

Asset Price Volatility and Monetary Policy Rules: A Dynamic Model and Empirical Evidence

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Abstract

A dynamic model is set up to explore monetary policy in the presence of asset price volatility. If the asset price bubble and the probability that the asset price increases or decreases next period are taken as exogenous variables, the optimal monetary policy rule turns out to be a linear function of the inflation deviation, output gap and asset price bubble. Unlike some other researchers, Bernanke and Gertler (2000) for example, who do not endogenize the probability that the asset price bubble breaks next period, we further explore the monetary policy rule by endogenizing such a probability and find that there may exist multiple equilibria for the economy. We also consider monetary policy and asset prices in the presence of deflation and a zero bound on the nominal interest rate. Our study shows that in the presence of a zero bound on the nominal rate, a financial market depression can make a deflation and an economic recession worse, implying that policy actions aiming at escaping a liquidity trap should not ignore the asset prices.

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1 Introduction

An interesting feature of the monetary environment in industrial countries in the 1990s is that inflation rates remained relatively stable and low, while the prices of equities, bonds, and foreign exchanges experienced a strong volatility with the liberalization of financial markets. Some central banks, therefore, have become concerned with such volatility and doubt whether those are justifiable on the basis of economic fundamentals. The question has arisen whether a monetary policy should be pursued that takes into account financial markets and asset price stabilization. In order to answer this question, it is necessary to model the relationship between asset prices and the real economy. An early study of such type can be found in Blanchard (1981) who has analyzed the relationship between the stock value and output in “good news” and “bad news” cases. Recent examples include Bernanke and Gertler (2000), Smets (1997), Kent and Lowe (1997), Chiarella et al. (2001), Mehra (1998), Vickers (1999), Filardo (2000), Okina, Shirakawa and Shirats (2000) and Dupor (2001).

Among these papers, the work by Bernanke and Gertler (2000) has attracted much attention. Bernanke and Gertler (2000) employ a macroeconomic model and explore how the macroeconomy may be affected by alternative monetary policy rules which may or may not take into account the asset price bubble. Based on some simulation and comparative analysis of the recent US and Japanese monetary policy, they conclude that it is desirable for central banks to focus on underlying inflationary pressures and that asset prices become relevant only to the extent they may signal potential inflationary or deflationary forces.

The shortcomings of the position by Bernanke and Gertler (2000) may, however, be expressed as follows. First, they do not derive monetary policy rules from certain estimated models, but instead design artificially alternative monetary policy rules which may or may not consider asset price bubbles and then explore the effects of these rules on the economy. Second, Bernanke and Gertler (2000) assume that the asset price bubble always grows at a certain rate before breaking. In actual asset markets the asset price bubble might not break suddenly, but instead may increase or decrease at a certain rate before becoming zero. Third, they assume that the bubble can exist for a few periods and will not occur any more after breaking. Therefore, they explore the effects of the asset price bubble on the real economy in the short-run. Fourth, they do not endogenize the probability that the asset price bubble breaks next period because little is known about the market psychology. Monetary policy with endogenized probability of the bubble to break may be different from that with an exogenous probability.

The difference of our model from that of Bernanke and Gertler (2000) consists in the following points. First, we employ an intertemporal framework to explore what the optimal monetary policy should be with and without the financial markets taken into account. Second, we assume that the bubble does not break suddenly and does not have to always grow at a certain rate, on the contrary, it may increase or decrease at a certain rate with some probability. The bubble does not have to break in certain periods and moreover, it can occur

again even after breaking. Third, we assume that the probability that the bubble increases or decreases next period can be endogenized. This assumption has also been made by Kent and Lowe (1997). They assume that the probability of the asset price bubble to break is a function of the current bubble size and monetary policy. The drawback of Kent and Lowe (1997), however, is that they explore only positive bubbles and that they assume a linear probability function, which is not necessarily bounded between 0 and 1. Following Bernanke and Gertler (2000), we consider both positive and negative bubbles and employ a nonlinear probability function which lies between 0 and 1.

What, however, complicates the response of monetary policy to asset price volatility is the relationship of asset prices and product prices, the latter being mainly the concern of the central banks. Low asset prices may be accompanied by low or negative inflation rates. Yet, there is a zero bound on the nominal interest rate. The danger of deflation and the so-called “Liquidity Trap” has recently attracted much attention because there exists, for example, a severe deflation and recession in Japan and monetary policy seems to be of little help since the nominal rate is almost zero and can hardly be lowered further. On the other hand, the financial market of Japan has also been in a depression for a long time. Although some researchers have discussed the zero interest-rate bound and the liquidity trap in Japan, little attention has been paid to the asset price depression in the presence of a zero bound on the nominal rate. We will explore this problem with some simulations of a simple model.

The remainder of the paper is organized as follows. In Section 2 we set up the basic model with the assumption that central banks pursue monetary policy to minimize a quadratic objective function. We first derive a monetary policy rule from the basic model without asset price bubbles and then extend the model by assuming that the output can be affected by the asset price bubbles. The asset price bubbles are taken as exogenous variables. The probability of the asset price bubble to increase or decrease next period is also assumed to be constant. Based on some empirical evidence we then take the asset price bubble as an endogenous variable and explore the difference between the monetary policy rules with endogenous and exogenous bubbles. Section 3 explores evidence of the monetary policy with asset price in the Euro-area with a model set up by Clarida, Gali and Gertler (1998). Section 4 extends the model by assuming that the probability that the asset price bubble increases or decreases next period is influenced by the size of the current bubble and the level of the current interest rate and derives monetary policy rule in such a case. Section 5 explores how the asset price may affect the real economy in the presence of the danger of deflation and a zero bound on the nominal rate. The last section concludes the paper.

2 The Basic Model

2.1 Monetary Policy Rule from a Traditional Model

Following Svensson (1997), we assume that central banks pursue a monetary policy in the following model:

$$\underset{\{r_t\}_0^\infty}{Min} \sum_{t=0}^{\infty} \rho^t L_t$$

with

$$L_t = (\pi_t - \pi^*)^2 + \lambda y_t^2, \quad \lambda > 0,$$

subject to

$$\pi_{t+1} = \alpha_1 \pi_t + \alpha_2 y_t, \quad \alpha_i > 0 \tag{1}$$

$$y_{t+1} = \beta_1 y_t - \beta_2 (r_t - \pi_t), \quad \beta_i > 0, \tag{2}$$

where π_t denotes the inflation rate, y_t the output gap, r_t the gap between the nominal short-term rate R_t and the long-run level of the short-term rate \bar{R} , namely $r_t = R_t - \bar{R}$. ρ is the discount factor bounded between 0 and 1 and π^* is the inflation target which is assumed to be zero in our model.

To solve the optimal control problem above, we can follow Svensson (1997; 1999) and ignore the constraint on y_{t+1} first.¹ Therefore we have a standard linear quadratic (LQ) dynamic programming problem which reads

$$V(\pi_t) = \underset{y_t}{Min} [(\pi_t^2 + \lambda y_t^2) + \rho V(\pi_{t+1})] \tag{3}$$

subject to

$$\pi_{t+1} = \alpha_1 \pi_t + \alpha_2 y_t \tag{4}$$

Equation (3) is the so-called Hamilton-Jacobi-Bellman (HJB) equation and $V(\pi_t)$ is the value function, with y_t being the control variable now. For an LQ problem like ours, we know that the value function must be quadratic. Therefore, we assume that the value function takes the form

$$V(\pi_t) = \Omega_0 + \Omega_1 \pi_t^2, \tag{5}$$

where Ω_0 and Ω_1 remain to be determined (we need only to determine Ω_1 since the goal is to derive the interest rate rule, not to get the optimal value of the objective function). The first-order condition turns out to be

$$\lambda y_t + \rho \alpha_2 \Omega_1 \pi_{t+1} = 0,$$

from which we get

$$\pi_{t+1} = -\frac{\lambda}{\rho \alpha_2 \Omega_1} y_t. \tag{6}$$

¹The reader is referred to the appendices in Svensson (1997; 1999).

