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Stepwise Calibration of a Higher-Order Keynes-Metzler-Goodwin Model

by

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Abstract

The paper puts forward a deterministic macrodynamic model of the business cycle that allows for sluggish price and quantity adjustments in response to disequilibrium on product and labour markets. Based on regular oscillations of two exogenous variables, 14 reaction coefficients are calibrated such that the cyclical patterns of the endogenous variables are broadly compatible with stylized facts. The calibration procedure is organized in a hierarchical structure, so that subsets of the parameters can be determined step by step. Subsequently, the exogenous variables are endogenized and the additional parameters are chosen. The resulting dynamic system, which in its reduced form is of dimension six, generates persistent cyclical behaviour with similar time series properties of the variables as found before.

JEL classification: E 12.

Keywords: Calibration, business cycle, disequilibrium adjustments, wage-price dynamics, aggregate demand.

Contents

1 Introduction

The paper takes up a deterministic macrodynamic modelling framework from the literature; Chiarella and Flaschel (2000, Ch. 6), Chiarella et al. (2000, Chs 3 and 4), Flaschel et al. (2001). Allowing for disequilibrium on the product and labour markets, which gives rise to sluggish price and quantity adjustments, it incorporates elements of economic theory that are, in particular, connected with the names of Keynes, Metzler and Goodwin. Briefly, Keynesian elements are encountered in the treatment of aggregate demand (besides an LM-sector), Metzler has stimulated the modelling of production and inventory investment decisions, and Goodwinian ideas are reflected in the income distribution dynamics. Although each modelling block is quite simple, the model in its entirety is of dimension six. It is thus still possible to carry out a mathematical analysis that delivers meaningful conditions for local stability (see Köper, 2000), but an investigation of the global dynamics of the system has to rely on computer simulations. This, in turn, raises the problem of setting the numerical parameters, especially the reaction coefficients. After all, even without the investment function there are 14 parameters to be determined.

One approach to numerical parameter setting is, of course, econometric estimation. Using single equation or subsystem estimations, this approach has been employed for a slightly different version of the model by Flaschel et al. (20001). We do not, however, think that this study has already settled the issue. The presentation is not always transparent, and not all coefficients seem credible.[1] Supplementarily to this kind of work, we therefore choose another approach. That is, referring to a business cycle context we seek to calibrate the model.

A few words may be in order to clarify the concept of calibration as we understand it here. The aim of calibrating a model economy is to conduct (computer) experiments in which its properties are derived and compared to those of an actual economy. In this respect calibration procedures can be viewed as a more elaborate version of the standard back-of-the-envelope calculations that theorists perform to judge the validity of a model. The underlying notion is that every model is known to be false. A model is not a null hypothesis to be tested, it is rather an improper or simplified approximation of the true data generating process of the actual data. Hence, a calibrator is not interested in verifying whether the model is true (the answer is already known from the outset), but in identifying which aspects of the data a false model can replicate.[2]

Our investigation of how well the model-generated trajectories match the data follows the usual practice. We select a set of stylized facts of the business cycle, simulate the model on the

[1] For example, in the working paper version (October 2000) the time unit underlying the fluctuations in the time series diagrams is not made explicit; the stock adjustment speed is implausibly low; or a discussion of the cyclical implications for the real wage dynamics is missing.

[2] See also the introductory discussion in Canova and Ortega (2000, pp. 400–403).

computer, and assess the corresponding cyclical properties of the resulting time series in a more or less informal way. Since a (false) model is chosen on the basis of the questions it allows to ask, and not on its being realistic or being able to best mimic the data, we share the point of view that rough reproduction of simple statistics for comovements and variability is all that is needed to evaluate the implications of a model.[3] In sum, our philosophy of setting the numerical parameters is similar to that of the real business cycle school, though the methods will be different in detail.

It turns out that the model gives rise to a hierarchical structure in the calibration process. Some variables which are exogenous in one model building block are endogenous within another module at a higher level. Thus, the parameters need not all be chosen simultaneously, but fall into several subsets that can be successively determined. This handy feature makes the search for suitable parameters and the kind of compromises one has to accept more intelligible.

The evaluation of the numerical parameters takes place at three stages. Most of the work is done at the first stage. Here we suppose exogenous motions of two exogenous variables that drive the rest of the model. These are capacity utilization and, synchronously with it, the capital growth rate. Since random shocks are neglected in our framework, the exogenous motions may well be of a regular and strictly periodic nature, most conveniently sine waves. This perhaps somewhat unusual approach can be viewed as a heuristic device. It is more carefully defended later in the paper.

Tying ourselves down to a base scenario, it is then checked at a second stage whether the previous results are seriously affected if the exogenous sine waves are replaced with the more noisy time paths of the empirical counterpart of the utilization variable and the thus induced capital growth rate.

The decisive test to which the numerical parameters are put is, however, stage three. Here we endogenize capacity utilization and propose an investment function. Setting the parameters thus newly introduced, the model is now fully endogenous and we can study the properties of the time series it generates. The calibration will have passed this test if the model produces persistent cyclical behaviour with similar features as found before.

The remainder of the paper is organized as follows. Section 2 expounds the stylized facts of the business cycle that will be used as guidelines. Section 3 presents the model at calibration level 1 – 3, which determine the wage-price dynamics. Section 4 turns to the interest rate and then to demand and the quantity adjustments on the goods market, with the parameters to be set at level 4 – 6. The main calibration is undertaken in Section 5. It puts forward the numerical coefficients

[3] As Summers (1991, p. 145) has expressed his skepticism about decisive formal econometric tests of hypotheses, "the empirical facts of which we are most confident and which provide the most secure basis for theory are those that require the least sophisticated statistical analysis to perceive."

and discusses their cyclical implications, and the kind of compromises we make, at stage one and two of the analysis. The complete endogenous model, stage three, is examined in Section 6. Section 7 concludes. An appendix makes explicit the details concerning the construction of the empirical time series we are referring to.

2 Stylized facts

Our measure of the business cycle is capacity utilization u. As we use it, this notion rests on an output-capital ratio y^n that would prevail under 'normal' conditions. With respect to a given stock of fixed capital K, productive capacity is correspondingly defined as $Y^n = y^n K$. Y being total output and y the output-capital ratio, capacity utilization is thus given by $u = Y/Y^n = y/y^n$. Against this theoretical background, we may take the motions of the output-capital ratio in the firm sector (nonfinancial corporate business) as the empirical counterpart of the fluctuations of u.

In the models' production technology, y^n is treated as a constant. In reality, there are some variations in y at lower than the business cycle frequencies. We therefore detrend the empirical series of y and, treating the 'normal' output-capital ratio as variable over time, set $y^n = y^n_t$ equal to the trend value of y at time t. In this way, the model's deviations from normal utilization, $u{-}1$, can be identified with the empirical trend deviations $(y_t - y^n_t)/y^n_t$.

To correct for the low frequency variation of y, the Hodrick-Prescott (HP) filter is adopted. Choosing a smoothing parameter $\lambda = 1600$ for the quarterly data and looking at the resulting time series plot, one may feel that the trend line nestles too closely against the actual time path of y. This phenomenon is not too surprising since the HP 1600 filter amounts to defining the business cycle by those fluctuations in the time series that have periodicity of less than eight years (cf. King and Rebelo, 1999, p. 934), whereas the US post-war economy experienced two trough-to-trough cycles that exceed this period.[4] Other filters, such as HP with values of $\lambda = 6400$ or higher, or a segmented linear trend, correspond better to what one may draw freehand as an intuitive trend line in a diagram. However, the cyclical pattern of the trend deviations is in all cases very similar,

[4] According to the NBER reference data, one is from February 1961 to November 1970, the other from November 1982 to March 1991. In recent times, the band-pass (BP) filter developed by Baxter and King (1996) has gained in popularity. On the basis of spectral analysis, this procedure is mathematically more precise about what constitutes a cyclical component. The BP(6,32) filter preserves fluctuations with periodicities between six quarters and eight years, and eliminates all other fluctuations, both the low frequency fluctuations that are associated with trend growth and the high frequencies associated with, for instance, measurement error. More exactly, with finite data sets the BP(6,32) filter approximates such an ideal filter. As it turns out, for the time series with relatively low noise (little high frequency variation) the outcome of the HP 1600 and the BP(6,32) filter is almost the same. For real national US output, this is exemplified in King and Rebelo (1999, p. 933, fig. 1).

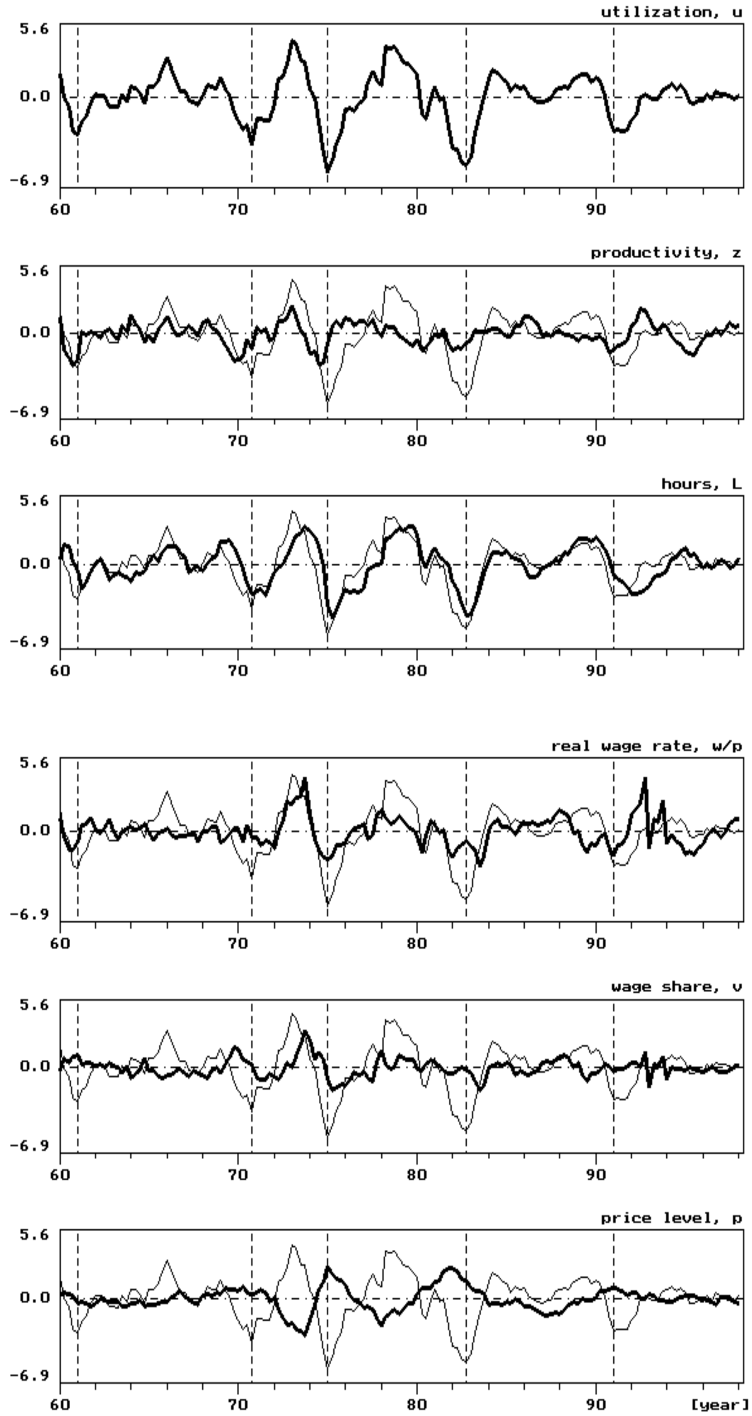

Figure 1: Cyclical components of empirical series 1.

Note: Variables measured in per cent of their trend values (HP 1600). The thin line is the cyclical component of utilization.

only the amplitudes are somewhat larger. Because in the literature the HP filter is based on $\lambda = 1600$ with almost no exception, we may just as well follow this conventional practice. The trend deviations of the output-capital series thus obtained, or of capacity utilization $u-1$, for that matter, are exhibited in the top panel of Figure 1.

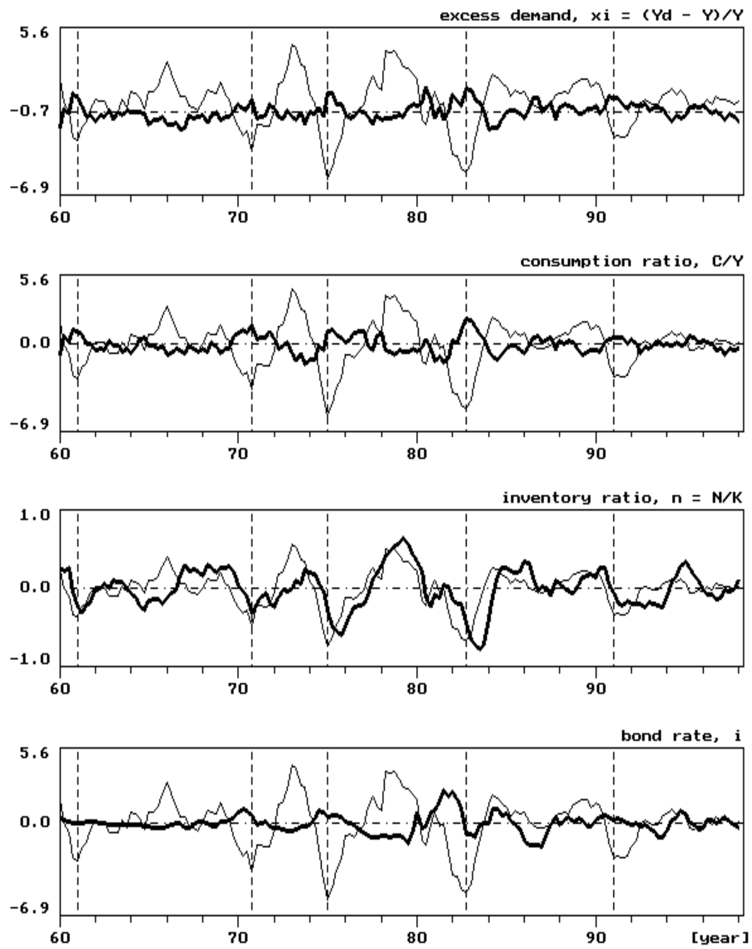

Figure 2: Cyclical components of empirical series 2.

