a higher growth rate of production does not affect the rate of growth of effective pollution, a shift of preferences towards less pollution does not affect the balanced growth rate in the economy. That is obvious because pollution does not affect production possibilities directly. Further, the growth rate of effective pollution does not affect the rate of growth of instantaneous utility. Therefore, a shift of preferences towards less pollution will not affect the household's saving-consumption decision and leave the balanced growth rate unchanged.

Let us, in a next step, investigate the effects of using a less polluting production technology, which is modelled by a lower value for φ.

To analyze increases in φ we differentiate g with respect to that parameter giving

$$\frac{\partial g}{\partial \varphi} = h^{-\alpha}\tau_p(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K) - z\alpha h^{-\alpha-1}\frac{\partial h}{\partial \varphi},$$

with $z = (\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)(\tau_w(1-\alpha)+\tau_K\alpha))$. $\partial h/\partial\varphi$ is obtained by implicit differentiation from $f(\cdot) = 0$ (from the proof of proposition 1) as

$$\frac{\partial h}{\partial \varphi} = \frac{\tau_p(1+\xi(1-\beta)\frac{1-\sigma}{\sigma})(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K)+\tau_p(1-\tau_K)\alpha h/\sigma}{\alpha(1-\tau_K)(1-\varphi\tau_p)((1-\alpha)/\sigma)+\alpha h^{-1}z(1+\xi(1-\beta)(1-\sigma)/\sigma)}.$$

For $(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K)=0$ we get $\partial g/\partial\varphi < 0$. To get results for $(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K) \neq 0$ we insert $\partial h/\partial\varphi$ in $\partial g/\partial\varphi$. That gives

$$\begin{aligned}\frac{\partial g}{\partial \varphi} &= h^{-\alpha}\tau_p(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K)\cdot \\ &\left(1-\frac{z[(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K)(1+\xi(1-\beta)(1-\sigma)/\sigma)+h\alpha(1-\tau_K)/\sigma]}{(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K)[z(1+\xi(1-\beta)\frac{1-\sigma}{\sigma})+h(1-\tau_K)(1-\tau_p\varphi)\frac{1-\alpha}{\sigma}]}\right).\end{aligned}$$

From that expression it can be seen that the expression in brackets is always positive for $(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K) < 0$ such that $\partial g/\partial\varphi < 0$. For $(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K) > 0$ it is immediately seen that

$$\frac{\partial g}{\partial \varphi} >=< 0 \Leftrightarrow (1-\varphi\tau_p)(1-\alpha)(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K) >=< \alpha\, z,$$

which simplifies to

$$\frac{\partial g}{\partial \varphi} >=< 0 \Leftrightarrow (1-\eta)(1-\alpha) >=< \varphi\tau_p(1-\eta)+(1-\varphi\tau_p)(\tau_w(1-\alpha)+\alpha\tau_K).$$

The r.h.s. of that expression is equivalent to I_P/Y. Thus we have proved the following proposition.

Proposition 3 *If $(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K) \leq 0$ the use of a less polluting technology raises the balanced growth rate. For $(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K) > 0$ the use of a less polluting technology raises (leaves unchanged, lowers) the balanced growth rate if*

$$\frac{I_p}{Y} > (=,<)(1-\alpha)(1-\eta).$$

To interpret that result we first note that a cleaner production technology (i.e. a lower φ) shows two different effects: on the one hand it implies that less resources are needed for abatement activities leaving more resources for public investment. That effect leads to a higher ratio H/K, thus raising the marginal product of private capital r in (8), that is the return on investment rises. Further, a less polluting technology implies that the firm has to pay less pollution taxes (the term $(1-\tau_p\varphi)$ rises) which has also a stimulating effect on r, which can be seen from (8) and which also raises the incentive to invest. On the other hand, however, less pollution implies that the tax revenue resulting from the taxation of pollution declines and, thus, productive public spending. That effect tends to lower the ratio H/K and, therefore, the marginal product of private capital which tends to lower the balanced growth rate.