Substituting (6) into (4) gives

$$y_t = -\frac{\rho\alpha_1\alpha_2\Omega_1}{\lambda + \rho\alpha_2^2\Omega_1}\pi_t, \quad (7)$$

and after substituting this equation back into (6), we have

$$\pi_{t+1} = \frac{\alpha_1\lambda}{\lambda + \rho\alpha_2^2\Omega_1}\pi_t. \quad (8)$$

By applying (3), (5) and (7), the envelop theorem gives us the following equation

$$V_\pi(\pi_t) = 2\left(1 + \frac{\alpha_1^2\rho\lambda\Omega_1}{\lambda + \rho\alpha_2^2\Omega_1}\right)\pi_t,$$

and from (5), we know that

$$V_\pi(\pi_t) = 2\Omega_1\pi_t,$$

these two equations tell us that

$$\Omega_1 = 1 + \frac{\alpha_1^2\rho\lambda\Omega_1}{\lambda + \rho\alpha_2^2\Omega_1}.$$

The right-hand side of this equation has the limit $1 + \frac{\alpha_1^2\lambda}{\alpha_2^2}$ as $\Omega_1 \rightarrow \infty$. The root of Ω_1 larger than one can therefore be solved from the equation

$$\Omega_1^2 - \left[1 - \frac{(1 - \rho\alpha_1^2)\lambda}{\rho\alpha_2^2}\right]\Omega_1 - \frac{\lambda}{\rho\alpha_2^2} = 0,$$

which gives the solution of Ω_1 :

$$\Omega_1 = \frac{1}{2} \left(1 - \frac{\lambda(1 - \rho\alpha_1^2)}{\rho\alpha_2^2} + \sqrt{\left(1 - \frac{\lambda(1 - \rho\alpha_1^2)}{\rho\alpha_2^2}\right)^2 + \frac{4\lambda}{\rho\alpha_2^2}} \right). \quad (9)$$

By substituting $t + 1$ for t into (7), we have

$$y_{t+1} = -\frac{\rho\alpha_1\alpha_2\Omega_1}{\lambda + \rho\alpha_2^2\Omega_1}\pi_{t+1}. \quad (10)$$

Substituting (1) and (2) into (10) with some computation, optimal decision rule for the short-term interest rate:

$$r_t = f_1\pi_t + f_2y_t, \quad (11)$$

with

$$f_1 = 1 + \frac{\rho\alpha_1^2\alpha_2\Omega_1}{(\lambda + \rho\alpha_2^2\Omega_1)\beta_2}, \quad (12)$$

$$f_2 = \frac{\beta_1}{\beta_2} + \frac{\rho\alpha_2^2\alpha_1\Omega_1}{(\lambda + \rho\alpha_2^2\Omega_1)\beta_2}; \quad (13)$$

Equation (11) shows that the optimal short-term interest rate should be a linear function of the inflation rate and output gap. It is similar to the Taylor rule (Taylor 1993). Semmler, Greiner and Zhang (2003, Ch. 6) undertake some simulations and find that the inflation deviation, output gap and r_t may converge to zero as time goes to infinity.

2.2 Monetary Policy Rule with Asset Price Bubbles

Above we have derived a monetary policy rule from a traditional model which does not take into account the effects of the financial market on the output or inflation. Recently, however, some researchers argue that the financial markets can probably influence the inflation and output. Filardo (2000), for example, surveys some research which argues that the stock price may influence the inflation. Bernanke and Gertler (2000) explore how the asset price bubbles can affect the real economy with alternative monetary policy rules. Smets (1997) derives an optimal monetary policy rule from an intertemporal model under the assumption that the stock price can affect the output. In the research below we also take into account the effects of the financial markets on the output and explore what the monetary policy rule should be. Before setting up the model we explain some basic concepts.

In the research below we assume the stock price s_t consists of the fundamental value \tilde{s}_t and the asset price bubble b_t . We will not discuss how to compute the asset price or the fundamental value here, because this requires much work which may be out of the scope of the paper.² The stock price as a result reads

$$s_t = \tilde{s}_t + b_t.$$

We further assume that if the stock price equals its fundamental value, the financial market exerts no effects on the output gap, that is, the financial market affects the output gap only through the asset price bubbles. The asset price bubble can be either positive or negative. The difference between our bubble from those of Blanchard and Watson (1982), Bernanke and Gertler (2000) and Kent and Lowe (1997) lies in the following aspects.

The so-called “rational bubble” defined by Blanchard and Watson (1982) can not be negative because of the so-called limited liability of asset. A negative bubble can lead to negative expected stock price. Another difference of our bubble from that of a rational bubble is that the rational bubble can only increase before it breaks. Therefore, the bubble is non-stationary. Bernanke and Gertler (2000) also define the bubble as the gap between the stock price and its fundamental value. It can be positive or negative. The reason that they do not assume a rational bubble is that the non-stationarity of a rational bubble leads to technical problems in their framework. Kent and Lowe (1997) explore only positive bubbles.

²Alternative approaches have been proposed to compute the fundamental value and bubbles of the asset price. One example can be found in Shiller (1984).

Bernanke and Gertler (2000) and Kent and Lowe (1997), however, have something common: They all assume that the bubble will break in a few period periods (4 or 5 periods) from a certain value to zero *suddenly*, not step by step. Moreover, if the bubble is broken, it will not appear again. This is in fact not really consistent with the reality. In reality, the bubble usually does not necessarily break suddenly from a large or low value, but may decrease or increase step by step before becoming zero with a high or low speed. Especially, if the bubble is negative, it is implausible that the stock price will return to its fundamental value suddenly. A common assumption of the rational bubble and those definitions of Bernanke and Gertler (2000) and Kent and Lowe (1997) is that they all assume that the bubble will grow at a certain rate before it bursts.

Although we also define the asset bubble as the deviation of the asset price from its fundamental value, the differences between our bubble and those mentioned above are clear: (1), it can be positive or negative, (2), it can increase or decrease before it becomes zero or even change from a positive (negative) one to a negative (positive) one and does not have to burst suddenly, (3), nobody knows when it bursts and (4) it can occur again next period even if it becomes zero in the current period. Therefore we assume the asset price bubble evolves in the following way

$$b_{t+1} = \begin{cases} b_t(1 + g_1) + \varepsilon_{t+1}, & \text{with probability } p \\ b_t(1 - g_2) + \varepsilon_{t+1}, & \text{with probability } 1 - p \end{cases} \quad (14)$$

where $g_1, g_2 (\geq 0)$ are the growth rate or decrease rate of the bubble. g_1 can of course equal g_2 . ε_t is an iid noise with zero mean and a constant variance. *Eq. (14) indicates that if the current asset price bubble b_t is positive, it can increase at rate g_1 with probability p and decrease at rate g_2 with probability $1 - p$ next period. If the bubble is negative, however, it will decrease at rate g_1 with probability p and increase at rate g_2 with probability $1 - p$ next period.* The probability p is assumed to be constant at the moment but is assumed to be state-dependent in the following sections. From this equation we find that even if the bubble is zero in the current period, it may not be zero in the next period.

Before exploring the monetary policy with asset price bubbles theoretically, we explore some empirical evidence of the effects of the share bubbles on the output gap. Exactly speaking, we estimate the following equation with the OLS for several European countries and the US with quarterly data:

$$y_t = c_0 + c_1 y_{t-1} + c_2 b_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2) \quad (15)$$

with y_t denoting the output gap. Following Clarida, Gali and Gertler (1998) we use the industrial production index (IPI) to measure the output. The output gap is computed as the percent deviation of the IPI from its Band-Pass filtered trend.³ Similarly the asset price bubble is measured by the percent deviation of

³The reader is referred to Baxter and King (1995) for the Band-Pass filter. As surveyed by Orphanides and van Norden (2002), there are many methods to measure the output gap. We find that filtering the IPI using the Band-Pass filter leaves the measure of the output gap essentially unchanged from the measure with the HP-filter. The Band-Pass filter has also been used by Sargent (1999).

Parameter	Sample	US	UK*	France	Germany	Italy
c_1	80.1-99.1	0.902 (22.218)	0.827 (16.821)	0.879 (19.170)	0.855 (19.313)	0.912 (22.024)
	90.1-99.1	0.925 (15.790)	0.918 (22.362)	0.836 (12.153)	0.808 (16.267)	0.843 (11.666)
c_2	80.1-99.1	0.064 (5.158)	0.050 (2.898)	0.005 (0.713)	0.021 (2.506)	0.002 (0.385)
	90.1-99.1	0.0005 (0.035)	0.099 (5.517)	0.032 (2.328)	0.075 (6.085)	0.020 (1.921)
R^2	80.1-99.1	0.875	0.824	0.845	0.864	0.869
	90.1-99.1	0.886	0.953	0.849	0.928	0.819

Table 1: Estimation of Eq. (15)

*The estimation of the UK is undertaken for 80.1-97.1 and 90.1-97.1 because the share price index after 1999 is unavailable. Data source: International Statistics Yearbook.

the share price index from its Band-Pass filtered trend just for simplicity. The estimation of Eq. (15) is shown in Table 1 with t-statistics in parentheses. The estimate of c_0 is not shown just for simplicity. The estimation is undertaken for two samples: 1980-1999 and 1990-1999.