Note: Differences between variables and their trend values (HP 1600), both measured in percentage points. The thin line is the cyclical component of utilization (in the third panel it is scaled down by the standard deviation of n).

The HP 1600 filter is also applied to the other empirical series we are studying. The fact that the trend deviations of these cyclical components might likewise appear somewhat narrow need not be of great concern to us. It will serve our purpose to express their standard deviations in terms of the standard deviation of u.

In the calibration procedure, we are concerned with the cyclical behaviour of nine endogenous variables. Regarding the wage-price dynamics, these are the employment rate e, labour productivity z, the (productivity-deflated) real wage rate ω, the wage share v, and the price level, p. In addition, with respect to the goods and money markets we are interested in excess demand ξ (in relative terms, $\xi = (Y^d - Y)/Y$, where Y^d are real sales), in the consumption ratio C/Y, the inventory ratio $n = N/K$ (N the stock of inventories), and the bond rate of interest i. The empirical counterparts of these variables are depicted as the bold lines in Figures 1 and 2. For a first assessment of their cyclical properties and the size of their variation, the thin lines reproduce the reference series of capacity utilization. Source and construction of the empirical data are described in the appendix.

Note that in Figure 1 the cyclical components are measured in per cent of the trend values, that is, a variable x_t with trend values x_t^o is represented as $100 \cdot (x_t - x_t^o)/x_t^o$. By contrast, the trend deviations of the variables in Figure 2, which are already themselves expressed in percentage points, are just the differences $x_t - x_t^o$ between the original values and the trend values. Relative excess demand ξ is plotted directly. Here the reference line is not the zero level but is drawn at -0.657%, which is the time average of the series. It is explained in the theoretical part that on average ξ should indeed be slightly less than zero since a small fraction of production is excepted from being sold on the market and put to inventories to keep them growing with the rest of economy.

The first endogenous variable, labour productivity z, has to be dealt with since in the modelling framework it connects, on the one hand, the employment rate with utilization and, on the other hand, the real wage rate with the wage share. Labour productivity has since long been counted a procyclical variable. May it suffice to mention that Okun (1980, pp. 821f) lists it among his stylized facts of the business cycle. Procyclical variations of z can to some degree also be recognized in the second panel in Figure 1, perhaps with a slight lead before u. The cross-correlation coefficients quantifying the comovements of z with u are given in Table 1, whose sample period 1961 – 91 covers four major trough-to-trough cycles. Reckoning in a lead of z between one and three quarters, these statistics indicate a stronger relationship between z and u than one might possibly infer from a visual inspection of the time series alone.[5]

To get information about the employment rate, we refer to total working hours L. For simplicity, we directly interpret the trend line, $L^o = L_t^o$, as labour supply, i.e., as supply of normal

[5]Unfortunately, the statistics cannot be compared with the most recent comprehensive compilation of stylized business cycle facts by Stock and Watson (1999), since they employ real GDP as a measure of the business cycle. Over the sample period 1953 – 96, they report a cross-correlation coefficient as large as $\rho(z_{t-k}, \mathrm{GDP}_t) = 0.72$ for a lead of $k = 2$. Curiously enough, we could not reproduce a similar number with the trend deviations of the GDP series taken from Ray Fair's database (see the appendix), which is due to the fact that (especially) over the subperiod 1975 – 82 this series is quite different from the Citibase GDP series used by Stock and Watson (statistically, it shows less first-order autocorrelation).

Series x	σ_x/σ_u	cross correlations between u at time t and x at time $t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$
u	$--$	0.48	0.70	0.89	1.00	0.89	0.70	0.48
z	0.44	0.56	0.58	0.53	0.46	0.17	−0.06	−0.27
L	0.83	0.03	0.30	0.57	0.79	0.88	0.86	0.77
w/p	0.51	0.31	0.48	0.57	0.61	0.56	0.48	0.34
v	0.38	−0.21	−0.05	0.09	0.21	0.42	0.53	0.57
p	0.51	−0.59	−0.70	−0.73	−0.70	−0.62	−0.49	−0.32
ξ	0.32	−0.29	−0.39	−0.49	−0.62	−0.52	−0.35	−0.17
C/Y	0.35	0.07	−0.17	−0.43	−0.68	−0.69	−0.62	−0.51
n	0.13	0.01	0.17	0.36	0.59	0.74	0.81	0.79
i	0.36	−0.59	−0.59	−0.50	−0.37	−0.27	−0.18	−0.09
g_k	0.29	−0.06	0.20	0.48	0.72	0.84	0.86	0.80

Table 1: Descriptive statistics for cyclical components of quarterly series, 1961:1 – 91:4.

Note: All series detrended by Hodrick-Prescott (with smoothing factor $\lambda = 1600$). g_k is the capital growth rate, notation of the other variables as in Figures 1 and 2. σ designates the standard deviation.

working hours. In this view, the normal employment rate is given by $e = 1$, and the deviations from normal employment are proxied by $e_t - 1 = (L_t - L_t^o)/L_t^o \approx \ln(L_t - L_t^o)$, which is the series displayed in the third panel in Figure 1. The juxtaposition with utilization in the same panel makes clear that this employment rate is markedly procyclical. The third line in Table 1 details that it lags one or two quarters behind u.

The controversy surrounding the comovements of the real wage rate is usually summarized by saying that, if anything, it moves (weakly) procyclical, rather than countercyclical. Results about the cyclical properties of the real wage appear to be quite sensitive to precisely how it is constructed, depending on whether the numerator (w) includes various compensation items and on the index in the denominator (p). Since our modelling context is a one-good economy, we adopt the deflator of total output as our price level, so that w/p denotes the product real wage. On the other hand, we follow Ray Fair's procedure (see the appendix) and include a uniform 50% wage premium as a rough measure for overtime payment.

On the basis of this specification, Figure 1 (fourth panel) shows that the real wage rate is fairly close connected to the motions of capacity utilization, while quantitative evidence for its procyclicality is given in Table 1. Although this finding is in some contrast to what is reported in the literature, it should play an important role in the calibration later on.[6]

The variable that more directly describes the distribution of income between workers and capital owners is the wage share v. It is only rarely mentioned in the discussion of typical features of a business cycle. This might in part also be due to the special difficulties that one encounters for this variable in separating the cyclical from some intermediate quasi-trend behaviour. The HP 1600 trend deviations depicted in the fifth panel in Figure 1 may therefore be taken with some care.

Accepting them as they are, we see another explanation for the infrequent reference to the wage share: it does not exhibit a distinctive and unique cyclical pattern. Over the 1960s, v looks rather countercyclical, whereas from 1970 to 1990 it appears to be more or less procyclical. In fact, over the 1960s the highest (in modulus) correlation coefficient is negative, as large as $\rho(u_t, v_{t-1}) = -0.71$. Over the period 1970 – 91 the maximal coefficient is positive; at a lag of three quarters it amounts to $\rho(u_t, v_{t+3}) = 0.67$. For this reason the cross-correlations given in Table 1 over the full period 1961 – 91 have to be cautiously interpreted. They do not summarize a general law of a systematic relationship between the business cycle and income distribution, but they sort of average these different relationships.

As far as price inflation is concerned, it has to be noticed that time series of inflation rates are relatively noisy and so cannot be easily related to the motions of utilization with its high persistence.[7] It is therefore more convenient to study the variations of the price level directly. While prices were formerly treated as procyclical, there seems now to be general consensus that their cyclical component moves countercyclical; see, for example, Cooley and Ohanian (1991), Backus and Kehoe (1992), Fiorito and Kollintzas (1994). With respect to the price index for total output, this phenomenon is plainly visible in the bottom panel of Figure 1. According to Table 1, the inverse relationship between p and u is strongest at a lead of the price level by one quarter.

[6] For example, King and Rebelo (1999, p. 938) obtain a contemporaneous correlation of compensation per hour with output of $\rho = 0.12$, and the coefficient for the correlation with GDP that is presented by Stock and Watson (1999, Table 2) is similarly low. As regards the present data, with no overtime payment in the wage rate the contemporaneous correlation is reduced to 0.34 (and no lagged coefficients are higher), even though the correlation of the trend deviations of the two real wage time series is as high as 0.93. On the other hand, considering the issue more carefully, Barsky et al. (1994) argue that real wage indexes may fail to capture changes in the composition of employment over the cycle. They conclude that real wages are procyclical if the composition is held constant.

[7] Quarterly inflation rates have first-order serial correlation in the region of 0.35, which may be compared to the AR(1) coefficients for the trend deviations of u and p, which are 0.89 and 0.92, respectively.

Given the tightness of the relationship, countercyclical prices are a challenge for any theory of inflation within a business cycle context.[8]

The next set of variables are related to the goods market. The crucial point is that we here allow for disequilibrium, which is buffered by inventories. It is well-known that in low-dimensional versions of our Metzlerian modelling approach, inventory investment can possibly be strongly destabilizing through an accelerator mechanism. Because the motions of inventories and their feedbacks on the rest of the economy are determined by the variations of excess demand, it is important to have a representation of this latter variable with reliable cyclical properties. The top panel of Figure 2 shows that relative excess demand $\xi = (Y^d - Y)/Y$ behaves in fact quite systematically. That is, ξ displays a fairly consistent countercyclical pattern, though at a much lower amplitude than utilization. This is numerically confirmed in Table 1.

Given that in other model variants some components of aggregate demand may be more flexible than they presently are, we may also study consumption on its own. Referring to the consumption ratio C/Y, it is seen that this series exhibits similar properties as ξ.

The state variable in the model that keeps track of inventories is the inventory ratio $n = N/K$. The third panel in Figure 2 indicates that the motions of the capital stock and excess demand give rise to a markedly procyclical behaviour of this ratio, with a short lag of two or three quarters. The variation of n is, however, quite small (note the different scale of n in Figure 2).

The final endogenous variable is the bond rate i in the bottom panel of Figure 2. Since the modelling of the financial sector and monetary policy will remain at a very elementary level, we should be content with meeting only some crude qualitative features of this variable.

Table 1 concludes the review of our business cycle variables with the growth rate of fixed capital g_k. It will be the second exogenous variable in the calibration study, whose cyclical properties will be considered further below.

On the basis of the statistics in Table 1, we summarize the cyclical features that one may ideally wish a small (deterministic) macrodynamic model to generate — at least insofar as it exhibits smooth and regular oscillations. Leaving some small play in the numbers, they are listed in Table 2. When it states a zero lag for productivity z, then this is already due to the simplifying modelling assumption on the production technology in the next section.

[8] A discussion of the issue of countercyclical prices should make clear what in (structural and descriptive) economic theory the trend line is supposed to reflect: (a) the evolution of prices on a deterministic long-run equilibrium path around which the actual economy is continuously fluctuating, or (b) the time path of an expected price level. From the latter point of view, Smant (1998) argues that other procedures than HP detrending should be adopted and, doing this, concludes that the so specified (unexpected) price movements are clearly procyclical (p. 159). By contrast, our theoretical background is notion (a).

variable x	σ_x/σ_u	lag x
dev z	0.40	0.00
e	0.75	0.00 – 0.75
dev ω	0.45 – 0.50	−0.50 – 0.50
dev v	0.30 – 0.40	— —
− dev p	0.45 – 0.50	−0.75 – 0.25
ξ	0.28 – 0.35	−0.50 – 0.50
C/Y	0.30 – 0.40	−0.25 – 0.75
n	0.10 – 0.15	0.00 – 0.75
i	0.30 – 0.40	— —

Table 2: Desirable features of macrodynamic oscillations.

Note: 'dev' means deviations from trend or steady state values in per cent, ω is the (productivity-deflated) real wage rate. The lags are measured in years.

A direct implication of the specification of technology is that, independently of the rest of the model, any standard deviation of z can be achieved. The reason for fixing σ_z somewhat lower than the coefficient 0.44 given in Table 1 is the apparently lower amplitude of z in the recent past. In fact, over the sample period 1975 – 91, the ratio σ_z/σ_u falls to 0.33 (and the relationship with utilization becomes weaker). The reduction of σ_z/σ_u should carry over to the variations of employment, hence the proportionately lower value of σ_e/σ_u.

We should not be too definitive about the variation of the wage share, either, because the precise empirical construction of this variable and the outcome of the specific detrending mechanism may not be overly robust against alternative procedures. By the same token, it would not be appropriate to commit oneself to a particular phase shift of v. This is all the more true when the lead in labour productivity is neglected (the relationship between the wage share and productivity is made explicit in eq. (4) below). Given that $\sigma_v/\sigma_u = 0.31$ over the subperiod 1975 – 91, we content ourselves with proposing the range $0.30 - 0.40$ for that ratio and leave the issue of desirable lags of v open.[9]

The desired statistics of the remaining four endogenous variables are straightforward. Our reduced ambitions regarding the cyclical pattern of the bond rate have already been mentioned.

[9] The ratios $\sigma_{w/p}/\sigma_u$ and σ_p/σ_u are more stable. For the same subperiod 1975 – 91, they amount to 0.46 and 0.50, respectively.

3 Wage-price dynamics

Wage and price adjustments are represented by two Phillips curves. Besides the standard arguments, which are the employment rate e for the wage Phillips curve and capacity utilization u for the price Phillips curve, both curves will also include the wage share v as an additional factor. As is shortly made explicit, e as well as v are connected with capacity utilization through average labour productivity $z = Y/L$. So the evolution of z has to be dealt with first.

While we wish to account for the procyclicality of productivity, for a small macrodynamic model to be analytically tractable this should be done in a simplified way. We therefore neglect the lead of z in the comovements with u and postulate a direct positive effect of u on the percentage deviations of z from its trend value z^o.[10] Like the functional specifications to follow, we assume linearity in this relationship,

$$z/z^o \quad = \quad f_z(u) \;:=\; 1 + \beta_{zu}\,(u-1) \tag{1}$$

β_{zu} and all other β-coefficients later on are positive constants.