If $\eta \geq 1 - \tau_w(1-\alpha) - \alpha\tau_K$, i.e. if much of the pollution tax is used for abatement activities a cleaner technology always raises the balanced growth rate. In that case, the negative growth effect of a decline in the pollution tax revenue is not too strong since most of that revenue is used for abatement activities which are non-productive anyway. If, however, $\eta < 1 - \tau_w(1-\alpha) - \alpha\tau_K$, i.e. a good deal of the pollution tax is used for productive government spending, a cleaner technology may either raise or lower economic growth. It increases the balanced growth rate if the share of public investment per GDP is larger than a constant which positively depends on the elasticity of aggregate output with respect to public capital and negatively on η, and vice versa.

Let us next study growth effects of increasing the tax rates. First, we will analyze the impact of variations in the capital income tax rate. Proposition 4 demonstrates that a rise in that tax may have positive or negative growth effects and that there exists a growth maximizing capital income tax rate.

Proposition 4 *The capital income tax rate maximizing the balanced growth rate is given by*

$$\tau_K = (1-\alpha)(1-\tau_w) - \varphi\tau_p(1-\eta)/(1-\varphi\tau_p).$$

Proof: To calculate growth effects of varying τ_K we take the balanced growth rate g from (14) and differentiate it with respect to that parameter. Doing so gives

$$\frac{\partial g}{\partial \tau_K} = h^{-\alpha}\alpha(1-\tau_p\varphi)\left(1 - \frac{z}{1-\tau_p\varphi}\frac{\partial h}{\partial \tau_K}\frac{1}{h}\right),$$

with $z = (\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)(\tau_w(1-\alpha) + \tau_K\alpha))$. $\partial h/\partial\tau_K$ is obtained by implicit differentiation from $f(\cdot) = 0$ leading to

$$\frac{\partial h}{\partial \tau_K}\frac{1}{h} = \frac{(1-\varphi\tau_p)(1+\xi(1-\beta)\frac{1-\sigma}{\sigma} + h/\sigma)}{h(1-\tau_K)(1-\varphi\tau_p)((1-\alpha)/\sigma) + z(1+\xi(1-\beta)(1-\sigma)/\sigma)}.$$

Inserting $h^{-1}\partial h/\partial\tau_K$ in $\partial g/\partial\tau_K$ we get

$$\frac{\partial g}{\partial \tau_K} = h^{-\alpha}\alpha(1-\tau_p\varphi)\left(1 - \frac{z(h/\sigma) + z(1+\xi(1-\beta)(1-\sigma)/\sigma)}{h(1-\tau_K)(1-\varphi\tau_p)((1-\alpha)/\sigma) + z(1+\xi(1-\beta)(1-\sigma)/\sigma)}\right),$$

showing that

$$\frac{\partial g}{\partial \tau_K} >=< 0 \Leftrightarrow (1-\tau_K)(1-\varphi\tau_p)(1-\alpha) >=< \varphi\tau_p(1-\eta) + (1-\varphi\tau_p)(\tau_w(1-\alpha) + \tau_K\alpha).$$

Solving for τ_K gives

$$\frac{\partial g}{\partial \tau_K} >=< 0 \Leftrightarrow \tau_K <=> (1-\alpha)(1-\tau_w) - \varphi\tau_p(1-\eta)/(1-\varphi\tau_p)$$

That shows that the balanced growth rate rises with increases in τ_K as long as τ_K is smaller than the expression on the r.h.s. which is constant. □

That proposition shows that the growth maximizing capital income tax rate does not necessarily equal zero in our model which was to be expected since the government finances productive public spending with the tax revenue. There are two effects of the capital income tax rate: on the one hand, the capital income tax lowers the marginal product of private capital and, therefore, is a disincentive for investment. On the other hand, the government finances productive public spending with its tax revenue leading to a rise in the ratio H/K, which raises the marginal product of private capital r and which has, as a consequence, a positive effect on economic growth. However, boundary solutions, i.e. $\tau_K = 0$ or $\tau_K = 1$, cannot be excluded, too. Whether there exists an interior or a boundary solution for the growth maximizing capital income tax rate depends on the numerical specification of the parameters φ, τ_p, and η. Only for $\varphi\tau_p = 0$ or $\eta = 1$ the growth maximizing tax rate is always in the interior of $(0, 1)$.