From Table 1 we find that c_2 is significant enough in most of the cases. For the sample 1990-99 it is significant enough for all countries except the US, but for the sample 1980-99 it is very significant for the US. For the sample 1980-99 it is insignificant for France and Italy, but significant enough for both countries for the period 1990-99. In short, the evidence in Table 1 does show some positive relationship between the share bubbles and the output gap.

In the estimation above we have considered only the effect of the bubble lag on the output for simplicity, but in reality the expectation of financial markets may also influence the output. As for through what mechanism the financial variables may influence the output, the basic argument is that the changes of the asset price may influence the consumption (see Ludvigson and Steindel (1998) for example) and investment, which may in turn affect the inflation and output. The investment, however, can be affected by both current and forward-looking behaviors.

Therefore in the model below we assume that the the output gap can be influenced not only by the asset price bubble lag but also by the expectation of asset price bubble formed in the previous period, namely we assume

$$y_{t+1} = \beta_1 y_t - \beta_2 (r_t - \pi_t) + \beta_3 b_t + (1 - \beta_3) E b_{t+1|t}, \quad 1 > \beta_3 > 0, \quad (16)$$

where $E b_{t+1|t}$ denotes the expectation of b_{t+1} formed at time t . From Eq. (14) and with $E \varepsilon_{t+1|t} = 0$ we know

$$E b_{t+1|t} = [1 - g_2 + p(g_1 + g_2)] b_t. \quad (17)$$

As a result Eq. (16) turns out to be

$$y_{t+1} = \beta_1 y_t - \beta_2 \{r_t - \pi_t\} + \{1 + (1 - \beta_3)[p(g_1 + g_2) - g_2]\} b_t. \quad (18)$$

Because the bubble is taken as an exogenous variable, we can follow the same procedure as in the previous subsection to solve the optimal control problem. After substituting Eq. (18) for Eq. (2) we obtain the following monetary policy rule for the central bank

$$r_t = f_1\pi_t + f_2y_t + f_3b_t, \quad (19)$$

with f_1, f_2 given by (12)–(13) and

$$f_3 = \frac{1}{\beta_2}\{1 + (1 - \beta_3)[p(g_1 + g_2) - g_2]\}. \quad (20)$$

This rule is similar to the one obtained before except that there is an additional term of the bubble. The effect of p on the monetary policy rule can be explored from the following derivative

$$\frac{df_3}{dp} = \frac{1}{\beta_2}[(1 - \beta_3)(g_1 + g_2)] \geq 0. \quad (21)$$

The interpretation of (21) depends on whether the bubble is positive or negative. If the bubble is positive, a larger p leads to a higher f_3 and as a result, a higher the r_t . This is consistent with intuition: In order to eliminate a positive bubble which is likely to continue to increase, it is necessary to raise the interest rate since it is usually argued that there exists a negative relationship between the stock price and the interest rate.⁴ If the bubble is negative, however, a larger p also leads to a higher f_3 but a lower r_t , since b_t is negative. This is also consistent with intuition: In order to eliminate a negative bubble which is very likely to continue to decrease further, the interest rate should be decreased because a rise in the interest rate is usually supposed to decrease the stock price and vice versa. As stated before, although p can be a variable we do not assume it to be state-dependent until next section.

2.3 The Asset Price Bubble as an Endogenous Variable

In the previous subsection we have considered the effect of the asset price bubbles on the output gap and monetary policy with the asset price bubble taken as an exogenous variable. The asset price in reality, however, can be influenced by the monetary policy. Therefore the asset price bubble is probably not an exogenous variable but instead an endogenous one. In this subsection we consider this possibility. Before exploring this problem theoretically we present some evidence of the effect of the short-term interest rate on the asset price bubbles. Namely we estimate the following equation with the OLS with quarterly data

$$b_t = \gamma_0 + \gamma_1b_{t-1} - \gamma_2r_t + \xi_t, \quad \gamma_2 > 0, \quad \xi_t \sim N(0, \sigma_\xi^2). \quad (22)$$

The reason that we assume a minus sign of γ_2 is that it is usually supposed that the increase of the short-term interest rate may lead to a decrease of the

⁴Some empirical evidence of this argument will be shown in the next subsection.

asset price and vice versa. The estimation results are shown in Table 2, with t-statistics in parentheses.⁵

Parameter	US	UK	France	Germany	Italy
γ_1	0.902 (21.877)	0.874 (20.125)	0.909 (24.227)	0.898 (23.050)	0.921 (25.636)
γ_2	-0.288 (2.738)	-0.450 (2.591)	-0.363 (2.583)	-0.339 (2.067)	-0.070 (0.410)
R^2	0.809	0.815	0.838	0.831	0.854

Table 2: Estimation of Eq. (22)

Data source: International Statistics Yearbook. Sample 1970.1-99.1. (UK 70.1-97.1). The short-term interest rates of the US, UK, France, Germany and Italy are the federal funds rate, the treasury bill rate (the UK and France), call money rate and discount rate respectively.

From Table 2 we find that the estimates of γ_2 always have the right sign with significant t-statistics except Italy. If we try the sample 1990-99 for Italy, however, we obtain a significant t-statistics (2.923) with $R^2 = 0.886$ for γ_2 with the right sign. Therefore the evidence above on the whole indicates that the interest rate does exert some effect on the asset price.

Assuming that the asset price bubble can be affected by the monetary policy in a linear manner for simplicity, Eq. (14) can be then changed as

$$b_{t+1} = \begin{cases} b_t(1 + g_1) - \gamma r_t + \varepsilon_{t+1}, & \text{with probability } p \\ b_t(1 - g_2) - \gamma r_t + \varepsilon_{t+1}, & \text{with probability } 1 - p \end{cases} \quad (23)$$

with $\gamma > 0$.⁶

With the asset price bubble process defined by Eq. (23) the optimal monetary policy rule in Eq. (19) can then be correspondingly modified as

$$r_t = f'_1 \pi_t + f'_2 y_t + f'_3 b_t, \quad (24)$$

with

$$f'_1 = \frac{1}{\beta_2 + (1 - \beta_3)\gamma} \left[\beta_2 + \frac{\rho \alpha_1^2 \alpha_2 \Omega_1}{\lambda + \rho \alpha_2^2 \Omega_1} \right]; \quad (25)$$

$$f'_2 = \frac{1}{\beta_2 + (1 - \beta_3)\gamma} \left[\beta_1 + \frac{\rho \alpha_2^2 \alpha_1 \Omega_1}{\lambda + \rho \alpha_2^2 \Omega_1} \right]; \quad (26)$$

$$f'_3 = \frac{1}{\beta_2 + (1 - \beta_3)\gamma} \{1 + (1 - \beta_3)[p(g_1 + g_2) - g_2]\}. \quad (27)$$

⁵We have also estimated Eq. (22) with r_{t-1} instead of r_t and find that the estimates γ_2 have right signs with smaller t-statistics than those shown in Table 2, lying between 1.65 and 2.59 for all countries except Italy.

⁶Here we assume a lag effect of the monetary policy on the bubble to be consistent with our IS curve in Eq. (2), although the estimation of Eq. (22) indicates that the current r_t has more significant t-statistics than the lag of r_t .

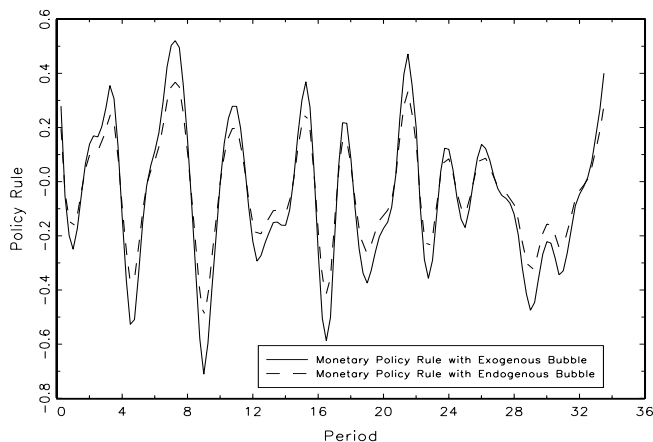


Figure 1: Monetary Policy Rules with Endogenous and Exogenous Asset Price Bubbles

Comparing the monetary policy rule in (24) with that given by (19) we find that the responses to the inflation, output gap and asset price bubbles are smaller in the case of an endogenous b_t than when it is taken as exogenous.⁷ That is, the monetary policy need not to be so “active” as in the case of an exogenous asset price bubble. Moreover, the larger the γ is, that is, the more sensitive the asset price to the short-term interest rate, the lower the responses of the optimal monetary policy rule to the inflation, output and the financial markets.