Trend productivity is assumed to grow at an exogenous constant rate g_z. To deal with dynamic relationships, it is convenient to work in continuous time (where for a dynamic variable $x = x(t)$, $\dot{x}$ is its time derivative, $\hat{x}$ its growth rate; $\dot{x} = dx/dt$, $\hat{x} = \dot{x}/x$). Thus,

$$\hat{z}^o \quad = \quad g_z \tag{2}$$

Trend productivity also serves to deflate real wages, or to express them in efficiency units. We correspondingly define

$$\omega \quad = \quad w\,/\,p\,z^o \tag{3}$$

For short, ω itself may henceforth be referred to as the real wage rate. Obviously, if w/p grows steadily at g_z, the rate of technical progress, ω remains fixed over time. Since $v = wL/pY = (w/pz^o)\,(z^oL/Y) = (w/pz^o)\,(z^o/z)$, the wage share and the real wage rate are linked together by

$$v \quad = \quad \omega\,/\,f_z(u) \tag{4}$$

To express the employment rate by variables which in a full model would constitute some of the dynamic state variables, we decompose it as $e = L/L^s = z^o\,(L/Y)\,(Y/Y^n)\,(Y^n/K)\,(K/\;z^oL^s)$, where L^s is the labour supply (which in the previous section was proxied by the trend values of working hours, L^o). As indicated before, productive capacity is given by $Y^n = y^nK$ with y^n a

[10] Leaving aside (suitably scaled and autocorrelated) random shocks to the technology, an immediate explanation of the comovements of z and u may be overhead labour and labour hoarding.

fixed technological coefficient, and utilization is $u = Y/Y^n$. Hence, if we denote capital per head in efficiency units by k^s,

$$k^s \;=\; K \,/\, z^o L^s \tag{5}$$

the employment rate can be written as

$$e \;=\; y^n \, u \, k^s \,/\, f_z(u) \tag{6}$$

Assuming a constant growth rate g_ℓ for the labour supply,

$$\hat{L}^s \;=\; g_\ell \tag{7}$$

and denoting the (variable) capital growth rate by g_k, the motions of k^s are described by the differential equation

$$\dot{k}^s \;=\; k^s \,(g_k - g_z - g_\ell) \tag{8}$$

It has been mentioned in the introduction that our investigations are based on exogenous oscillations of utilization together with the capital growth rate. Once the time paths $u = u(t)$ and $g_k = g_k(t)$ are given, the time path of the employment rate is determined as well, *via* (8) and (6) — independently of the rest of the economy. The only parameter here involved is β_{zu} from the hypothesis on labour productivity in eq. (1). This constitutes the first level in the hierarchy of calibration steps. We summarize:

<u>*Level 1:*</u> employment rate e (parameter β_{zu})

$$\dot{k}^s \;=\; k^s \,(g_k - g_z - g_\ell) \tag{8}$$

$$e \;=\; y^n \, u \, k^s \,/\, [1 + \beta_{zu} \, (u-1)] \tag{6}$$

We can thus turn to the Phillips curve mechanism for the nominal wage rate w. The usual positive feedback from the employment rate is here augmented by a negative feedback from the wage share, an effect that will turn out to be essential in the calibration of the real wage dynamics. The theoretical content of this extension is discussed in Franke (2001).[11] Apart from that, the changes in w are measured against the changes in prices and trend labour productivity. Regarding

[11] It may directly be argued that at relatively low levels of the wage share, workers seek to catch up to what is considered a normal, or 'fair', level, and that this is to some degree taken up in a wage bargaining process. More rigorously, the negative wage share effect can also be derived from the wage setting model by Blanchard and Katz (1999, p. 6), which makes reference to the workers' reservation wage and interpret it as depending on labour productivity and lagged wages.

benchmark inflation, we allow for an influence of current inflation, $\hat{p}$, as well as a general "inflation climate", which is designated π. So we have

$$\hat{w} \;=\; \hat{z}^o \,+\, \kappa_{wp}\hat{p} + (1-\kappa_{wp})\pi \;+\; f_w(e,v) \tag{9}$$

$$f_w \;=\; f_w(e,v) := \;\beta_{we}\,(e-1) \;-\; \beta_{wv}\,(v-v^o)/v^o \tag{10}$$

where the abbreviation f_w will simplify the presentation below, κ_{wp} is a weighting parameter between 0 and 1, unity is the normal rate of employment, and v^o serves as a reference value for the wage share.

As for the price adjustments, price Phillips curves are a flexible concept which is at the theoretical core of a variety of macroeconometric models.[12] We employ the following version:

$$\hat{p} \;=\; \kappa_{pw}(\hat{w}-\hat{z}^o) + (1-\kappa_{pw})\pi \;+\; f_p(u,v) \tag{11}$$

$$f_p \;=\; f_p(u,v) := \;\beta_{pu}\,(u-1) \;+\; \beta_{pv}\,[(1+\mu^o)v-1] \tag{12}$$

The parameter κ_{pw} $(0 \le \kappa_{pw} \le 1)$ weights the influence of wage inflation (corrected for technical progress) and the inflation climate, which provides a benchmark. As utilization u reflects the pressure of demand, the term $\beta_{pu}(u-1)$ signifies a demand-pull term. The final component, $\beta_{pv}\,[(1+\mu^o)v-1]$, can be viewed as a cost-push term proper, which goes beyond taking the present inflationary situation into account. Devising μ^o as a target markup rate, we mean by this that prices tend to rise by more than what is captured by the other terms if labour costs are so high that, at current prices, $p < (1+\mu^o)wL/Y$, which is equivalent to $0 < (1+\mu^o)wL/pY - 1 = (1+\mu^o)v - 1$. For reasons of consistency it is assumed that the target markup is compatible with the normal level v^o of the wage share in (2), i.e., $(1+\mu^o)v^o = 1$.[13]

Since in (1) and (3), $\hat{w}$ and $\hat{p}$ are mutually dependent on each other, in the next step the two equations have to be solved for $\hat{w}$ and $\hat{p}$. In the resulting reduced-form expressions for wage and price inflation it has to be ruled out that the weights κ_{pw} and κ_{wp} are both unity. Obviously, wage inflation depends on the core terms in the price Phillips curve, and price inflation on the core terms in the wage Phillips curve:

$$\hat{w} \;=\; \hat{z}^o \,+\, \pi \,+\, \kappa\,[\kappa_{wp}f_p(u,v) + f_w(e,v)] \tag{13}$$

$$\hat{p} \;=\; \pi \,+\, \kappa\,[f_p(u,v) + \kappa_{pw}f_w(e,v)] \tag{14}$$

$$\kappa \;=\; 1\,/\,(1-\kappa_{pw}\kappa_{wp}) \tag{15}$$

It is then seen that in the growth rate of the real wage, $\hat{\omega} = \hat{w} - \hat{p} - \hat{z}^o$, not only trend productivity growth but also the inflation climate π cancels out. This independence of the income distribution

[12] For an elaboration of this point see Chiarella et al. (2000, pp. 52ff).

[13] Empirical support for a positive impact of v on $\hat{p}$ can be inferred from Brayton et al. (1999, pp. 22–27). This is more clearly explained in Franke (2001).

dynamics from inflationary expectations may be considered a particularly attractive feature of the approach with two Phillips curves. On the other hand, in general six parameters are entering at this level, which is also emphasized by the notation of the functional expressions f_p and f_w:

<u>*Level 2:*</u> real wage ω, wage share v (κ_{pw}, κ_{wp}, β_{pu}, β_{pv}, β_{we}, β_{wv})

$$\dot{\omega} = \omega\,\kappa\,[\,(1-\kappa_{pw})f_w(e,v;\beta_{we},\beta_{wv}) \;-\; (1-\kappa_{wp})f_p(u,v;\beta_{pu},\beta_{pv})\,] \tag{16}$$

$$v = \omega \,/\, f_z(u) \tag{4}$$

$$\kappa = 1\,/\,(1-\kappa_{pw}\kappa_{wp}) \tag{7}$$

The inflation climate does have a bearing on the rate of inflation. The law of governing the variations of π is specified as a mix of two simple mechanisms. One of them, adaptive expectations, often proves destabilizing if the speed of adjustment is high enough. The other rule, regressive expectations, constitutes a negative feedback. Introducing the weight $\kappa_{\pi p}$ and adopting π^o as a 'normal' value of inflation (or the steady state value in a full model), and β_π as the general adjustment speed, we posit

$$\dot{\pi} = \beta_\pi\,[\kappa_{\pi p}(\hat{p}-\pi) \,+\, (1-\kappa_{\pi p})(\pi^o-\pi)] \tag{9}$$

Though after the intellectual triumph of the rational expectations hypothesis, working with adaptive expectations has become something of a heresy, in a disequilibrium context there are a number of theoretical and empirical arguments which demonstrate that adaptive expectations make more sense than is usually attributed to them (see Flaschel et al., 1997, pp. 149–162; or more extensively, Franke, 1999). This is all the more true if π is not inflation expected for the next quarter, but if it is employed as a benchmark value in wage and price decisions, alternatively to current inflation. Since, on the other hand, π will not be decoupled from the recent time path of inflation, it makes sense if π adjusts gradually in the direction of $\hat{p}$. The regressive mechanism in (1), by contrast, expresses a 'fundamentalist' view, in the sense that the public perceives a certain tendency of inflation to return to normal after some time.[14]

Taken on their own, both principles ($\kappa_{\pi p}=1$ or $\kappa_{\pi p}=0$) are of course rather mechanical. They are, however, easy to integrate into an existing macrodynamic framework and, in their combination of stabilizing and destabilizing forces, already allow for some flexibility in modelling the continuous revision of benchmark rates of inflation.

[14] The general idea that an inflation expectations mechanism, which includes past observed rates of inflation only (rather than observed increases in the money supply), may contain an adaptive and a regressive element is not new and can, for example, already been found in Mussa (1975). The specific functional form of eq. (1) is borrowed from Groth (1988, p. 254).

The time paths of $\pi(\cdot)$ from (1) will evidently lag behind actual inflation $\hat{p}(\cdot)$. This, as such, is no reason for concern, it is even consistent with inflationary expectations themselves that are made in the real world. Here forecast errors are found to be very persistent, and forecasts of inflation often appear to be biased (see, e.g., Evans and Wachtel, 1993, fig. 1 on p. 477, and pp. 481ff).

The time paths of $\omega(\cdot)$ and $v(\cdot)$ being computed at level 2, eq. (6) for $\hat{p}$ can be plugged in the dynamic equation (1) for the adjustments of π. Subsequently, the solution of $\pi(\cdot)$ can be used in (6) to record the time path of the inflation rate. Apart from the two parameters β_π, $\kappa_{\pi p}$, all parameters have already been set at level 2. We review these operations in one step at calibration level 3:

<u>*Level 3:*</u> price inflation $\hat{p}$, inflation climate π (parameters β_π, $\kappa_{\pi p}$)

$$\dot{\pi} = \beta_\pi \left[\kappa_{\pi p}(\hat{p} - \pi) + (1-\kappa_{\pi p})(\pi^o - \pi)\right] \qquad (1)$$

$$\hat{p} = \pi + \kappa \left[f_p(u,v) + \kappa_{pw} f_w(e,v)\right] \qquad (6)$$

4 Supply and demand on goods and money markets

4.1 The money market

Financial markets are treated at a textbook level. Three assets are considered: money, government bonds and equities, but the price for equities remains in the background and the only financial variable with which we are concerned is the bond rate of interest i. Given the money supply M, i is determined by an LM equation of the form

$$M = pY\,(\beta_{mo} - \beta_{mi}\, i) \qquad (2)$$

In intensive form with output-capital ratio $y = Y/K = (Y/Y^n)(Y^n/K)$ and real balances normalized by the capital stock, $m = M/pK$, eq. (1) is readily solved as

$$i = (\beta_{mo} - m/y)\,/\,\beta_{mi} \qquad (3)$$

$$y = u\,y^n \qquad (4)$$

The responsiveness of money demand is best measured by the interest elasticity $\eta_{m,i}$, which may be conceived as a positive number. Referring to an equilibrium position with output-capital ratio $y^o = y^n$, a real balances ratio m^o and bond rate i^o, the elasticity is defined as $\eta_{m,i} = \beta_{mi}\, i^o/(\beta_{mo} - \beta_{mi}\, i^o) = \beta_{mi}\, i^o/(m^o/y^n)$. Hence, if for the calibration we choose a value of the interest elasticity, the two coefficients β_{mo} and β_{mi} are computed as

$$\beta_{mi} = \eta_{m,i}\, m^o\,/\,y^n\, i^o \qquad (5)$$

$$\beta_{mo} \;=\; \beta_{mi}\, i^o \,+\, m^o/y^n \tag{6}$$

Monetary policy itself is supposed to be completely neutral by keeping up a constant growth rate g_m of the money supply,

$$\hat{M} \;=\; g_m \tag{7}$$

By logarithmic differentiation of $m = M/pK$, real balances therefore evolve according to the differential equation

$$\dot{m} \;=\; m\,(g_m - \hat{p} - g_k) \tag{8}$$

Since $g_k(\cdot)$ is exogenous and the time path of $\hat{p}(\cdot)$ is obtained at level 3 of the calibration, no further parameter is needed to determine the solution of (7). On this basis, we can then study the implications of different values of the interest elasticity $\eta_{m,i}$ for the motions of the interest rate i. In sum:

<u>*Level 4:*</u> interest rate i (parameter $\eta_{m,i}$)

$$\begin{aligned}
\dot{m} &= m\,(g_m - \hat{p} - g_k) && (7)\\
i &= (\beta_{mo} - m/y)\,/\,\beta_{mi} && (2)\\
y &= u\,y^n && (3)\\
\beta_{mi} &= \eta_{m,i}\,m^o\,/\,y^n\,i^o && (4)\\
\beta_{mo} &= \beta_{mi}\,i^o + m^o/y^n && (5)
\end{aligned}$$

4.2 Excess demand for goods

In modelling disequilibrium on the goods market, it is assumed that demand for final goods is always realized. This demand is satisfied from current production and the existing stocks of inventories, while any excess of production over sales replenishes inventories. The thus implied motions of inventories are discussed below. Let us first discuss aggregate demand Y^d, which is made up of consumption C, net investment in fixed capital I, replacement investment δK (δ the constant rate of depreciation), and real government spending G,

$$Y^d \;=\; C + I + \delta K + G \tag{8}$$

Among the endogenous components of demand, the most important feedback effects are contained in consumption demand of private households. Here we differentiate between workers and asset owners, or more precisely, between consumption financed out of wage income and consumption financed out of rental income. As for the former, it is assumed for simplicity that disposable wage income is exclusively spent on consumption. With respect to a tax rate τ_w and hours worked L, this component of (nominal) consumption expenditures is given by $(1-\tau_w)wL$.