Concerning the impact of the wage tax rate on the growth maximizing capital income tax rate, it can easily be seen that the latter negatively varies with the tax on labour. As to the tax on pollution the growth maximizing capital income tax rate negatively varies with that tax if $\eta < 1$. For $\eta > 1$ the growth maximizing capital income tax rate is the higher the higher the tax on pollution τ_p. The interpretation of that result is as follows: if $\eta < 1$ the government uses a part of the pollution tax revenue for the creation of public capital which has positive growth effects. Increasing the tax on pollution implies in that case that a part of the additional tax revenue is used for productive investment in the creation of public capital. Consequently, the capital income tax rate can be reduced without having negative growth effects. It should be noticed that a decrease in the capital income tax rate shows an indirect positive growth effect because it implies a reallocation of private resources from consumption to investment. In contrast to that, if $\eta > 1$ the whole pollution tax revenue is used for abatement activities. Raising the pollution tax rate in that situation implies that the additional tax revenue is used only for abatement activities but not for productive public spending. Consequently, the negative indirect growth effect of a higher pollution tax (through decreasing the return on capital r) must be compensated by an increase in the capital income tax rate. It should be noted that the latter also has a negative indirect growth effect but that one is dominated in this case by the positive direct growth effect of higher productive public spending.

Let us next analyze growth effects of a rise in the wage tax rate and the pollution tax rate. Proposition 5 gives the result.

Proposition 5 *Raising the wage tax rate always increases the balanced growth rate. For* $(1 - \eta - \tau_w(1 - \alpha) - \alpha\tau_K) \leq 0$ *a rise in the pollution tax rate always lowers the balanced growth rate. If* $(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K) > 0$ *the pollution tax rate maximizing the balanced growth rate is determined by*

$$\tau_p = \left(\frac{1}{\varphi}\right)\left(\frac{1 - \eta - \tau_w(1 - \alpha) - \alpha\tau_K - \alpha(1 - \eta)}{1 - \eta - \tau_w(1 - \alpha) - \alpha\tau_K}\right)$$

which is equivalent to

$$\frac{I_p}{Y} = (1-\alpha)(1-\eta).$$

Proof: To calculate growth effects of varying τ_w we take the balanced growth rate g again from (14) and differentiate it with respect to that parameter. Doing so gives

$$\frac{\partial g}{\partial \tau_w} = h^{-\alpha}(1-\alpha)(1-\tau_p\varphi)\left(1 - \frac{\alpha\, z}{(1-\tau_p\varphi)(1-\alpha)}\frac{\partial h}{\partial \tau_w}\frac{1}{h}\right),$$

with $z = (\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)(\tau_w(1-\alpha)+\tau_K\alpha))$. $\partial h/\partial\tau_w$ is obtained by implicit differentiation from $f(\cdot) = 0$ leading to

$$\frac{\partial h}{\partial \tau_w}\frac{1}{h} = \frac{(1-\varphi\tau_p)(1-\alpha)(1+\xi(1-\beta)(1-\sigma/\sigma))}{h(1-\tau_K)(1-\varphi\tau_p)\alpha((1-\alpha)/\sigma) + \alpha z(1+\xi(1-\beta)(1-\sigma)/\sigma)}.$$

Inserting $h^{-1}\partial h/\partial\tau_w$ in $\partial g/\partial\tau_w$ we get

$$\frac{\partial g}{\partial \tau_w} = h^{-\alpha}(1-\alpha)(1-\tau_p\varphi)\left(1 - \frac{z(1+\xi(1-\beta)(1-\sigma)/\sigma)}{z(1+\xi(1-\beta)(1-\sigma)/\sigma) + h(1-\tau_K)(1-\tau_p\varphi)(1-\alpha)/\sigma}\right)$$

which demonstrates that $\partial g/\partial\tau_w > 0$.