Numerical Example Next we undertake some simulations. Let $\alpha_1 = 0.85$, $\alpha_2 = 0.2$, $\beta_1 = 0.9$, $\beta_2 = 0.3$, $\rho = 0.985$, $\lambda = 0.5$, $\beta_3 = 0.5$, $g_1 = 0.15$, $g_2 = 0.2$, $p = 0.4$ and $\gamma = 0$, we obtain $f_1 = 2.964$, $f_2 = 3.462$ and $f_3 = 3.233$. This is the situation analyzed in the previous subsection. If we take the asset price bubble as an endogenous variable and let $\gamma = 0.25$, for example, we have $f'_1 = 2.092$, $f'_2 = 2.444$ and $f'_3 = 2.282$, which are smaller than f_1 , f_2 and f_3 respectively, implying that the monetary policy rule does not have to be so active. With the US time series data of b_t from 1966.1-1999.2 and $\pi_0 = y_0 = 0.03$ we show the monetary policy rule with b_t as endogenous and exogenous variables in Figure 1. It is clear that the monetary policy rule with an endogenous bubble fluctuates less than that with an exogenous variable. The former has mean -0.037 with standard deviation (S.D.) 0.187, while the latter has mean -0.052 with S.D. 0.265.

⁷If g_2 is much larger than g_1 , f'_3 can be negative. In this case f'_3 is smaller in absolute value than when b_t is assumed as an exogenous variable.

3 Monetary Policy Rule in Practice: The Case of the Euro-Area

So far we have explored theoretically the monetary policy rule with the asset price volatility considered. The question is then whether the asset price bubbles have been taken into account in practice. This section presents some evidence on this problem.

Following Clarida, Gali and Gertler (1998) (referred to as CGG98 afterwards), Smets (1997) estimates the monetary reaction function of Canada and Australia by adding three financial variables into the CGG98 model, namely, the nominal trade-weighted exchange rate, ten-year nominal bond yield and a broad stock market index. His conclusion is that an appreciation of the exchange rate induces a significant change in the interest rates of the Bank of Canada. Moreover, he finds that changes in the stock market index are also significant in the policy reaction function. For Australia, the coefficients are insignificant.

Bernanke and Gertler (2000) also follow CGG98 by adding stock returns into the model to test whether interest rates respond to stock returns in the US and Japan. Their conclusion is that the federal funds rate does not show a significant response to stock returns from 1979-97. For Japan, however, they find different results: For the whole period 1979-97, there is little evidence that the stock market plays a role in the interest rate setting, but for the two subperiods, 1979-89 and 1989-97, the coefficients of stock returns are significant enough, but with different signs. Rigobon and Sack (2001), however, claim that the US monetary policy reacts significantly to stock market movements: With a 5% rise (fall) in the S & P 500 index increasing the likelihood of a 25 basis point tightening (easing) by about a half.

In this section we also follow CGG98 to test whether the Euro-area monetary policy shows a significant response to the stock market.⁸

CGG98 assumes that the short-term interest rate has the following path:

$$R_t = (1 - \kappa)R_t^* + \kappa R_{t-1} + v_t, \quad (28)$$

where R_t denotes the short-term interest rate, R_t^* the interest rate target, v_t an iid noise with mean zero and constant variance, and κ captures the degree of the interest rate smoothing. The target interest rate is assumed to be determined in the following way:

$$R_t^* = \bar{R} + \beta(E[\pi_{t+n}|\Omega_t] - \pi^*) + \gamma(E[y_t|\Omega_t] - y_t^*),$$

⁸We use the aggregate data of the the three main Euro countries (Germany, France and Italy, referred to as EU3 afterwards) for the estimation. The inflation rate of EU3 is measured by the GDP weighted sum of the inflation rates of the three countries. The output is measured by the GDP weighted sum of the industrial production index and the output gap is measured by the percent deviation of the industrial production index from its HP filtered trend. We also try to compute the potential output by the Band-Pass filter and find that the result is very similar to that computed by the HP filtered trend. The short-term interest rate is measured by the call money rate of Germany. Data source: International Statistics Yearbook.

where \bar{R} is the long-run equilibrium nominal rate, π_{t+n} is the rate of inflation between periods of t and $t+n$, y_t is the real output and π^* and y^* are target levels of the inflation and output respectively. E is the expectation operator and Ω_t is the information available to the central bank at the time it sets the interest rate. After adding the stock market into the equation above we obtain

$$R_t^* = \bar{R} + \beta(E[\pi_{t+n}|\Omega_t] - \pi^*) + \gamma(E[y_t|\Omega_t] - y_t^*) + \theta(E[s_{t+n}|\Omega_t] - \tilde{s}_{t+n}), \quad (29)$$

where s_{t+n} is the asset price in period $t+n$ and \tilde{s}_t denotes the fundamental value of the asset price. We expect θ to be positive, since we assume that central banks try to stabilize the stock market with the interest rate as the instrument. If we define $\alpha = \bar{R} - \beta\pi^*$, $x_t = y_t - y_t^*$ and $b_{t+n} = s_{t+n} - \tilde{s}_{t+n}$ (namely the asset price bubble), equation (29) can be rewritten as

$$R_t^* = \alpha + \beta E[\pi_{t+n}|\Omega_t] + \gamma E[x_t|\Omega_t] + \theta E[b_{t+n}|\Omega_t], \quad (30)$$

after substituting equation (30) into (28), we have the following path for R_t :

$$R_t = (1 - \kappa)\alpha + (1 - \kappa)\beta E[\pi_{t+n}|\Omega_t] + (1 - \kappa)\gamma E[x_t|\Omega_t] + (1 - \kappa)\theta E[b_{t+n}|\Omega_t] + \kappa R_{t-1} + v_t. \quad (31)$$

After eliminating the unobserved forecast variables from the expression, we get the following presentation:

$$R_t = (1 - \kappa)\alpha + (1 - \kappa)\beta\pi_{t+n} + (1 - \kappa)\gamma x_t + (1 - \kappa)\theta b_{t+n} + \kappa R_{t-1} + \eta_t, \quad (32)$$

where $\eta_t = -(1 - \kappa)\{\beta(\pi_{t+n} - E[\pi_{t+n}|\Omega_t]) + \gamma(x_t - E[x_t|\Omega_t]) + \theta(b_{t+n} - E[b_{t+n}|\Omega_t])\} + v_t$ is a linear combination of the forecast errors of the inflation, output gap, asset price bubbles and the iid v_t . Let μ_t be a vector of variables within the central bank's information set at the time it chooses the interest rate that are orthogonal to η_t , we have

$$E[R_t - (1 - \kappa)\alpha - (1 - \kappa)\beta\pi_{t+n} - (1 - \kappa)\gamma x_t - (1 - \kappa)\theta b_{t+n} - \kappa R_{t-1} | \mu_t] = 0. \quad (33)$$

The generalized methods of moments (GMM) will be applied to estimate this equation with quarterly data for the EU3.⁹ Let $\pi_{t+n} = \pi_{t+4}$, as for b_{t+n} we will try with different n (0,1,..4).¹⁰ The estimates with different n of b_{t+n} are presented in Table 3, with t-statistics in parentheses.

⁹As for the details of GMM, the reader is referred to Hamilton (1994). In order to get the initial estimates of the parameters, we estimate the equation with traditional non-linear 2SLS methods first, since η_t is correlated to the independent variables. The instruments include the 1-4 lags of the output gap, inflation rate, Germany call money rate, asset price bubbles, nominal USD/ECU exchange rate and a constant. The instruments are pre-whitened before the estimation. Data source: International Statistics Yearbook.

¹⁰Correction for MA(4) autocorrelation is undertaken, and j-statistics are also presented to see the validity of the over-identifying restrictions.