Next, let B be variable-interest bonds outstanding, whose price is fixed at unity. Disposable income of asset owners consists of interest payments iB plus dividends from firms, minus taxes pT^c. A fraction s_c of this income is saved, the remainder is consumed. Regarding dividends, firms are supposed to pay out all net earnings to the shareholders, where the earnings concept may be based on expected sales, Y^e. Another assumption is that equities are the only external source of financing fixed investment, so that firms incur no interest on debt. Hence dividends are given by $pY^e - wL - \delta\, pK$, and (nominal) consumption spending out of total rental income is $(1-s_c)\,(pY^e - wL - \delta pK + iB - pT^c)$.

In addition to consumption out of wage and rental income, we identify consumption by that part of the population who do not earn income from economic activities, like people living on welfare or unemployment benefits, or retired people drawing on a pension. Since these expenditures are not too closely linked to the business cycle, they may be assumed to grow with the capital stock pK, as governed by a coefficient $c_p > 0$.[15] We expect this type of consumption to help account for the observed countercyclical consumption ratio C/Y.

Collecting the terms of the three consumption components, total consumption expenditures sum up to

$$pC \;=\; c_p\, pK \;+\; (1-\tau_w)wL \;+\; (1-s_c)\,(pY^e - wL - \delta\, pK + iB - pT^c) \tag{9}$$

Fiscal policy, too, should presently play a neutral role, with minimal feedbacks on the private sector. This most conveniently means that government spending G and taxes T^c, which are net of real interest receipts, are postulated to remain in a fixed proportion to the capital stock:

$$G \;=\; \gamma\, K \tag{10}$$

$$T^c \;=\; \theta_c\, K + iB/p \tag{11}$$

On this basis basis aggregate demand, normalized by the capital stock, is now fully determined. Defining the constant term a_y,

$$a_y \;:=\; c_p + \gamma + s_c\delta - (1-s_c)\theta_c \tag{12}$$

[15] $c_p pK$ can be thought of as being financed by taxes. In a full model this expression would also have to show up in the government budget restraint, which lends $c_p pK$ the same formal status as government expenditures. A part of the tax collections could be conceived of as payments into a pension fund, which are directly passed on to retired people. Admittedly, this interpretation neglects the fact that pension funds accumulate financial assets and actively operate on the financial markets, which might be an issue for a more elaborated financial sector.

dividing (1) by K, using (2) – (4), and denoting $y^e = Y^e/K$, $y^d = Y^d/K$, we arrive at

$$y^d \;=\; (1-s_c)y^e \,+\, (s_c-\tau_w)vy \,+\, g_k \,+\, a_y \tag{13}$$

The parameters entering (5) and (6), however, cannot all be freely chosen. We shall later directly set the equilibrium values g^o for the real growth rate, v^o for the wage share, and $y^o = y^n$ for the output-capital ratio. When discussing the production decisions of firms in the next subsection, it will also be shown that equilibrium demand $(y^d)^o$ is slightly less than output y^o. We here anticipate that the two are connected through g^o and another parameter β_{ny}, which is related to inventories: $(y^d)^o = y^n/(1+\beta_{ny}g^o)$ (cf. eqs (12) and (1) below). β_{ny} will be equally determined in advance, as will be the parameters δ, γ and θ_c in (5). Considering (6) in the steady state with $y^e = y^d$ and solving this equation for a_y, we have therefore only two 'free' parameters left on which this magnitude depends, namely, the tax rate on wages τ_w and the propensity s_c to save out of rental income. In explicit terms, a_y and subsequently c_p result like

$$a_y = a_y(s_c,\tau_w) \;=\; s_c y^n/(1+\beta_{ny}g^o) \,-\, (s_c-\tau_w)v^o y^n \,-\, g^o \tag{14}$$
$$c_p = c_p(s_c,\tau_w) \;=\; a_y(s_c,\tau_w) \,-\, \gamma \,-\, s_c\delta \,+\, (1-s_c)\theta_c \tag{15}$$

As we are concerned with the motions of relative excess demand $\xi = (Y^d - Y)/Y$, it remains to put

$$\xi \;=\; y^d/y \,-\, 1 \tag{16}$$

In particular in models where the rigid rule (3) for government expenditures is relaxed, one may also be interested in the cyclical pattern of the consumption ratio C/Y. Using (2) and (4), it is given by

$$C/Y \;=\; (s_c-\tau_w)\,v \;+\; [\,(1-s_c)\,y^e + c_p - (1-s_c)(\delta+\theta_c)\,]\,/\,y \tag{17}$$

4.3 Production and inventory decisions

The modelling of stock management and production of firms follows the production-smoothing / buffer-stock approach, which was initiated by Metzler (1941). Although in recent times its economic significance has been questioned (cf. the survey article by Blinder and Maccini 1991), it was demonstrated in Franke (1996) that it can be made compatible with the main stylized facts of the inventory cycle.

The approach distinguishes between actual and desired changes in inventories. The actual change is just the difference between production Y and sales = demand Y^d,

$$\dot{N} \;=\; Y \,-\, Y^d \tag{18}$$

Desired inventory changes are based on a ratio β_{ny} of inventories over expected sales. Correspondingly, the desired level N^d of inventories is given by

$$N^d \;=\; \beta_{ny}\, Y^e \tag{19}$$

N^d generally differs from N, and firms seek to close this gap gradually with speed β_{nn}. That is, if everything else remained fixed, the stock of inventories would reach its target level in $1/\beta_{nn}$ years. In addition, firms have to account for the overall growth of the economy, for which they employ the long-run equilibrium growth rate g^o. The desired change in inventories, designated I_N^d, thus reads

$$I_N^d \;=\; g^o N^d \;+\; \beta_{nn}\,(N^d - N) \tag{20}$$

Eq. (13) is the basis of the so-called production-smoothing model; e.g., see Blinder and Maccini (1991, especially p. 81).

Production of firms takes care of these desired inventory changes. Otherwise, of course, firms produce to meet expected demand,

$$Y \;=\; Y^e \;+\; I_N^d \tag{21}$$

Eq. (14) represents the buffer-stock aspect. In fact, by inserting (14) into (11), which yields $\dot N = I_N^d + (Y^e - Y^d)$, it is seen that sales surprises are completely buffered by inventories.

In specifying the formation of sales expectations, we assume adaptive expectations as a straightforward device. Invoking growth similarly as in (13), they take the form[16]

$$\dot Y^e \;=\; g^o Y^e \;+\; \beta_y\,(Y^d - Y^e) \tag{22}$$

The time rate of change of the expected sales ratio $y^e = Y^e/K$ is then obtained from $\hat y^e = \hat Y^e - \hat K = g^o + \beta_y[(Y^d - Y^e)/K]\cdot(K/Y^e) - g_k$. The implied evolution of inventories, equally studied in the intensive form of the inventory ratio $n = N/K$, derives from (11) and $\hat n = \hat N - \hat K = (\dot N/K)\cdot(K/N) - g_k = [(Y - Y^d)/K]/n - g_k$.

On the whole, the goods market dynamics is represented by the following set of equations. Although they require no more input variables, computed at a higher level, than the motions of the rate of interest at level level 4, we assign them level 5. Not only would other numbering

[16] As an alternative to the usual interpretation of partial adjustments of expected sales Y^e towards realized sales Y^d, (15) can also be viewed as an approximation to the results of extrapolative forecasts on the basis of a rolling sample period. If the latter has length T, the speed of adjustment β_y is related to T by $\beta_y = 4/T$ (Franke 1992). Such extrapolative predictions are in the same spirit as the simple extrapolative forecasts that Irvine (1981, p. 635) reports to be common practice in real-world retailer forecasting.

conventions be more cumbersome, later extensions of the present model might also include the interest rate as another argument in private consumption.

<u>*Level 5:*</u> excess demand ξ, consumption ratio C/Y, inventory ratio n
(parameters s_c, τ_w, β_y)

$$\xi \;=\; y^d/y \,-\, 1 \tag{9}$$

$$C/Y \;=\; (s_c - \tau_w)\, v \;+\; [\,(1-s_c)\, y^e + c_p - (1-s_c)(\delta + \theta_c)\,]\,/\,y \tag{10}$$

$$y^d \;=\; (1-s_c)y^e \,+\, (s_c - \tau_w)vy \,+\, g_k \,+\, a_y \tag{6}$$

$$\dot{y}^e \;=\; (g^o - g_k)\, y^e \,+\, \beta_y\, (y^d - y^e) \tag{16}$$

$$\dot{n} \;=\; y \,-\, y^d \,-\, n g_k \tag{17}$$

$$a_y = a_y(s_c, \tau_w) \;=\; s_c y^n/(1 + \beta_{ny} g^o) \,-\, (s_c - \tau_w) v^o y^n \,-\, g^o \tag{7}$$

$$c_p \,=\, c_p(s_c, \tau_w) \;=\; a_y(s_c, \tau_w) \,-\, \gamma \,-\, s_c\delta \,+\, (1-s_c)\theta_c \tag{8}$$

4.4 Endogenous utilization

It may have been noticed that one behavioural parameter has not yet been made use of, namely, the stock-adjustment speed β_{nn} from eq. (13). Even more important, the previous subsection has put forward a theory of production that so far has not been fully exploited. The point is that the output level in (14) implies an endogenous determination of utilization. So we face the following situation: the exogenous variations of utilization u and the capital growth rate g_k give rise to variations in income distribution (and inflation), which in turn determine aggregate demand, which in turn determines sales expectations and the motions of inventories, from which then firms derive their production decisions and, thus, the utilization of their present productive capacity.

Denoting the endogenously determined value of utilization by u^{endo}, the crucial problem is how such an endogenous time path of $u^{endo}(\cdot)$ compares to the exogenous time path $u(\cdot)$ from which it has been ultimately generated. Ideally, we would like the two trajectories $u^{endo}(\cdot)$ and $u(\cdot)$ to coincide. That is, we are looking for a set of parameters that not only produce acceptable cyclical patterns for the variables already discussed, but which also imply that the underlying motions of utilization exhibit a fixed-point property. We will certainly be content if the time paths of $u^{endo}(\cdot)$ and $u(\cdot)$ are close, while too large discrepancies between the two would clearly be dubious.

In detail, using (14), (13), (12), u^{endo} is determined from $Y = Y^e + I^d_N = Y^e + (g^o + \beta_{nn})\beta_{ny} Y^e - \beta_{nn} N$. Division by K gives the endogenous output-capital ratio y^{endo} as a function of y^e and n,

$$y^{endo} \;=\; f_y(y^e, n) := \,[\,1 + (g^o + \beta_{nn})\beta_{ny}\,]\, y^e \,-\, \beta_{nn} n \tag{3}$$

where now also the abovementioned parameter β_{nn} comes in. β_{nn} can therefore be set at level 6 of the calibration procedure.

<u>*Level 6:*</u> endogenous utilization u^{endo} (parameter β_{nn})

$$u^{endo} = f_y(y^e, n) \,/\, y^n \qquad (4)$$

$$f_y(y^e, n) = [\,1 + (g^o + \beta_{nn})\beta_{ny}\,]\, y^e - \beta_{nn} n \qquad (1)$$

At the end of the section, we may provide the argument determining the steady state value of y^d, which entered the coefficient a_y in (7) above. In the same step, the equilibrium value for the inventory ratio n can be derived. Note first that $\dot{y}^e = 0$ and $g_k = g^o$ in (1) gives $y^d = y^e$ in the steady state. Then, putting $y = f_y(y^e, n)$ and, in eq. (2), $\dot{n} = 0$, we obtain $0 = y - y^d - n g_k = [1 + (g^o + \beta_{nn})\beta_{ny}]\, y^e - \beta_{nn} n - y^e - n g^o = (g^o + \beta_{nn})\beta_{ny}\, y^e - (g^o + \beta_{nn})\, n$; hence $n = \beta_{ny} y^e$. Inserting this in $y = f_y(y^e, n)$ leads to $y = (1 + \beta_{ny} g^o) y^e$. In sum,

$$(y^d)^o = (y^e)^o = y^n \,/\, (1 + \beta_{ny} g^o) \qquad (3)$$

$$n^o = \beta_{ny}\, y^n \,/\, (1 + \beta_{ny} g^o) \qquad (4)$$

5 Calibration of the model

5.1 The exogenous oscillations

As indicated in Table 2, on the whole we are interested in the cyclical behaviour of nine endogenous variables. In the calibration procedure itself, two variables will be exogenous: utilization u and the capital growth rate g_k. Once their time paths are given, the motions of the endogenous variables follow, successively, from the equations summarized under 'level 1' to 'level 6'. To this end, we assume regular oscillations of u and g_k . For convenience, they may take the form of sine waves.