Doing the same procedure for the pollution tax rate we get

$$\begin{aligned}\frac{\partial g}{\partial \tau_p} &= h^{-\alpha}\varphi(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K)\cdot \\ &\left(1 - \frac{z[(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K)(1+\xi(1-\beta)(1-\sigma)/\sigma) + h\alpha(1-\tau_K)/\sigma]}{(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K)[z(1+\xi(1-\beta)\frac{1-\sigma}{\sigma}) + h(1-\tau_K)(1-\tau_p\varphi)\frac{1-\alpha}{\sigma}]}\right).\end{aligned}$$

From that expression it is immediately seen that the expression in brackets is always positive for $(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K) < 0$ such that $\partial g/\partial\tau_p < 0$. For $(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K) = 0$ the result can directly be seen by multiplying out the expression above. For $(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K) > 0$ it is seen that

$$\frac{\partial g}{\partial \tau_p} >=< 0 \Leftrightarrow (1-\varphi\tau_p)(1-\alpha)(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K) >=< \alpha\, z,$$

which simplifies to

$$\frac{\partial g}{\partial \tau_p} >=< 0 \Leftrightarrow \tau_p <=> \left(\frac{1}{\varphi}\right)\left(\frac{1-\eta-\tau_w(1-\alpha)-\alpha\tau_K-\alpha(1-\eta)}{1-\eta-\tau_w(1-\alpha)-\alpha\tau_K}\right)$$

and is equivalent to

$$\frac{\partial g}{\partial \tau_p} >=< 0 \Leftrightarrow \frac{I_p}{Y} <=> (1-\eta)(1-\alpha).$$

Thus, the proposition is proved. □

The interpretation of that result is straightforward. Since the wage tax rate is a non-distortionary tax an increase in that tax does not affect the household's decision how much to save and how much to consume, i.e. it does not affect the allocation of private resources. As a consequence, there is only the positive indirect growth effect of an increase in the tax revenue which leads to more public investment in public capital which raises the ratio H/K and, thus, the marginal product of private capital. That effect spurs private investment and, therefore, economic growth, too.

An increase in the pollution tax rate always lowers the balanced growth rate if $(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K) \leq 0$. In that case, too much of the additional tax revenue (gained through the increase in τ_p) goes in abatement activities such that the positive growth effect of a higher pollution tax revenue (i.e. the increase in the creation of the stock of public capital) is dominated by the negative indirect one of a reduction of the rate of return to physical capital r. The latter effect namely implies a reallocation of private resources from investment to consumption which reduces economic growth. For $(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K) > 0$, however, there exists a growth maximizing pollution tax rate. In that case, the pollution tax has to be set such that public investment per GDP equals the elasticity of aggregate output with respect to public capital multiplied with that share of the pollution tax revenue which is not used for abatement activities but for productive public spending. It should be noticed that the growth maximizing value of τ_p[6] is the higher the less of the pollution tax revenue is used for abatement activities. In the limit ($\eta = 0$) we get the same result as in Barro (1990) and Futagami et.al. (1993) that the growth maximizing share of public investment per GDP equals the elasticity of aggregate output with respect to public capital.

[6]Note that I_p/Y positively varies with τ_p for $(1-\eta-\tau_w(1-\alpha)-\alpha\tau_K) > 0$.

It should also be noted that the conditions for a positive growth effect of an increase in the pollution tax rate are just reverse to the conditions which must be fulfilled such that the introduction of a less polluting technology raises economic growth.

5 Conclusion

In this paper we have analyzed an endogenous growth model with productive public spending and pollution and seen how fiscal policy may influence the balanced growth rate of the decentralized economy. The main novelty is the analysis of growth effects of various tax rates as well as the assumption that pollution only affects the utility of the household but not production possibilities directly. We showed which conditions must be fulfilled such that variations in the tax rates considered lead to a higher balanced growth rate.

We should also like to point out that all of our results derived in the paper remain valid if we assume that public investment as a flow variable enters the aggregate production function, instead of public capital as a stock. A prerequisite, however, is that the condition mentioned in proposition 1 is fulfilled. In this case, no transitional dynamics occur and our economy immediately jumps on the balanced growth path.

As to future research it would be interesting to study welfare effects of fiscal policy. Given the fact that pollution is an inevitable by-product of production and negatively affects utility it is intuitively clear that maximum economic growth does not necessarily maximize welfare. Further, it would be interesting to study the effects of fiscal policy on the transition path. However, that task seems to be extremely difficult and it is questionable whether useful results could be obtained from that analysis.

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