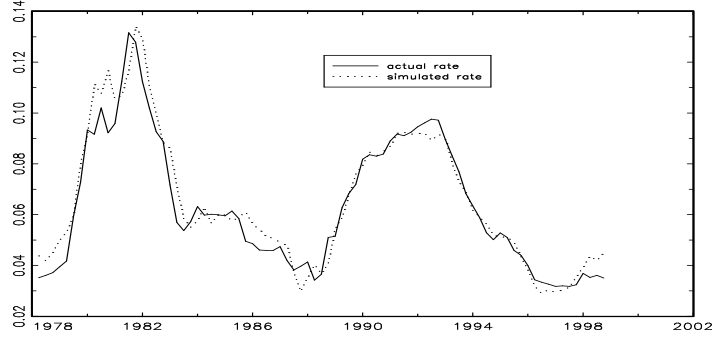


Figure 2: Actual and Simulated Interest Rates of EU3 (1978.1-98.4)

Parameter	Estimates				
	n=0	n=1	n=2	n=3	n=4
κ	0.813 (19.792)	0.811 (18.561)	0.894 (30.224)	0.833 (15.870)	0.832 (17.089)
α	0.030 (4.581)	0.028 (3.920)	0.007 (0.466)	0.020 (1.918)	0.021 (2.074)
β	0.748 (5.446)	0.777 (5.343)	1.522 (3.921)	0.940 (4.410)	0.890 (4.567)
γ	2.046 (5.679)	2.011 (5.300)	1.626 (3.234)	2.345 (3.990)	2.363 (4.203)
θ	0.014 (0.509)	0.030 (0.927)	0.240 (2.328)	0.081 (1.264)	0.082 (1.100)
$J - stat.$	0.088	0.087	0.111	0.069	0.074

Table 3: GMM Estimates of Eq. (33) with Different n for b_{t+n}

As shown in Table 3, β and γ always have the right signs and significant t-statistics no matter which forward value of b_{t+n} is taken, indicating that the inflation and output always play important roles in the interest rate setting. As for θ we find that it always has the right sign, but the t-statistics is not always significant enough. When $n=0,1$ it is insignificant, when $n=3,4$ it is not very significant, but when $n=2$ it is significant enough. Therefore we may say that the asset price may have played some role (although not necessarily an important one) in the interest rate setting in the Euro-area. The simulated interest rate with $b_{t+n} = b_{t+2}$ is presented together with the actual interest rate in Figure 2. It is clear that the two rates are close to each other, especially after the second half of the 1980s.

4 Endogenization of P and Multiple Equilibria

Up to now we have explored monetary policy with the probability of the asset price bubble to increase or decrease next period as a constant. This is in fact a simplified assumption. Monetary policy and other variables can probably influence the path of p . Although Bernanke and Gertler (2000) take it as an exogenous variable because so little is known about the effects of policy actions on p that it is hard to be endogenized. Kent and Lowe (1997), however, endogenize the probability of the bubble to break as follows:

$$p_{t+1} = \phi_0 + \phi_1 b_t + \phi_2 r_t, \quad \phi_i > 0. \quad (34)$$

This function implies that the probability of the asset price bubble to break next period depends on three factors, an exogenous probability ϕ_0 , the size of the current bubble and the level of the current interest rate. The larger the size of the current bubble and the higher the current interest rate, the larger the probability of the bubble to break next period. Note that as mentioned before Kent and Lowe (1997) analyze only positive asset price bubbles. Kent and Lowe (1997) describes the effect of the size of the current bubble on p as follows:

“... as the bubble becomes larger and larger, more and more people identify the increase in asset prices as a bubble and become increasingly reluctant to purchase the asset; this makes it more likely that a correction will occur.” (Kent and Lowe, 1997)

The effect of the current interest rate level on p is clear: As the interest rate increases, the economic agents may expect the asset price to decrease, which as a result raises the probability that the bubble breaks next period.

In this section we will also endogenize the p . Although the function given by Eq. (34) is a reasonable setting-up, we will not adopt it below for several reasons: First, as stated above, Kent and Lowe (1997) explore only positive bubbles, while we consider both positive and negative ones. When the asset price bubble is positive, Eq. (34) is a reasonable choice. But if the bubble is negative, this function should be changed. Second, a probability function should be bounded between 0 and 1, but Eq. (34) is an increasing function without bounds. Thirdly, Eq. (34) is a linear function, indicating that p changes proportionally to the changes of the bubble size and the interest rate. This, however, may not be completely consistent with the practice. The same can be true of the interest rate. Last, the p in our model describes the probability that the bubble will increase (if the bubble is positive) or decrease (if the bubble is negative) next period, while that in the model of Kent and Lowe (1997) describes the probability that the bubble breaks next period.

Before defining the probability function we introduce a function $h(x)$ that will be used below. Exactly speaking we define

$$h(x) = \frac{1}{2}[1 - \tanh(x)]. \quad (35)$$

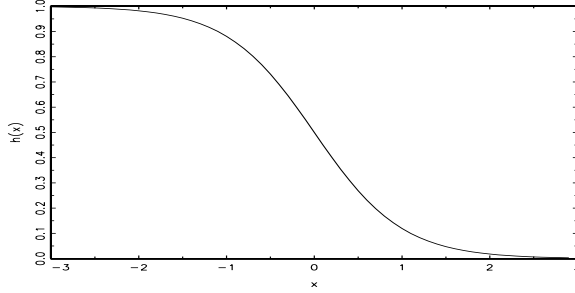


Figure 3: $h(x)$

It is clear that $\frac{dh(x)}{dx} = -\frac{1}{2 \cosh^2(x)} < 0$, with $\lim_{x \rightarrow \infty} h(x) = 0$ and $\lim_{x \rightarrow -\infty} h(x) = 1$. The function $h(x)$ is shown in Figure 3.

Next we define the probability function p_{t+1} as

$$p_{t+1} = \frac{1}{2} \{1 - \tanh[\vartheta(b_t, r_t)]\}, \quad (36)$$

with

$$\vartheta(b_t, r_t) = \phi_1 f(b_t) + \phi_2 \text{sign}(b_t) r_t, \quad \phi_i > 0,$$

where $\text{sign}(b_t)$ is the sign function which reads

$$\text{sign}(b_t) = \begin{cases} 1, & \text{if } b_t > 0; \\ 0, & \text{if } b_t = 0; \\ -1, & \text{if } b_t < 0, \end{cases} \quad (37)$$

and $f(b_t)$ is the so-called LINEX function which is nonnegative and asymmetric around 0. The LINEX function can be found in Varian (1975) and Nobay and Peel (1998) and reads

$$f(x) = \kappa [e^{\varphi x} - \varphi x - 1], \quad \kappa > 0, \varphi \neq 0. \quad (38)$$

κ scales the function and φ determines the asymmetry of the function. An example of $f(x)$ with $\kappa = 0.1$ and $\varphi = \pm 1.2$ is shown in Figure 4. In the work below we take $\kappa = 1$ and $\varphi > 0$. The function $f(x)$ with a positive φ is flatter when x is negative than when it is positive.

It is clear that

$$\frac{\partial p_{t+1}}{\partial b_t} = -\frac{\phi_1 \varphi (e^{\varphi b_t} - 1)}{2 \cosh^2[\vartheta(b_t, r_t)]} \begin{cases} < 0, & \text{if } b_t > 0, \\ > 0, & \text{if } b_t < 0. \end{cases} \quad (39)$$

Therefore, the probability function given by Eq. (36) indicates that the effects of the current asset price bubble b_t on p_{t+1} depends on whether the bubble is positive or negative. In fact, the probability function defined above is asymmetric around $b_t = 0$. If it is positive, a larger bubble in the current period implies

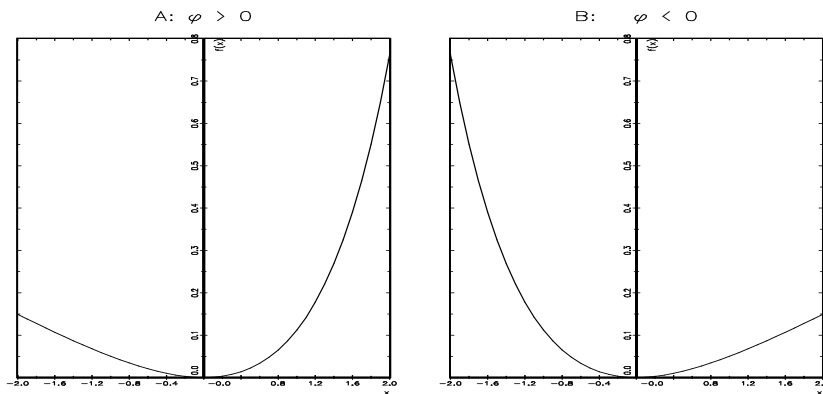


Figure 4: The LINEX Function

a lower probability that it increases next period. This is consistent with the implication of the model of Kent and Lowe (1997): As more and more economic agents realize the bubble and become reluctant to buy the asset as the stock price becomes higher and higher. This in turn prevents the stock price from increasing further. Please remember that if the bubble is negative, p represents the probability that b_t decreases next period. In the case of a negative bubble, Eq. (39) indicates that the larger the bubble in absolute value is (that is, the lower the stock price), the lower the probability that the (negative) bubble continues to decrease next period. The justification is the same as of the positive bubble. As the stock price becomes lower and lower, it is also closer and closer to its lowest point (stock price does not decrease without end!) and is therefore more and more likely to increase in the future. But we assume that the negative bubble does not influence p_{t+1} as greatly as a positive one, because in reality economic agents might usually be more pessimistic in a bear market than optimistic in a bull market.