Sine waves would be the outcome in a linear deterministic model, but such undampened and persistent oscillations will there only occur by a fluke. Self-sustained cyclical behaviour in a deterministic modelling framework will accordingly be typically nonlinear, so that even if the solution paths were quite regular, they would still be more or less distinct from a sine wave motion. Unfortunately, we have no clue in what form the endogenous oscillations are affected by these nonlinearities. Any proposal in this direction would have to introduce additional hypotheses, for which presently no solid indications exist. Note that the empirical time series in Figures 1 and 2 do not seem to exhibit any systematic asymmetries, a visual impression which is largely confirmed by the literature.[17] At least the symmetry in the sine waves would therefore be no

[17] A standard reference is DeLong and Summers (1986). For a more sophisticated appproach, see Razzak (2001).

counter-argument.

It may, on the other hand, be argued that the exogenous variables be driven by a random process. An obvious problem with this device is that our approach has not intended to mimic the random properties of the time series under study. The model could therefore not be evaluated by statistical methods, unless it were augmented by some random variables (cf. Gregory and Smith, 1993, p. 716). Similar as with the nonlinearities just mentioned, however, there are no clear options for such stochastic extensions. Hence, exogenous stochastic fluctuations would here be no less arbitrary than the deterministic sine waves.[18]

There is also another point why random perturbations cannot be readily introduced into the present deterministic framework. It relates to the fact that the sine waves generate (approximately) symmetrical oscillations of the endogenous variables around the steady state values, provided the initial conditions are suitably chosen. This phenomenon is more important than it might seem at first sight, because it allows us to maintain $v^o, 1, \mu^o, \pi^o$ as constant benchmark values in the adjustment functions (2), (4), (1). By contrast, in a stochastic setting there may easily arise asymmetric fluctuations in the medium term, especially if, realistically, the exogenous random process has a near-unit root. The asymmetry that over a longer time horizon utilization, for example, would be more above than below unity would lead to systematic distortions in the adjustment mechanisms. The distortions may be even so strong that they prompt the question if the adjustment rules still continue to make economic sense.[19]

Our methodological standpoint is that sine wave motions of the exogenous variables are a reasonable starting point to begin with. We will, however, not stop there. After deciding on a combination of reasonable parameter values, we will replace the sine waves with a special 'random' series of the exogenous variables, that is, with the empirical trend deviations. In this second step we will have to check if the basic properties of the endogenous variables are at least qualitatively preserved.

The ensuing third step is the decisive one. Here utilization as well as the capital growth

[18] To underline that stochastic simulations are no easy way out, we may quote from a short contribution to an econometric symposium: "Most econometricians are so used to dealing with stochastic models that they are rarely aware of the limitations of this approach", a main point being that "all stochastic assumptions, such as assumptions on the stochastic structure of the noise terms, are not innocent at all, in particular if there is no a priori reasoning for their justification" (Deistler, 2001, p. 72). More specifically, regarding a random shock term in a price Phillips curve, which (especially in the context of monetary policy) may possibly have grave consequences for the properties of a stochastic model, McCallum (2001, pp. 5f) emphasizes that its existence and nature is an unresolved issue, even when it is only treated as white noise.

[19] To avoid dubious adjustments in these circumstances, the benchmark values might themselves be specified as (slowly) adjusting variables, similar as, for example, a time-varying NAIRU in empirical Phillips curve estimations. While this device may be appealing, it would add further components — and parameters — to the model.

rate are endogenized, which, in particular, means we still have to set up an investment function. Once starting values of the dynamic state variables are given, the evolution of the economy will then be completely determined. Satisfactory cyclical patterns of the variables generated within the full (deterministic) model will be the final proof for the proposed parameter scenario. In this perspective, the initial sine wave experiments are a heuristic device to find, step by step, or level by level, promising numerical values for the many parameters in the model.

After these introductory methodological remarks, we can turn to the numerical details of the sine wave oscillations. As the US economy went through four cycles between 1961 and 1991, and another cycle seems to have expanded over roughly the last ten years, we base our investigations on a cycle period of eight years. For utilization, we furthermore assume an amplitude of $\pm 4\%$, so that we have

$$u(t) \quad = \quad 1 \; + \; 0.04 \cdot \sin(2\pi t/8) \tag{5}$$

The amplitude amounts to a standard deviation of $u(\cdot)$ over the eight-year cycle of 2.84%, while the corresponding empirical value is 2.05%. We opt for the higher amplitude because of our feeling expressed in Section 2, that the HP 1600 trend line of the empirical output-capital ratio absorbs too much medium frequency variation. The choice of the amplitude is, however, only for concreteness and has no consequences for setting the parameters since the amplitudes, or standard deviations, of the endogenous variables will always be related to that of utilization.

In contrast, it should be pointed out that for some variables the duration of the cycle does matter. It obviously makes a difference for the amplitude whether, with respect to a fixed adjustment coefficient and thus similar rates of change per unit of time, a variable increases for 24 months or only for, say, 18 months.

Regarding the motions of the capital growth rate, we see in Table 1 that it lags utilization by one or two quarters. In economic theory, this delay is usually ascribed to an implementation lag, according to which investment decisions might respond quite directly to utilization or similarly fluctuating variables, but it takes some time until the investment projects are completely carried out and the plant and equipment has actually been built up. For simplicity, most macro models neglect the implementation lag, so that utilization and the capital growth rate tend to move in line (though this will have to be an endogenous feature of any particular model). For this reason, we assume that g_k is perfectly synchronized with u. According to the ratio of the two standard deviations reported in Table 1, the amplitude of g_k is a fraction of 0.29 of the 4% in (3). Thus,

$$g_k(t) \quad = \quad g^o \; + \; 0.29 \cdot [u(t) - 1] \tag{6}$$

where g^o is the long-run equilibrium growth rate introduced in eqs (13) and (15) in Section 4.3. By a most elementary growth accounting identity, g^o is given by adding up the (constant) growth rates of labour supply g_ℓ and trend productivity g_z. As 3% is the order of magnitude of the average growth rate of real output over the period 1960 – 98, we specify,

$$g_z \;=\; 0.02\,, \qquad g_\ell \;=\; 0.01\,, \qquad g^o \;=\; g_z + g_\ell \;=\; 0.03 \tag{7}$$

5.2 Steady state values and other constant relationships

Before beginning with the calibration of the adjustment parameters of level 1 to level 6, a number of more 'technical' coefficients have to be set, which presumably have a lesser bearing on the dynamics. These are the steady state ratios and certain coefficients in the demand relationships. (Incidentally, they do not enter the calibration until level 4.) Continuing to denote steady state values by a superscript 'o', our numerical choice is as follows:

$$\begin{array}{rclrclrclrcl} y^n &=& 0.70 & v^o &=& 70\% & \mu^o &=& 0.429 & \delta &=& 9.5\% \\ (k^s)^o &=& 1.429 & \pi^o &=& 3\% & i^o &=& 7\% & m^o &=& 0.140 \\ \gamma &=& 0.077 & \theta_c &=& 0.025 & \beta_{ny} &=& 0.220 & n^o &=& 0.153 \end{array} \tag{8}$$

To check the data we use the package of empirical time series of the US economy that is provided by Ray C. Fair on his home page (see the appendix), which is particularly helpful since it also contains a capital stock series of the nonfinancial firm sector. As concerns the output-capital ratio, the ratio of the empirical real magnitudes, Y/K, is in the region of 0.90. The price ratio p_y/p_k of the output and capital goods is, however, systematically different from unity. It varies around 0.75 until the early 1980ies and then steadily increases up to around 1 at the end of the 90ies. Correspondingly, the nominal output-capital ratio, p_yY/p_kK, first varies around 0.65 and then steadily increases up to 0.90. On the grounds that in a two- or multi-sectoral context the relevant ratio would be p_yY/p_kK, we prefer to make reference to the nominal magnitudes and choose an equilibrium value $y^o = y^n = 0.70$, which is slightly higher than 0.65.

When employer social security contribution is included in the definition of the wage share, $v = wL/pY \approx 0.70$ results as the time average between 1952 and 1998. Insofar as wages are a cost on the part of firms, entering the definition of profits, this is an obvious convention. Insofar as, implicitly, these receipts from social insurances are included in the theoretical model, they are taxed at the same rate as wages and the rest is likewise fully spent on consumption. Taking v^o for granted, the target markup rate μ^o derives from the consistency condition $(1+\mu^o)v^o = 1$.

The physical depreciation rate of the capital stock given by Fair is lower than the value of δ here proposed. However, what Fair calls (nominal) 'capital consumption' in his identity for profits in the firm sector yields a higher ratio when related to the nominal capital stock p_kK. In this way we decide on $\delta = 9.5\%$. Note that the implied equilibrium (gross) rate of return on real capital is

$(1 - v^o)\, y^o - \delta = 11.5\%$, which does not appear too unreasonable.

In the second row of (6), the equilibrium value of k^s is inferred from (6). With $e^o = 1$, the solution of this equation for k^s is $(k^s)^o = e^o/y^n = 1.42857$.

Setting the equilibrium values of inflation and the bond rate takes into account that over the period 1960 – 98, the real rate of interest is nearly 4% on average. The real balances ratio m is based on a value of 0.20 for M/pY, which is roughly the time average of M1 to nominal output in the last twenty years, when this ratio was relatively stable (as compared to the steady decline until the end of the 70ies). It remains to calculate $m = M/pK = (M/pY)(pY/pK)$, i.e., $m^o = 0.20 \cdot y^n$. In a similar manner, the government spending coefficient γ is decomposed as $\gamma = G/K = (G/Y)(Y/K)$. Here we take for G/Y the average ratio of nominal government demand to nominal output between 1960 and 1998, which amounts to 0.11 (though the ratio varies considerably over different subperiods).

To get an idea of the order of magnitude of the tax parameter θ_c, view taxes on rental income net of interest receipts, $pT^c - iB = \theta_c pK$ from (4), as a fraction τ_c of the profit flow $pY - wL - \delta pK$. Dividing the equation $\tau_c\,(pY - wL - \delta pK) = \theta_c pK$ by pK allows us to express θ_c as $\theta_c = \tau_c\,[(1 - v^o)y^n - \delta]$. Setting $\tau_c = 0.20$ yields 0.023 for θ_c, and $\tau_c = 0.25$ increases this value up to 0.02875. Against this background we settle for the value given in (6).

Regarding the ratio β_{ny} of desired inventories to expected sales in (12), we have the steady state relationship $Y/Y^d = 1 + \beta_{ny} g^o$ from (1). On the other hand, in commenting on Figure 2 the time average of $\xi = (Y^d - Y)/Y$ was reported to be $\bar{\xi} = -0.657\%$. Rearranging these terms as $Y/Y^d = 1/(1 + \bar{\xi})$, we may equate $1 + \beta_{ny} g^o$ to $1/(1 + \bar{\xi})$, which solving for β_{ny} gives $\beta_{ny} = -\bar{\xi}/(1 + \bar{\xi})g^o = 0.220$. The steady state value of the inventory ratio n^o is then directly computed from (2).

5.3 Calibration of the wage-price dynamics

The calibration of the wage-price dynamics, level 1 to level 3, can be taken over from one of the wage-price modules investigated in Franke (2001). We briefly report the results relevant for the present model, which emerged as a compromise of different issues. Parameters and cyclical statistics are given in the running text in the course of discussion. For better display, they are collected in an extra compilation in Section 5.6 below.

We begin with the desired standard deviation, $\sigma_{dev\,z}/\sigma_u = 0.40$, of the trend deviations of labour productivity in Table 2. It is achieved by setting $\beta_{zu} = 0.40$. The induced amplitude of the employment rate is then, however, lower than desired: $\sigma_e/\sigma_u = 0.69$ rather than 0.75. With three quarters, the lag of e is at the upper end of the range given in Table 2.

Subsequently, a battery of simulation runs led to the following choice of the six parameters at level 2: $\kappa_{pw} = \kappa_{wp} = 0$, $\beta_{pu} = 0.15$, $\beta_{pv} = 1.50$, $\beta_{we} = 0.55$, $\beta_{wv} = 0.50$. In this way, the desired standard deviation of the real wage can be met, $\sigma_{dev\,\omega}/\sigma_u = 0.47$, while the lag is somewhat longer than we aspired to, lag $\omega = 0.75$. With $\sigma_{dev\,v}/\sigma_u = 0.26$, the oscillations of the wage share are (necessarily, as it turns out) lower than in Table 2. They are shifted by about a quarter of a cycle with respect to utilization, lag $v = 2.08$. It is worth pointing out that this type of comovements between measures of economic activity and income distribution is equally obtained in Goodwin's (1967) seminal growth cycle model and its various extensions. Hence the present framework is well compatible with Goodwin's basic approach and could, indeed, provide a richer underpinning of its income distribution dynamics.

The coefficients at level 3 were set freehand at $\beta_\pi = 1.00$, $\kappa_{\pi p} = 0.50$. This choice proves to be justified by the good cyclical pattern of the price level, which is precisely countercyclical, lag $(-\operatorname{dev} p) = 0$, and displays a variability of $\sigma_{dev\,p}/\sigma_u = 0.48$.

The many simulation experiments undertaken in Franke (2001) showed that any improvement in the characteristics of one of the variables here discussed goes at the expense of some other variable(s). These trade-offs were judged worse than what has already been achieved. It was, in particular, worked out that a considerable influence of the wage share in the price as well as in the wage Phillips curve is indispensable for approximately procyclical real wages. As an aside, one might ask whether the present price Phillips curve with its dominant influence of the wage share, through the cost push / target markup argument in eq. (4), could still be reckoned a Phillips curve proper.