Moreover, it seems more difficult to activate a financial market when it is in recession than to hold it down when it is in booms. This is what the function $f(b_t)$ implies: It is flatter when $b_t < 0$ than when b_t is positive. An example of p_{t+1} with $\phi_1 = 0.4$, $\varphi = 10$ and $r_t = 0$ is shown in Figure 5, it is flatter when b_t is negative than when b_t is positive. Note that in Figure 5 we find if $b_t = 0$, $p_{t+1} = 0.5$. From the process of the bubble we know if $b_t = 0$ and $r_t = 0$, b_{t+1} is ε_{t+1} which can be either positive or negative. Because little is known about the sign of the noise ε_{t+1} , the economic agents then expect it to be positive or negative with equal probability 0.5.

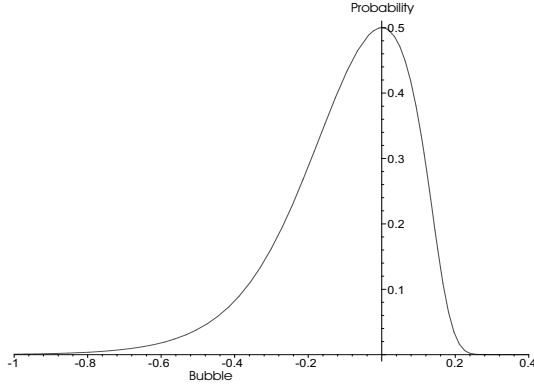


Figure 5: An Example of p_{t+1} with $r_t = 0$

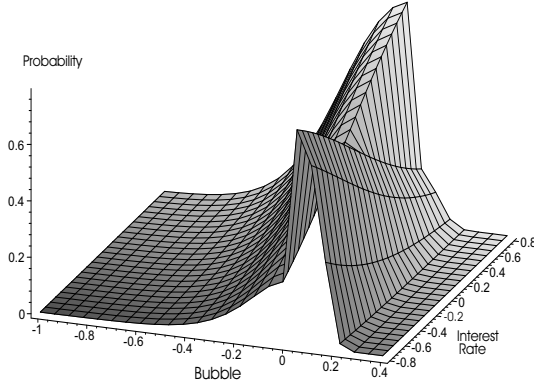


Figure 6: p_{t+1} with $\phi_1 = 0.4$, $\phi_2 = 0.8$ and $\varphi = 10$

The effect of r_t on p_{t+1} can be seen from below:

$$\frac{\partial p_{t+1}}{\partial r_t} = -\frac{\phi_2 \text{sign}(b_t)}{2 \cosh^2[\vartheta(b_t, r_t)]} \begin{cases} < 0, & \text{if } b_t > 0, \\ > 0, & \text{if } b_t < 0. \end{cases} \quad (40)$$

This indicates that if the asset price bubble is positive, an increase in the interest rate will lower the probability that the bubble increases next period. If the bubble is negative, however, an increase in r_t will increase the probability that the bubble decreases next period. This is consistent with the analysis in the previous section that an increase in the interest rate will lower the stock price. The probability function with $\phi_1 = 0.4$, $\phi_2 = 0.8$ and $\varphi = 10$ is shown in Figure 6.

With the probability function defined by Eq. (36) we know that

$$Eb_{t+1|t} = [1 - g_2 + \frac{1}{2}\{1 - \tanh[\vartheta(b_t, r_t)]\}(g_1 + g_2)]b_t - \gamma r_t. \quad (41)$$

Following the same procedure as in the previous sections we obtain the optimal monetary policy rule

$$r_t = f'_1 \pi_t + f'_2 y_t + \frac{2 + (1 - \beta_3)\{g_1 - g_2 - (g_1 + g_2)\tanh[\vartheta(b_t, r_t)]\}}{2[\beta_2 + (1 - \beta_3)\gamma]} b_t, \quad (42)$$

with f'_1 and f'_2 given in (25) and (26). r_t is a linear function of π_t and y_t but nonlinear in b_t . The effect of b_t on r_t is much more complicated than in the previous sections. It can be affected not only by the growth or decrease rate of the bubble, g_1 and g_2 , but also by the parameters, ϕ_1 , ϕ_2 and φ which measure the effects of the bubble and the interest rate on the probability function. It is not difficult to show that given an π_t , y_t and b_t there is a unique r_t . But since r_t is nonlinear in b_t , there can be multiple b_t corresponding to r_t , indicating that there might exist multiple equilibria in such a model. Note that in case $\gamma = 0$, the model collapses to the case of an exogenous asset price bubble. Therefore even if the interest rate does not affect the asset price bubble directly (namely $\gamma = 0$), there may exist multiple equilibria as long as the probability of the asset price to increase or decrease next period is an endogenous variable.

In Figure 7 we show Eq. (42) with $\pi_t = y_t = 0$ with alternative values of the parameters. It is clear that the response of r_t to b_t changes with the parameters. r_t is a monotonic function of b_t when the parameters are assigned some values (see Figure 7-(5) and (6)). When the parameters are assigned some other values, however, r_t is a non-monotonic function of b_t . In Figure 7-(1) and (4) the curve cuts the horizontal axis three times, indicating that there may exist multiple equilibria in the model. The parameters for Figure 7 are set as follows: $\beta_2 = 0.30$, $\phi_1 = 1.0$, $\phi_2 = 0.80$ and $\varphi = 10$. The other parameters β_3 , γ , g_1 and g_2 are assigned different values in different figures as follows: (1), $\beta_3 = 0.005$, $\gamma = 0.90$, $g_1 = 0.001$ and $g_2 = 1.05$; (2), $\beta_3 = 0.10$, $\gamma = 90$, $g_1 = 0.01$ and $g_2 = 0.90$; (3), $\beta_3 = 0.005$, $\gamma = 0.90$, $g_1 = 0.001$ and $g_2 = 0.95$; (4), $\beta_3 = 0.005$, $\gamma = 0.9$, $g_1 = 0.001$ and $g_2 = 1.50$; (5), $\beta_3 = 0.25$, $\gamma = 0.90$, $g_1 = 0.10$ and $g_2 = 6.50$; (6), $\beta_3 = 0.25$, $\gamma = 90$, $g_1 = 0.01$ and $g_2 = 0.70$. With other parameters unchanged, the values of g_1 and g_2 can determine whether r_t is a decreasing or increasing function of b_t . In case g_2 is relatively large, r_t is a decreasing function of b_t , and if g_2 is relatively small, r_t turns to be an increasing function of b_t .

This section endogenizes the probability that the asset price bubble increases or decreases next period. Defining p as a function of the asset price bubble and the current interest rate, we find that the monetary policy turns out to be a nonlinear function of the asset price bubble. Some simulations indicate that there may exist multiple equilibria in the economy.

5 The Zero Bound on the Nominal Interest Rate

Above we have discussed the monetary policy rule with asset prices considered. In the case of a constant probability (p) of the asset price to increase or decrease next period, the optimal monetary policy turns out to be a linear function of

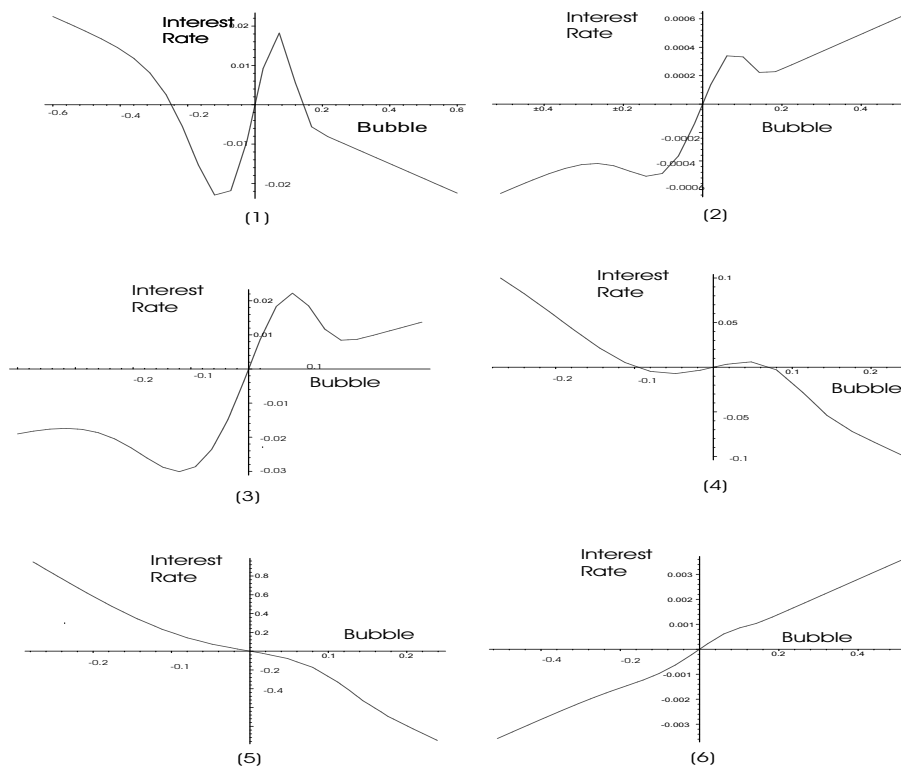


Figure 7: The Response of r_t to b_t with Alternative Values of Parameters

the inflation, output gap and asset price bubbles, similar to the simple Taylor rule except that the asset price bubble is added as an additional term. If p is assumed to be an endogenous variable depending on the monetary policy and the asset price, however, the monetary policy rule turns out to be nonlinear function of the asset price and, moreover, there might exist multiple equilibria in the economy.