5.4 Interest rate oscillations

On the basis of the price level dynamics obtained above, we can now turn to the interest rate elasticity $\eta_{m,i}$ at level 4. Given the equilibrium rates of growth, $g^o = 3\%$, and inflation, $\pi^o = 3\%$, the constant growth rate of the money supply in the real balances equation (7) has, of course, to be fixed at $g_m = 6\%$. The time path of $m = M/pK$ is then fully determined, and with a suitable initial value, this ratio oscillates around the steady state value m^o. Setting the parameters β_{mi} and β_{mo} of the money demand function as done in eqs (4), (5) ensures that the bond rate, which is calculated in (2), likewise oscillates around its equilibrium value $i^o = 7\%$.[20]

Inspection of equation (2) shows that the cyclical pattern of the interest rate is independent of the interest elasticity, as $\eta_{m,i}$ only affects the coefficients β_{mo} and β_{mi}, but not the time path

[20] To be precise, the time average of the inflation rate $\hat{p}$ over a cycle is (very) slightly less than π^o. There is hence a slight upward trend in the time path of m, and a slight downward trend in the time path of the bond rate. It takes, however, more than thirty years for this effect to become directly visible in the time series diagrams.

of m/y. Since $m(\cdot)$ shortly leads $y(\cdot)$ and the sign of the time derivative of i is given by the expression $m\dot{y} - y\dot{m}$, it follows that i still increases when y is already on the downturn (di/dt being still positive when $\dot{y}$ is already negative but so small that $|m\dot{y}| < -y\dot{m}$). Numerically, it turns out that the bond rate peaks 1.17 years after u or y, respectively. In this way the bond rate and utilization display less negative correlation than the empirical coefficients in Table 1, but at least the lag is sizeable. In fact, taking into account the extreme simplicity of the financial sector as well as monetary policy, this result may even be considered rather acceptable. That is, while a more elaborate financial sector is certainly an important task for future modelling, for the time being the LM-specification together with the constant money growth rate does not do too much harm.

As the only effect of the interest elasticity is on the amplitude of the bond rate oscillations, $\eta_{m,i}$ may be set at any level desired. Table 3 reports the outcome in terms of the relative standard deviation σ_i/σ_u.

$\eta_{m,i}$	0.08	0.10	0.12	0.14	0.16	0.20
σ_i/σ_u	0.38	0.30	0.25	0.21	0.19	0.15

Table 3: Standard deviation (σ_i) of the bond rate at calibration level 4.

A familiar order of magnitude of the elasticity is perhaps $\eta_{m,i} = 0.20$. However, this brings about a fairly low variation of the bond rate. On the other hand, to achieve a standard deviation in the empirical range of $\sigma_i/\sigma_u = 0.36$ of Table 1, $\eta_{m,i}$ has to be reduced as much as $\eta_{m,i} = 0.10$ or 0.08. The reason for this phenomenon is, of course, the relatively low variation in the real balances ratio M/pK, which is due to the constant growth rate of M. Incidentally, it may be noted that empirically in the pre-Volcker period the bond rate showed much less variation. For example, over the period 1961 – 75 (which excludes the soaring levels in the second half of the 70ies up to more than 14% at the beginning of the 80ies), we measure $\sigma_i/\sigma_u = 0.19$.

As $\eta_{m,i} = 0.10$ or 0.08 appears unusually small, a value between 0.10 and 0.20 may be chosen. Concretely, in the fully endogenous model in the next section it will be useful to employ $\eta_{m,i} = 0.14$.

5.5 Goods market dynamics

Because of the limited compatibility of our still relatively simple modelling framework with empirical data on the income flows of groups like 'workers' and 'rentiers', we have some freedom in choosing the numerical values for the latters' savings propensity s_c and the tax rate on wages τ_w. In particular, the presence of the term $c_p pK$ in the consumption function (2) allows us to set these parameters somewhat higher than is perhaps usually suggested. The range of *a priori* admissible values is nevertheless bounded. So we consider $s_c = 0.60, 0.80, 1.00$ for the savings propensity and $\tau_w = 0.30, 0.35$ for the tax rate. A finer subdivision is not necessary.

Before, it should be briefly checked how these values affect the coefficients a_y and c_p in (7) and (8). This is, however, no problem. a_y and c_p remain within a reasonable range and do not vary too much with changes in s_c and τ_w. Thus, with $\tau_w = 0.35$, a_y increase from 0.265 to 0.347 as s_c rises from 0.60 to 1.00, while c_p increases from 0.141 to 0.175. The effect is similar when $\tau_w = 0.30$ is underlying, only that the values are slightly lower.

Since the cyclical characteristics of the variables turn out to change in a monotonic and regular way, it also suffices to report the results for just two selected values of the adjustment speed β_y of sales expectations: $\beta_y = 4.0$ and 8.0. As discussed in Section 4.3, the three parameters s_c, τ_w, β_y constitute level 5 in the calibration hierarchy and determine the time paths of excess demand ξ, the consumption ratio C/Y, and the inventory ratio n.

Setting subsequently the stock adjustment speed β_{nn} at level 6 has some influence on the endogenous utilization variable u^{endo}. One may, however, be prepared that once the time paths of $u(\cdot)$ and also $y^e(\cdot)$ have been determined at level 5, the chances of suitable and meaningful variations of β_{nn} controlling for the cyclical features of $u^{endo}(\cdot)$ are restricted. For this reason, we set a value of β_{nn} simultaneously with s_c, τ_w, β_y and have then also a look at the characteristics of $u^{endo}(\cdot)$. Concretely, β_{nn} is fixed at 3.0. After dealing with these simulation runs, β_{nn} is changed and we examine if the previous results can thus be improved.

Our final choice of the four parameters s_c, τ_w, β_y, β_{nn} can be discussed on the basis of the results given in Table 4. With respect to $\tau_w = 0.35$, $\beta_y = 8$, $\beta_{nn} = 3$ underlying, it shows the consequences of variations of the savings propensity s_c. An increase in s_c raises the standard deviation of the consumption ratio C/Y and relative excess demand ξ. The increase is, however, not sufficient to reach the desired levels of Table 2, the gap being larger for excess demand than for consumption. This deficiency cannot be essentially reduced with other values of τ_w and β_y. If we are to maintain the model's otherwise convenient specifications of aggregate demand, then the variability of C/Y and ξ has to be accepted to be confined to the order of magnitude of Table 4.

Both the consumption ratio and excess demand display a certain tendency for countercyclical

	C/Y		ξ		n		u^{endo}	
s_c	$\tilde{\sigma}$	lag	$\tilde{\sigma}$	lag	$\tilde{\sigma}$	lag	$\tilde{\sigma}$	lag
0.60	0.22	3.25	0.14	2.50	0.12	0.00	1.04	0.50
0.80	0.26	3.42	0.16	2.92	0.12	0.42	0.95	0.25
1.00	0.28	3.50	0.17	3.08	0.12	0.67	0.91	0.08

Table 4: Cyclical features of variables at calibration level 5 and 6.

Note: Besides the parameters set at level 1–3, $\tau_w = 0.35$, $\beta_y = 8$, $\beta_{nn} = 3$ are underlying. $\tilde{\sigma}$ is the standard deviation of the respective variable in relation to σ_u.

movements, though this feature is weaker for excess demand. It is a bit surprising that despite the imperfections of excess demand, the cyclical features of the inventory ratio $n = N/K$ are within the desired range. This gives us some hope that in the fully endogenous model later on, the implications of the simplifying assumptions on aggregate demand are not too harmful to the inventory dynamics and its repercussion effects.

Regarding the variations of the savings propensity, higher values of s_c are favourable for the countercyclicality of C/Y and ξ and, weakly so, also for their amplitudes. s_c is, of course, bounded from above by unity. Since $s_c = 1$ appears too extreme, we may settle for $s_c = 0.80$. An additional argument for this value is that the associated oscillations, under $\beta_{nn} = 3$, of endogenous utilization u^{endo} are rather promising. The standard deviation of $u^{endo}(\cdot)$ is not too different from the standard deviation of the exogenous sine wave $u(\cdot)$, and the two series are almost synchronous. Note that the more desirable features of C/Y and ξ that can be brought about by increasing s_c go at the expense of a lower amplitude of $u^{endo}(\cdot)$.

As a preliminary conclusion it can thus be stated that, given $\tau_w = 0.35$, $\beta_y = 8$, $\beta_{nn} = 3$, setting $s_c = 0.80$ is a good compromise between the conflicting goals regarding C/Y and ξ on the one hand, and u^{endo} on the other. The value is also economically meaningful.

Taking this for granted, we can now ask for the effects of changing the numerical values of the underlying three parameters. A lower value of the tax rate, $\tau_w = 0.30$, slightly reduces the standard deviation of C/Y and ξ as well as their lags. The latter carries over to lag n. The amplitude of $u^{endo}(\cdot)$ is higher, a little above 1, but the lag is longer, lag $u^{endo} = 0.50$. On the whole, $\tau_w = 0.30$ may be reckoned slightly inferior to $\tau_w = 0.35$, whereas $\tau_w = 0.40$ not only seems too high a value to us but also reduces the standard deviation of $u^{endo}(\cdot)$ by too much.

A slower adjustment speed of expected sales, $\beta_y = 4$, results in a small increase in the amplitudes of C/Y, ξ, n and affects the lags of these variables only marginally. These improvements are, however, more than outweighed by the strong decrease in the standard deviation of $u^{endo}(\cdot)$, whose ratio to σ_u falls down to 0.77 (the lag becomes half a year). The original value $\beta_y = 8$ is therefore better maintained.

As pointed out before, changes in the stock adjustment speed β_{nn} have a bearing on $u^{endo}(\cdot)$ alone. While the impact on the lags of endogenous utilization turn out to be negligible, a reduction of β_{nn} lowers the standard deviation of $u^{endo}(\cdot)$, a phenomenon which could also be analytically inferred from the function $f_y = f_y(y^e, n)$ in (1). Numerically, $\tilde{\sigma}$, the ratio to σ_u, decreases to 0.90 if $\beta_{nn} = 1$. On the other hand, $\beta_{nn} = 5$ raises it to 1. For the moment being, we may nevertheless keep to $\beta_{nn} = 3$ for two reasons. This adjustment speed amounts to $1/3$ years $= 4$ months within which firms in (13) seek to close the gap between actual and desired inventories. By contrast, a lag of $1/5$ years $= 2.4$ months might already appear a bit short. Second, at least in low-dimensional models of the inventory cycle, β_{nn} proves to be destabilizing; cf. Franke (1996). It is to be feared for the endogenous model that the centrifugal forces evoked by $\beta_{nn} = 5$ are unpleasantly strong.

5.6 Summary of calibration results

For a better overview of what has been done and achieved, we collect the numerical parameter values, 14 in number, in an extra box and then, in Table 5, list the statistics of the cyclical features to which they give rise.

Synopsis of numerical parameters

Level 1:	$\beta_{zu} = 0.40$		
Level 2:	$\beta_{pu} = 0.15$	$\beta_{pv} = 1.50$	$\kappa_{pw} = 0.00$
	$\beta_{we} = 0.55$	$\beta_{wv} = 0.50$	$\kappa_{wp} = 0.00$
Level 3:	$\beta_{\pi} = 1.00$	$\kappa_{\pi p} = 0.50$	
Level 4:	$\eta_{m,i} = 0.14$		
Level 5:	$s_c = 0.80$	$\tau_w = 0.35$	$\beta_y = 8.00$
Level 6:	$\beta_{nn} = 3.00$		

We repeat that it could not have been our goal to obtain a perfect match of the cyclical statistics of the empirical series. And even if we came close to full success in this respect, we would not yet know what it would be worth since admittedly the exogenous sine wave motions

variable x	σ_x/σ_u	lag x
dev z	0.40	0.00
e	0.69	0.75
dev ω	0.47	0.75
dev v	0.26	2.08
$-$ dev p	0.48	0.00
i	0.21	1.17
ξ	0.16	2.92
C/Y	0.26	3.42
n	0.12	0.42
u^{endo}	0.95	0.25

Table 5: Cyclical statistics of variables under exogenous sine wave oscillations of utilization.

of utilization u are very stylized indeed. The results in Table 5 and the way we arrived at them being more of a heuristic value, we will have to see how the present set of numerical parameters performs under different conditions.

This brings us to the second test to which the parameters are subjected, where the regular sine waves of u are replaced with the empirical observations of this variable. To this end, we take the quarterly data on u (1960:1 – 91:4) depicted in Figure 1 and interpolate it to get a monthly series. As before, the simulation itself is run for the monthly discrete-time analogues of the model. Figure 3 selects six endogenous variables computed in this experiment (i.e., their deviations from the steady state values) and contrasts them with their empirical counterparts. The most remarkable result is that the simulated series follow the essential movements of the empirical variables. This finding supports the parameter choice.

Regarding the wage-price dynamics in the first three panels of Figure 3, one notices that in the first half of the 70ies the turning points of the real wage, the wage share as well as the price level have a lower amplitude than in reality. This phenomenon can be attributed to the shorter cycle over that period, so that here, with the same adjustment speeds (in, particularly, the two Phillips curves), the variables do not have enough time to reach the empirical peak or trough values.

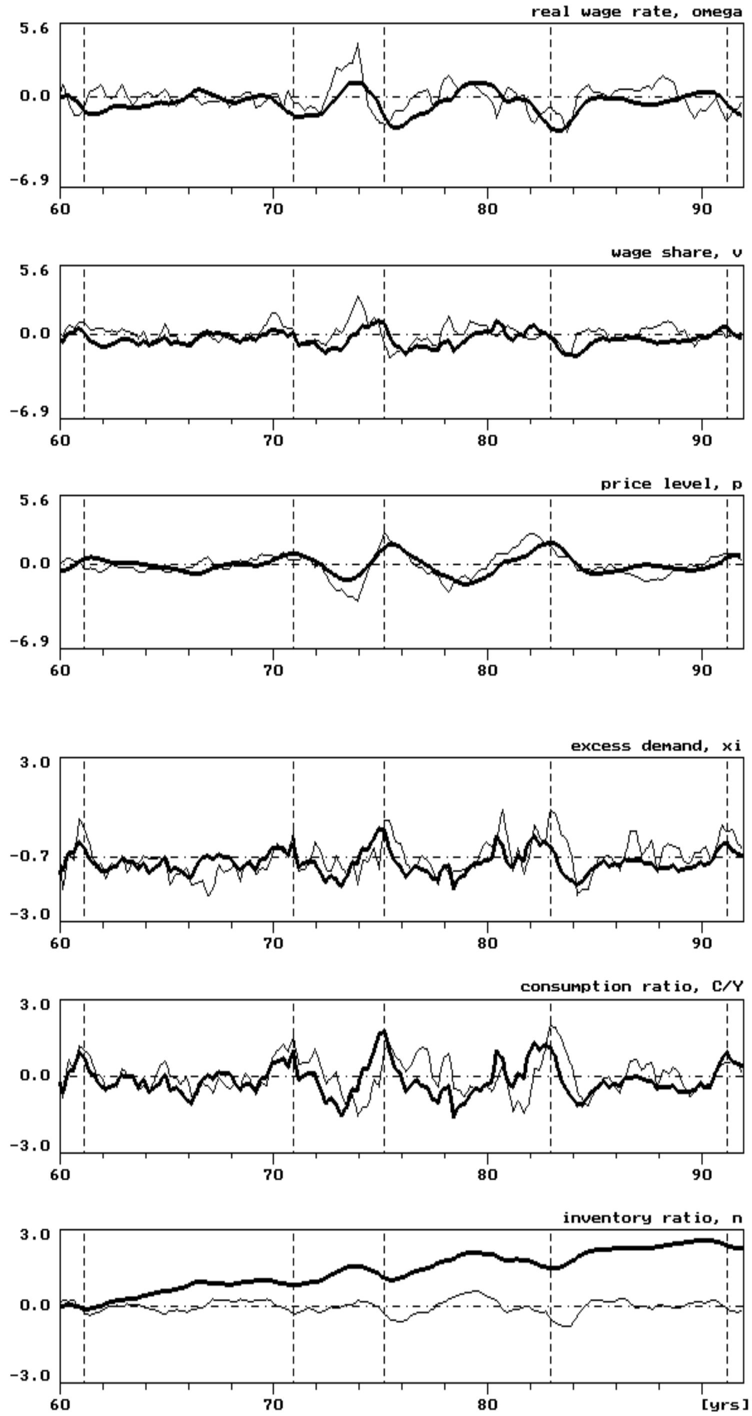

Figure 3: Endogenous variables under empirical fluctuations of u

Note: Bold lines are simulated time series, thin lines are the empirical counterparts.

This bias does not apply to the two demand variables ξ and C/Y. Over the whole sample period of Figure 3, their standard deviation is also somewhat higher than in Table 5; for ξ the ratio to σ_u increases to 0.20, for C/Y it increases to 0.30. Hence the assumptions on the components of aggregate demand are not too bad a simplification. Lastly, the slight upward trend in the inventory series, which precisely is $100 \cdot (n - n^o)$, is due to the fact that the capital growth rate from (4) is not perfectly tuned to the other growth components that make themselves felt in the ratio N/K.[21]

On the basis of these results it might now be argued that, maintaining the empirical data for the fluctuations of u, one should try further variations of some of the parameters in order to achieve better cyclical statistics in this framework. The significance of a good match of the empirical statistics is, however, an unsettled issue. The problem is that the historical moments have sampling variability and so can differ from the model's population moments — even if the model happened to be true. More specifically, given that a model cannot be expected to exactly duplicate reality, we can distinguish between a model variable, denote it by x_t^m, and its empirical counterpart x_t^e, with error $\varepsilon_t = x_t^e - x_t^m$. To compare the standard deviations of x^m and x^e, i.e. their variances, the identity $\mathrm{var}(x^e) = \mathrm{var}(x^m) + 2\,\mathrm{cov}(x^m, \varepsilon) + \mathrm{var}(\varepsilon)$ has to be taken into account. As a consequence, if the difference between $\mathrm{var}(x^m)$ and $\mathrm{var}(x^e)$ is viewed as a statement about $\mathrm{var}(\varepsilon)$, as it mostly is, this would require $\mathrm{cov}(x^m, \varepsilon) \approx 0$ to be fulfilled, which amounts to making an assumption that *a priori* is not really obvious. But if one allows for potential correlation between x^m and ε, it might even be possible that $\mathrm{var}(x^m) = \mathrm{var}(x^e)$ despite large errors ε_t. The problem here indicated is certainly beyond the scope of this paper.[22]

6 The fully endogenous model and its dynamics

The modelling equations so far provided can already be viewed as constituting a fully endogenous macrodynamic model if the exogenous motions of utilization are dropped and $u = u^{endo}$ is obtained from (2). Eq. (4) for the capital growth rate g_k would then have the status of an investment function. In this respect, however, we want to be more flexible, both on the grounds of greater conceptual richness and in order to gain some control over the stability of the system. We therefore

[21] This distorting effect is even stronger in the real balances ratio $m = M/pK$ and, thus, in the simulated time series of the rate of interest.

[22] The problem is hinted at in Kim and Pagan (1995, p. 371). The authors conclude, "the method of stylized facts really fails to come to grips with what is the fundamental problem in evaluating all small models, namely the assumptions that need to be made about the nature of the errors ζ_t" (ζ_t corresponds to ε_t in our notation). On pp. 378ff, Kim and Pagan elaborate more on the problems connected with the fact that generally the errors ζ_t cannot be recovered.

bring another variable into play.

Two motives for investment in fixed capital are considered. First, for reasons not explicitly taken into account in the model formulation, firms not only seek to avoid excess capacity but also desire no permanent overutilization of productive capacity. Hence investment increases with utilization u. The second motive refers to the profitability of firms. We may measure it by $r := (pY - wL - \delta pK)/pK = (1-v)uy^n - \delta$. Since investment is exclusively financed by equities, this rate of profit is seen in relation to the yields from holding bonds, which is the alternative of financial investment shareholders have, with the real interest rate, $i - \pi$, as the relevant rate of return. In sum, besides utilization, fixed investment is additionally a positive function of the differential returns q, defined as $q := r - (i-\pi)$, or

$$q \;=\; (1-v)\,uy^n \;-\; \delta \;-\; (i-\pi) \tag{9}$$

Our methodological approach to persistent cyclical behaviour in this paper is a deterministic one. We do not, however, wish to rely on a Hopf bifurcation.[23] Within a vicinity of the steady state position, the dynamics may rather be more or less destabilizing. Though there are a number of intrinsic nonlinearities in the model, they are only weak and 'dominated' by the many linear specifications in the behavioural functions. It thus turns out that the destabilizing forces are also globally operative. This means we have to build in some additional, extrinsic nonlinearities, which take effect in the outer regions of the state space and prevent the dynamics from totally diverging. For our present purpose, we can content ourselves with just one such nonlinearity, which we introduce into the investment function.

A simple idea will prove sufficient. Suppose utilization is steadily rising in an expansionary phase. The corresponding positive influence on the flow of investment may be reinforced or curbed by the differential returns q. If, however, utilization has become relatively high, firms will not expect the economy to grow at the same speed for too long. If moreover q is relatively low in that stage, so that this influence on investment is already negative, then the positive utilization motive may be further weakened. That is, we assume that under these circumstances the negative effect from q is stronger than it otherwise is at lower levels of capacity utilization. With signs reversed, the same type of mechanism applies when the economy is on the downturn. Introducing two positive reaction coefficients β_{Iu} and β_{Iq} and referring for simplicity directly to the growth rate g^o and the differential returns q^o in a long-run equilibrium, we specify this concept for g_k,

[23] One reason is that, *a priori*, we can by no means be sure that the periodic orbits of the Hopf bifurcation are attractive. But even then, meaningful cyclical trajectories would only exist over a very small range of parameter values.

the capital growth rate, as follows:

$$g_k \;=\; g_k(u,q) \;=\; g^o \;+\; \beta_{Iu}\,(u-1) \;+\; \alpha(u,q)\,\beta_{Iq}\,(q-q^o) \tag{10}$$

where with respect to given values d_1 and d_2, $0 < d_1 < d_2$, the function $\alpha = \alpha(u,q)$ is defined as[24]

$$\alpha \;=\; \alpha(u,q) \;=\; \begin{cases} 1 \;+\; [u-(1{+}d_1)]/(d_2-d_1) & \text{if } u \geq 1+d_1 \text{ and } q \leq q^o \\ 1 \;+\; [(1{-}d_1)-u]/(d_2-d_1) & \text{if } u \leq 1-d_1 \text{ and } q \geq q^o \\ 1 & \text{else} \end{cases} \tag{11}$$

Evidently, for this mechanism to work out it is necessary that the q-series peaks considerably before utilization, a property we have already checked in the sine wave experiments. Since it essentially depends on the relative amplitude of the bond rate and the rate of inflation, the mechanism cannot be expected to be effective under different circumstances. In this sense, (9) represents only a minimal nonlinearity to tame the centrifugal forces in the economy.

On the whole, we have now a self-contained differential equations system of dimension six. The state variables are $k^s = K/z^oL^s$: capital per head (measured in efficiency units), $\omega = w/pz^o$: the real wage rate (deflated by trend labour productivity), π: the inflation climate, $m = M/pK$: the real balances ratio, $y^e = Y^e/K$: the expected sales ratio, and $n = N/K$: the inventory ratio (where clearly the term 'ratio' refers to the stock of fixed capital). Collecting the equations of the laws of motions as they were presented at calibration level 1 – 6, the system reads:

$$\dot{k}^s \;=\; k^s\,(g_k - g_z - g_\ell) \tag{8}$$
$$\dot{\omega} \;=\; \omega\,\kappa\,[\,(1{-}\kappa_{pw})f_w(e,v) \;-\; (1{-}\kappa_{wp})f_p(u,v)\,] \tag{8}$$
$$\dot{\pi} \;=\; \beta_\pi\,[\kappa_{\pi p}(\hat{p}-\pi) \;+\; (1{-}\kappa_{\pi p})(\pi^o-\pi)] \tag{1}$$
$$\dot{m} \;=\; m\,(g_m - \hat{p} - g_k) \tag{7}$$
$$\dot{y}^e \;=\; (g^o - g_k)\,y^e \;+\; \beta_y\,(y^d - y^e) \tag{1}$$
$$\dot{n} \;=\; y \;-\; y^d \;-\; n g_k \tag{2}$$

To see that actually no more than these six dynamic variables are involved, note that κ is defined in (7) and $g_k = g_k(u,q)$ is determined in (8) and (9), $u = f_y(y^e,n)/y^n$ is determined in (2), $q = q(u,v,i,\pi)$ in (7), $v = v(u,\omega)$ in (4), $e = e(u,k^s)$ in (6), $\hat{p} = \hat{p}(u,e,v,\pi)$ in (6), $i = i(m,y)$ in (2), $y = uy^n$ in (3), $y^d = y^d(y^e,v,y,g_k)$ in (6).

To simulate this economy on the computer, it remains to set the investment parameters in (8) and (9). We choose

$$\beta_{Iu} \;= 0.260 \qquad \beta_{Iq} \;= 0.115 \qquad d_1 \;=\; 0.020 \qquad d_2 \;=\; 0.070 \tag{10}$$

[24] It may be noted that though the function α is not continuous in q, the multiplicative term $\alpha\,(q-q^o)$ in g_k is.

It seems that the influence of q on g_k tends to stabilize the system, while u gives rise to a positive feedback effect and so destabilizes it. The relatively high choice of the coefficient β_{Iu} *vis-à-vis* β_{Iq} renders the steady state unstable. The values of d_1 and d_2, on the other hand, make the nonlinearity in g_k sufficiently effective to keep the economy within realistic bounds. The system will therefore be characterized by persistent cyclical behaviour. The precise level of β_{Iu} is essential for the period of the fluctuations thus obtained.

All parameters being given, the endogenous model can now be numerically simulated. To set the system in motion, we start out from a steady state growth path and disturb it by a strong temporary shock. We do this by raising the growth rate of the money supply over one year from 6% up to 8%. Afterwards it is set back to its original level, from when on the economy is left to its own.

The short fall of the bond rate induced by the monetary impulse initiates an expansion, but after the economy has reached its peak, the economy steers into a severe recession four years after the suspension of the shock. That is, this negative deviation of u from normal is significantly larger than the previous positive deviation. With the recovery then setting in, the economy begins to move in an oscillatory fashion, such that the peak and trough values of the variables tend to level off. Some 15 or 20 years after the shock, the oscillations become quite regular. After a while, the trajectories are even almost periodic. We illustrate this phenomenon by the four phase diagrams in Figure 4.[25]

The first diagram in Figure 4 plots the wage share against utilization as the measure of economic activity (the '+' symbol indicates the steady state values of the variables). The picture is much the same as the income distribution dynamics in a Goodwin (1967) growth cycle model. The upper-right panel displays the pairs of inflation and utilization as they evolve over time. What results is not a Phillips curve relationship, but so-called Phillips loops. The real wage rate, too, forms no firm functional relationship with utilization. We rather observe a similar looping behaviour, though the shape of the loop is somewhat different from the previous two variables.

While the loops in these first three panels are fairly symmetric, the panel in the lower-right corner shows an example of a variable, namely the rate of interest, with less regularity. It indicates that the lag of the bond rate with respect to u is larger in the upper than in the lower turning point.

Having established the basically cyclical behaviour of the economy, we may turn to the time series characteristics of the trajectories. An introductory visual impression is given in Figure 5. Note first of all the period of the oscillations, which precisely is 8.33 years.

[25] Intuitively, with the steady state being unstable and only one essential nonlinearity, the resulting limit cycles should be unique. A more careful investigation of this issue is here, however, left aside.

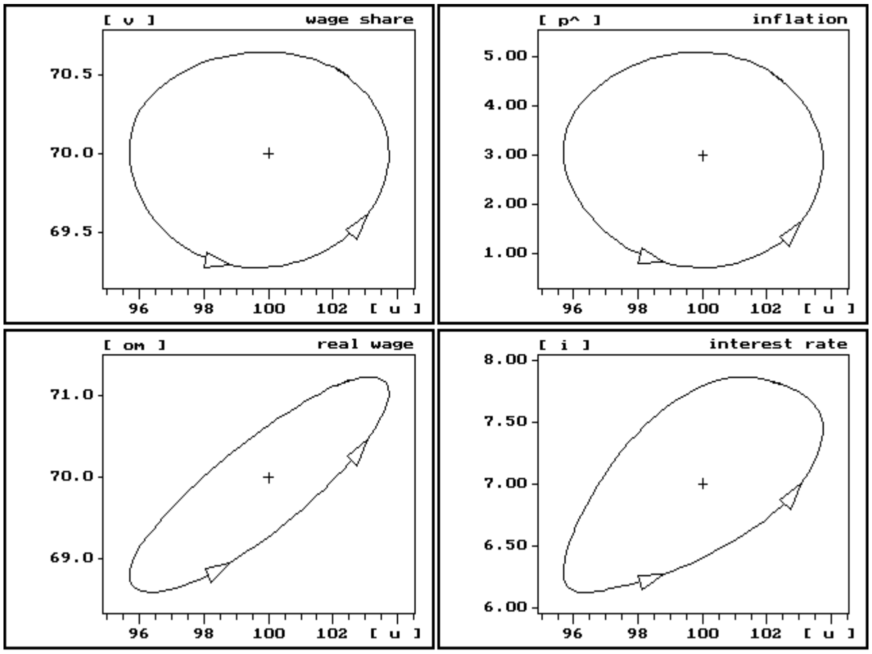

Figure 4: Selected phase diagrams of the calibrated endogenous model.