A drawback of the Taylor rule, and also of the monetary policy discussed above, is that the monetary policy instrument, the short-term interest rate, is assumed to be able to move without bounds. This is, however, not true in practice and one example is the so-called “Liquidity Trap” in which a monetary policy can not be of much help because the short-term nominal interest rate is almost zero and can not be lowered further. This problem has recently become important because of the “Liquidity Trap” in Japan and the low interest rate in the US. If, furthermore, there is deflation, the real interest rate will rise. Considering the zero bound on the short-term interest rate and the possibility of deflation at very low interest rates, the monetary policy can be very different from that without bounds on the interest rate.

Benhabbib and Schmitt-Grohé (2001), for example, argue that once the zero bound on nominal interest rates is taken into account, the active Taylor rule can easily lead to unexpected consequences. That is, there might exist an infinite number of equilibrium trajectories that originate close to the unique steady state converging to a Liquidity Trap.

Kato and Nishiyama (2001) analytically prove and numerically show that the

optimal monetary policy in the presence of the zero bound is highly nonlinear even in a linear-quadratic model. Eggertsson and Woodford (2003) simulate an economy with zero bound on the interest rate and argue that monetary policy will be effective only if interest rates can be expected to persistently stay low in the future. Coenen and Wieland (2003) also simulate the effect of zero bound of the interest rate on the inflation, output and exchange rate. Ullersma (2001) surveys several researchers' views (Krugman, Meltzer, Buiter and Goodfriend, and Svensson for example) on the zero lower bound.

Most of the recent research on the "Liquidity Trap" has been concerned with deflation, namely the decrease of the price level in the product markets. Yet most literature has ignored the depression in the financial markets. The depression of the financial markets can also be a problem in practice, if the financial markets can influence the output and as a result affects the inflation rate. Take Japan as an example, the share price index (1992=100) was about 200 in 1990 and decreased to lower than 80 in 2001. The industrial production index (IPI, 1992=100) was about 108 in 1990 and fluctuates between 107 and 92 afterwards. The inflation rate (percent changes in the consumer price index), IPI and share price index of Japan are shown in Figure 8A-C. The depression in the share markets seems to be as serious as the deflation. We find that the correlation coefficient between the IPI and share price index is as high as 0.72 from 1980-2001 and the correlation between the IPI and two-quarter lag of the share price index is even as high as 0.80. This seems to suggest that the influence of the financial markets on the output should not be overlooked.

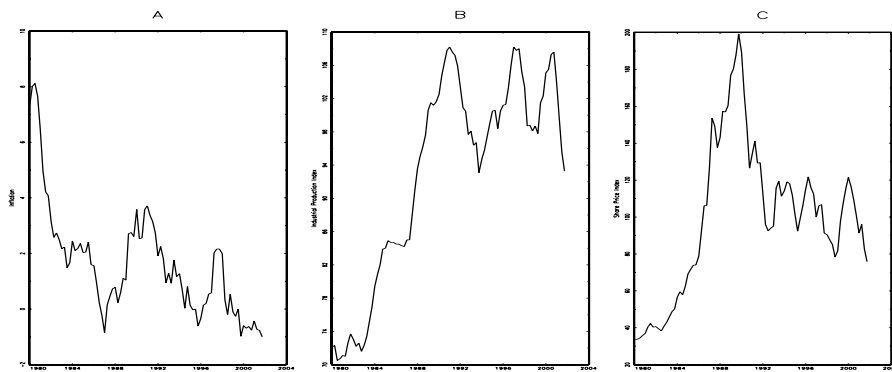


Figure 8: The Inflation Rate, IPI and Share Price Index of Japan 1980.1-2001.4
Data source: International Statistics Yearbook

Let us now return to the liquidity trap problem. The main difference of our research from that of others is that we will explore the zero bound on the nominal interest rate with depression in the financial markets as well as in the product markets (namely deflation).

In our model $r_t = R_t - \bar{R}$ with R_t being the nominal rate and \bar{R} the long-run level of R_t . In the research below we assume $\bar{R} = 0$ for simplicity. In the

presence of the zero bound on the nominal rate, we then assume

$$r_t = \begin{cases} r_o, & \text{if } r_o \geq 0; \\ 0, & \text{if } r_o < 0; \end{cases} \quad (43)$$

where r_o denotes the optimal monetary policy rule derived from the models in the previous sections. The equation above implies that if the optimal monetary policy rule is nonnegative, the central bank will adopt the optimal rule, if the optimal rule is negative, however, the nominal rate is set to zero, since it can not be negative.¹¹

We will first undertake some simulations without asset prices considered as the simple model (1)-(2). The parameters are set as follows: $\alpha_1 = 0.8$, $\alpha_2 = 0.3$, $\beta_1 = 0.9$, $\beta_2 = 0.3$, $\lambda = 0.5$ and $\rho = 0.97$. In order to explore the effect of the zero bound of the nominal rate on the economy, we assume there exists deflation. The starting values of π_t and y_t are set as -0.08 and 0.1 respectively. The optimal monetary policy rule from the basic model is given by eq. (11). The simulations with and without zero bound on the nominal rate are shown in Figure 9.

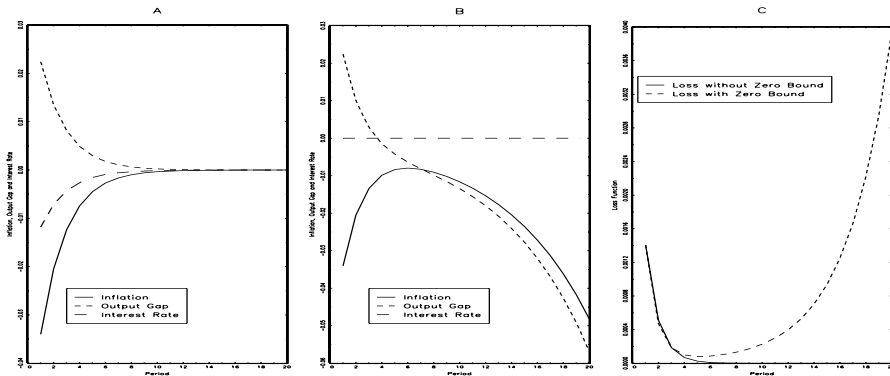


Figure 9: Simulation without Asset Price Considered

In Figure 9A we show the simulation of the inflation, output gap and r_t without zero bound on the nominal rate. Therefore r_t is always set according to (11). It is clear that all three variables converge to zero over time. The loss function can as a result be minimized to zero. Figure 9B shows the simulation with a zero bound on r_t . We find that the optimal nominal rate, which is negative as shown in Figure 9A, can not be reached and has to be set at zero. The inflation and output gaps, as a result, do not converge to zero, but instead evolve in a recession: The deflation becomes more and more severe and the output gap changes from positive to negative and continues to go down over time. Figure 9C shows the loss function $\pi^2 + \lambda y^2$ with and without a zero

¹¹There are some exceptional cases with negative nominal rates, see Cecchetti (1988) for example, but we will ignore these exceptional cases here.

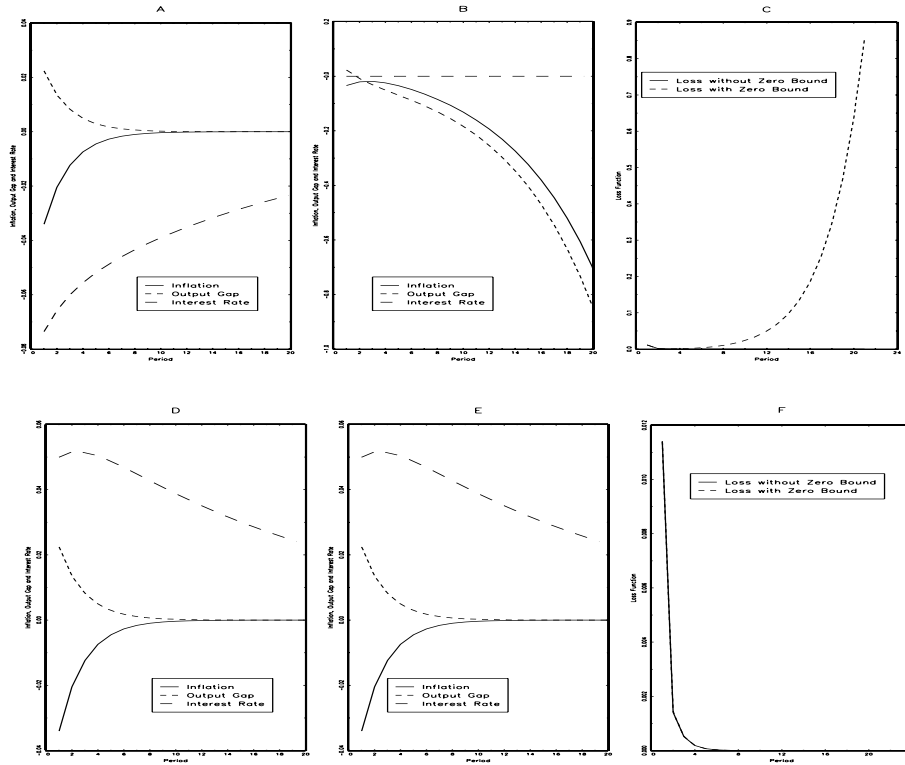


Figure 10: Simulation with Asset Price Considered

bound on the nominal rate. We observe that in the case of no zero bound the loss function converges to zero as π_t and y_t goes to zero. In the presence of a zero bound, however, the loss function increases fast over time because of the recession.

The simulation undertaken above does not consider the effect of the asset prices on the inflation and output. The simulation below assumes that the asset prices can influence the output as eq. (16) and the asset price bubble has the path (14). In order to simplify the simulation we just take $b_{t+1} = Eb_{t+1|t}$, therefore with an initial value of the bubble we can obtain a series of b_t . With other parameters the same as above, the remainder of the parameters is assigned the following values: $g_1 = 0.1$, $g_2 = 0.2$, $p = 0.5$ and $\beta_3 = 0.5$. The initial values of π_t and y_t are the same as above. The initial value of b_t is -0.02 , indicating a depression in the financial markets. The optimal rate r_o is given by eq. (19). The simulations with and without a zero bound on the nominal rate are shown in Figure 10A-C. In Figure 10A we present the simulation without a zero bound on r_t , this is similar to the case in Figure 9A where all three variables converge to zero except that r_t in Figure 10A is lower and converges more slowly than in Figure 9A. Figure 10B shows the simulation with a zero bound on r_t . Again we find that the optimal rate can not be reached and r_t has to be set at zero. The economy experiences a recession. This is similar to the case in Figure 9B, but the recession in Figure 10B is more severe than that in Figure 9B. In Figure 9B π_t and y_t decrease to about -0.06 with $t = 20$, but in Figure 10B, however, π_t and y_t experience larger and faster decreases and go down to about -0.8 in

the same period. This is because the output is affected by the depression in the financial markets (negative b_t) which also accelerates the deflation through the output. In Figure 10C we show the loss function with and without a zero bound on r_t . The loss function when no zero bound occurs converges to zero over time but increases very fast when there exists a zero bound. But the loss function with a zero bound in Figure 10C is higher than that in 9C because of the more severe recession in Figure 10B caused by the financial market depression.

Next we assume that the financial market is not in depression but instead in a boom, that is, the asset price bubble is positive. We set $b_0 = 0.02$ and obtain a series of positive bubbles. The simulation with the same parameters as above is shown in Figure 10D-F. In Figure 10D all three variables converge to zero when no zero bound on r_t is implemented. In Figure 10E, however, all three variables also converge to zero over time even if there exists a zero bound on the nominal rate. This is different from the cases in Figure 9B and 10B where a severe recession occurs. The reason is that in Figure 10E the asset price bubble is positive and the optimal interest rate turns out to be positive. The zero bound on the nominal rate is therefore not binding. As a result, Figure 10E is exactly the same as Figure 10D. The two loss functions with and without a zero bound on the nominal rate are therefore also the same, as shown in Figure 10F.

The simulations in this section indicate that in the presence of a zero bound on the nominal interest rate, a deflation can become more severe and the economy may move into a severe recession. Moreover, the recession can be worse if the financial market is also in a depression, because the asset price depression can then decrease the output and as a result makes the deflation more severe. Facing the zero bound on the nominal rate and a Liquidity Trap, some researchers have proposed some policy actions, see Clouse et al. (2000) for example. Our simulations above indicate that policy actions that aim at escaping a Liquidity Trap should not ignore the asset prices, since the financial market depression can make the economy recession worse.

On the other hand, a positive asset price bubble can make the zero bound on the nominal rate non-binding, since the optimal rate which takes the financial markets into account may be higher than zero even if there exists deflation. This case has been shown in Figure 10E.

Note that the simulations undertaken above are based on the simple model in which the probability (p) that the asset price increases or decreases next period is assumed to be exogenous. If p is taken as an endogenous variable, however, the analysis is more complicated. In the basic model we find that the optimal monetary policy rule turns out to be a linear function of b_t , but in the model with an endogenous p , the monetary policy rule is nonlinear in the asset price. This has been shown in the simulations in Figure 7. In the case of a linear rule it is clear that a negative asset price bubble lowers the optimal policy rule and therefore increases the likelihood of the zero bound on the nominal rate to be binding, while a positive asset price bubble increases the optimal nominal rate and therefore reduces the likelihood of the zero bound rate to be binding. When the optimal policy rule is a nonlinear function of the asset price, however,

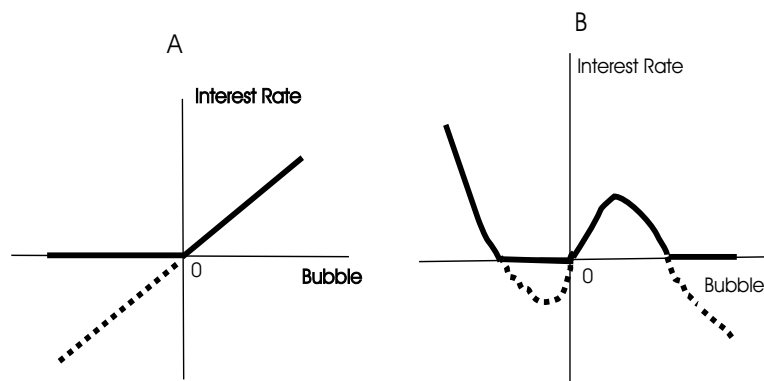


Figure 11: The Linear and Nonlinear Policy Rules in the Presence of the Zero Bound on the Nominal Rate

a positive bubble can also increase the likelihood of the zero bound of r_t to be binding, since the optimal rule can be negative even if the bubble is positive. Similarly, a negative bubble may reduce the likelihood of the zero bound to be binding because the optimal rule can be positive even if the bubble is negative. The linear and nonlinear policy rules in the presence of a zero bound on the nominal rate are shown in Figure 11A-B. Figure 11B looks similar to Figure 7-(1). In Figure 11 we set the optimal rule to be zero if it is negative. In some extreme cases, the endogenous p can make the optimal policy rule very different from that with a constant p . Figure 7-(5) is a good example: Unlike the linear rule which is an increasing function of the asset price bubble, r_t in Figure 7-(5) is a decreasing function of b_t and the effect of the zero bound of the nominal rate on the economy through the channel of financial markets can therefore be greatly changed.

6 Conclusion

A dynamic model has been set up to explore the monetary policy with asset prices. In the case of an endogenous asset price bubble, we find that the monetary policy does not have to be so active as in the case of an exogenous bubble. If the probability that the asset price bubble increases or decreases next period is assumed as constant, the monetary policy just turns out to be a linear function of the state variables. If such a probability is endogenized as a function of the asset price bubble and interest rate, however, the policy reaction function becomes nonlinear and there may exist multiple equilibria in the economy. Some empirical evidence has also been shown and it is found that the monetary policy rule in the Euro-area has taken into account the financial markets to some extent in the past two decades. We have also explored the zero bound on the nominal interest rate. The simulations indicate that a depression of the financial markets can make a recession economy worse in the presence of a lower

bound on the nominal rate. Therefore policy actions which aim at escaping a Liquidity Trap should not ignore the financial markets. We have also shown that the effect of the zero bound of the nominal rate on the economy can be greatly changed if the probability that the asset price increases or decreases next period is an endogenous variable instead of an exogenous one.

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