The top panel of Figure 5 shows utilization u as the central time series of the business cycle. It is contrasted with a sine wave motion — the dotted line — that has the same period and amplitude. The middle panel displays the capital growth rate g_k in a likewise fashion. It is thus seen that the endogenous dynamics lets u and g_k move almost synchronously. Moreover, at least at first sight both series do not differ very much from a sine wave. Hence, we may point out, the approach of specifying the exogenous variables as sine waves in the calibration procedure has not been too inappropriate after all. Besides, the amplitude of u is also of the same order of magnitude as in the calibration experiments.

At a closer look, the differences between u and the sine wave are greater in the expansion than in the contraction, and similarly so for g_k. The reason is that a contraction takes a bit longer: the time from peak to trough is 4.33 years, while the trough-to-peak period is 4.00 years. More consequential for the dynamic properties of the system is the fact that the peak and trough values are not exactly symmetric. Figure 5 demonstrates, and Table 6 makes it numerically precise, that for u as well as for g_k, the lower turning points deviate slightly more from the steady state values than the upper turning points. Given the linear specification of the behavioural functions and the strictly symmetric nonlinearity in the investment function, this asymmetry is somewhat surprising. In the end, it must originate with the intrinsic nonlinearities in the model, however weak they are. More directly it can be seen as being brought about by the asymmetric timing of

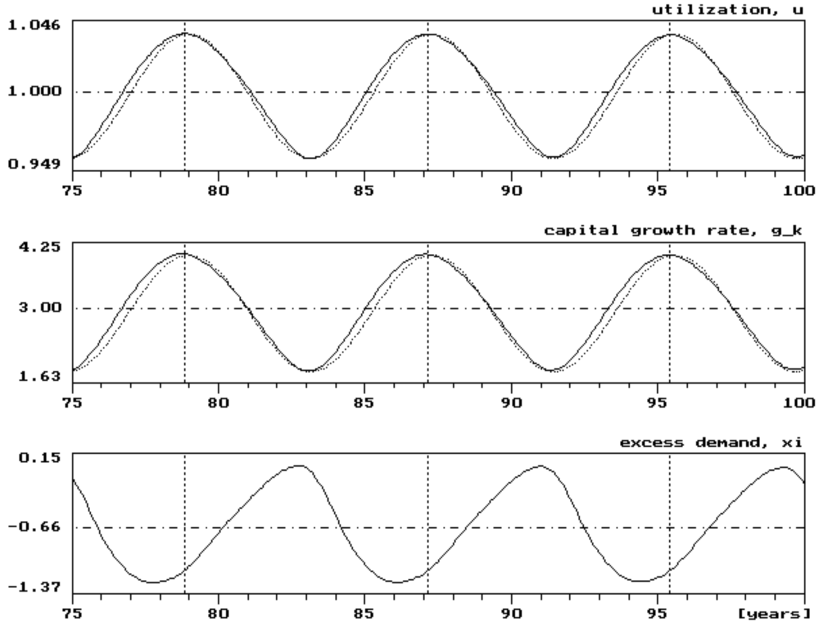

Figure 5: Selected time series of the calibrated endogenous model.

Note: Dotted lines are synchronous sine wave motions fitted in.

the turning points of the q-series documented in Table 6 (which can be traced back to the interest rate; see below) and its impact on investment.

The table states in addition that the capital growth rate has a short lead of one month with respect to utilization (a lag would have been 'preferable'). Comparing the peak and trough values of u with the coefficients d_1 and d_2 in (1) makes clear that the nonlinearity in (8), (9) does indeed take effect (observe the long lead of q in Table 6) . The particular choice of the two parameters accomplishes that the standard deviation of g_k relative to that of u is about the same as for the empirical series in Table 1.

Similarly as the phase diagram in Figure 4 has it already indicated for the bond rate, excess demand ξ is a second variable whose upper and lower turning points have different lags with respect to u. The peak in ξ moves quite close to the trough in u, whereas more time elapses from the trough in ξ until u reaches its peak. The lags given in Table 6 can also be read as saying that $-\xi$ leads utilization by 0.42 years in the trough, and by 1.08 years in the peak. Excess demand has thus become more countercyclical than in the sine wave simulations (cf. Table 5), where ξ also showed no apparent differences in its behaviour around the upper and lower turning points.

If we take the sine waves of u and g_k as a reference scenario, then the high sensitivity of the cyclical features of the excess demand variable to the relatively minor changes in the time series

variable x	x^o	in peak	in trough	σ_x/σ_u	lag (peak)	lag (trough)
u	100.00	+ 3.73	− 4.32	−−	−−	−−
g_k	3.00	+ 1.02	− 1.16	0.27	−0.08	−0.08
e	100.00	+ 2.59	− 2.97	0.70	0.75	0.42
ω	70.00	+ 1.23	− 1.40	0.47	0.75	0.58
v	70.00	+ 0.64	− 0.73	0.25	2.25	1.75
$-\operatorname{dev} p$	0.00	+ 1.90	− 2.04	0.50	0.08	0.08
$\hat{p}$	3.00	+ 2.08	− 2.28	0.56	2.25	1.92
π	3.00	+ 0.87	− 0.91	0.23	3.08	2.75
m	14.00	+ 0.44	− 0.46	0.82	−0.58	−0.58
i	7.00	+ 0.85	− 0.88	0.22	1.67	0.58
q	7.50	+ 0.51	− 0.52	0.13	−3.17	−2.33
ξ	−0.66	+ 0.67	− 0.58	0.16	3.83	2.92
C/Y	70.48	+ 1.11	− 0.92	0.25	3.83	3.33
n	15.30	+ 0.40	− 0.47	0.11	0.83	0.83

Table 6: Cyclical statistics of the calibrated endogenous model.

Note: All variables multiplied by 100. x^o denotes the steady state value of variable x, $-\operatorname{dev} p$ is deviation of price level from HP 1600 trend in per cent. The standard deviations of ω, v and m are divided by the respective steady state values.

of u and g_k can be explained by the very definition of ξ, which involves a *ratio* of two variables, y^d and y. Thus, $\xi = y^d/y - 1$ can be quite 'self-willed', although y is practically the same as u itself and y^d lags y by only one month in the peak as well as in the trough (the peaks and troughs are, however, 0.023 and 0.026 above and below $(y^d)^o = 0.695$). On the other hand, the implications of excess demand for the inventory ratio n, *via* the *difference* between y^d and y in (2) for $\dot{n}$, are restricted to asymmetric peak and trough values of n. Their timing is again symmetric, with the lags being a little longer than in Table 5, or than desired in Table 2.

A similar result as for ξ holds for the consumption ratio C/Y (which likewise has somewhat improved in its countercyclicality) and for the rate of interest, where the differences in the lags at peak and trough times are even greater. The latter effect is equally remarkable as it was for

ξ, since i depends on the ratio m/y according to (2), but m displays no asymmetry at all in the timing of its turning points. The irregularity in the cyclical pattern of the bond rate is also mainly responsible for a similar phenomenon in the abovementioned differential returns $q = r - (i - \pi)$, though it is shifted in time. Interestingly, the mean value of the lags 1.67 and 0.58 of the bond rate is nearly the same as its lag of 1.17 years in the sine wave scenario, which generated fairly symmetrical motions of i.

The inflation rate, too, exhibits different lags of its turning points. They are, however, completely washed out in the integrated series of the price level, which is almost perfectly countercyclical. Besides, the difference in the peak and trough lags of $\hat{p}$ is the same as for the inflation climate π, even though, as eq. (6) and the standard deviations of $\hat{p}$ and π show, the inflation rate is predominantly influenced by the price Phillips curve term $f_p = f_p(u, v)$ (the term $f_w = f_w(e, v)$ does not feed back on $\hat{p}$ because of $\kappa_{pw} = 0$).

All the asymmetries that have been pointed out have no effect on the amplitude of the variables. The standard deviations in Table 6 are therefore practically the same as they resulted from the sine wave calibration in Table 5. This is another aspect corroborating this methodological approach.

7 Conclusion

The paper has put forward a complete deterministic macro model of the business cycle that takes up elements which may be connected with, in particular, the names of Keynes, Metzler and Goodwin. The aim of the paper was a calibration of the model. This procedure was organized in a hierarchical structure, so that the numerical coefficients need not all be determined simultaneously but could be chosen step by step. Given stylized oscillations of two exogenous variables, capacity utilization and the capital growth rate, each step gave rise to motions of some endogenous variables. Their cyclical pattern could then be compared to the behaviour of their empirical counterparts.

The calibration analysis has ended up with numerical values of, on the whole, 14 parameters. Subsequently, the hitherto exogenous variables were endogenized, which added another four parameters in the investment function thus introduced. They were set such that the steady state position of the fully endogenous model became unstable, while a suitable nonlinearity in the investment function prevented the system from totally diverging. Hence, the model produces persistent cyclical behaviour, actually in the form of a limit cycle.

The main characteristics of the model's time series, their variability and comovements, may be judged to be by and large satisfactory. Specifically, this concerns more or less procyclical movements of the capital growth rate, the employment rate, the (productivity-deflated) real wage and the inventory ratio (relative to the capital stock), as well as countercyclicality in the price level, relative excess demand and the consumption ratio (the latter two relative to total output). Of course, the cyclical statistics are not always perfect. Within the present modelling framework a single statistic could also hardly be improved any further without seriously affecting another one. In a brief summary we may nevertheless claim that the results we arrived at can stand comparison with the properties generated by the competitive equilibrium models of the real business cycle school.

To keep a curb on the system's centrifugal forces it was sufficient to introduce just one extrinsic nonlinearity in the investment function. The efficiency of this mechanism is, however, rather sensitive to the choice of the two investment reaction intensities β_{Iu} and β_{Iq}, and possibly also to changes in other parameters (especially those that have a direct bearing on the motions of the differential returns q). Extensions of the model may therefore in the first instance include additional nonlinearities in other parts of the models, which can contribute to a better containment of the instabilities at the outer boundaries of the state space. On the basis of further exploratory simulations we feel that fixed investment is still the most important, from a constructivist point of view even indispensable, point of intervention for a global stabilization, but future investigations of the dynamics should systematically study what other mechanisms can support the present nonlinear investment schedule.

A conceptual weakness of the model is the financial sector. On the one hand, markets for other, nonsubstitutable financial assets need to be introduced, such that they play a more active role than is admitted by the textbook LM-sector. Our interest in this respect lies in equities and bank loans to firms. A good candidate of a financial sector that besides money and bonds takes these assets into account is the temporary equilibrium approach by Franke and Semmler (1999), which should be relatively easy to integrate into the present framework. On the other hand, in addition to equities, fixed investment of firms may also be financed by retained earnings and, as just mentioned, bank loans. This will allow firms' reactions to strong disequilibria to be more flexible as it is presently the case. Regarding global stabilization, supplementary and more robust nonlinearities in the investment function may thus arise in quite a natural way.

8 Appendix: the empirical time series

The time series examined in table 1 are constructed from the data that are made available by Ray Fair on his homepage (`http://fairmodel.econ.yale.edu`), with a description being given in Appendix A of the US Model Workbook. Taking over Fair's abbreviations, the following time series of his database are involved. They all refer to the firm sector, i.e., non-financial corporate business.

CD	real consumption expenditures for durable goods
CD	real consumption expenditures for nondurable goods
CD	real consumption expenditures for services
HN	average number of non-overtime hours paid per job
HO	average number of overtime hours paid per job
JF	number of jobs
KK	real capital stock
PF	output price index
RB	bond rate (percentage points)
SIFG	employer social insurance contributions paid to US government
SIFS	employer social insurance contributions paid to state and local governments
V	real stock of inventories
WF	average hourly earnings excluding overtime of workers (but including supplements to wages and salaries except employer contributions for social insurance).
X	real sales
Y	real output

The variables in table 1 are then specified as follows. For Fair's assumption of a 50% wage premium for overtime hours, see, e.g., his specification of disposable income of households (YD in eq. (115), Table A.3, The Equations of the US Model).

$$u = \mathrm{Y} / \mathrm{KK}$$
$$z = \mathrm{Y} / [\mathrm{JF} \times (\mathrm{HN} + \mathrm{HO})]$$
$$L = \mathrm{JF} \times (\mathrm{HN} + 1.5{\times}\mathrm{HO})$$
$$w = \mathrm{WF} \times (\mathrm{HN} + 1.5{\times}\mathrm{HO}) / (\mathrm{HN} + \mathrm{HO})$$
$$p = \mathrm{PF}$$
$$v = [\mathrm{WF} \times (\mathrm{HN} + 1.5{\times}\mathrm{HO}) \times \mathrm{JF} + \mathrm{SIFG} + \mathrm{SIFS}] / [\mathrm{Y} \times \mathrm{PF}]$$
$$\xi = 100 \times (\mathrm{X} - \mathrm{Y}) / \mathrm{Y}$$
$$C/Y = 100 \times (\mathrm{CD} + \mathrm{CN} + \mathrm{CS}) / \mathrm{Y}$$
$$n = \mathrm{V} / \mathrm{KK}$$
$$i = \mathrm{RB}$$

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