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**Liquidity, Credit and Output:  
A Regime Change Model and Empirical  
Estimations**

by

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# Liquidity, Credit and Output: A Regime Change Model and Empirical Estimations <sup>\*</sup>

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## Abstract

There is a long tradition which maintains that liquidity and credit impact aggregate economic activity. Recent events seem to give fresh support to this line of research. Economic theory on credit and financial markets is in search of mechanisms that might explain the strong propagation effect of real, monetary and financial shocks. We employ a simple macrodynamic model of threshold and regime change type to provide such a propagation mechanism. We estimate the model by transforming our continuous time form into an estimable discrete time form using the Euler approximation and a method proposed by Ozaki. We also approximate the model by employing the discrete time Smooth Transition Regression (STR) methodology. Our estimation procedures are applied to U.S. time series data. We find essential nonlinearities and regime changes in the data. The change of the dynamic properties of the estimated model occur as the variables pass through certain thresholds. Locally unstable but globally bounded fluctuations as well as asymmetric responses to shocks are detected.

**Keywords:** regime change models, Smooth Transition Regression models, financial-real interaction, thresholds, asymmetry in business cycles.

**JEL Classification:** C32(Time Series Models), E32(Business Fluctuations, Cycles), E44(Financial Markets and the Macroeconomy)

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**Confidence:** "As credit by growing makes itself grow, so when distrust has taken the place of confidence, failure and panic breed panic and failure." (Marshall, 1879);

**Financial practices:** "Success breeds a disregard of the possibility of failures. The absence of serious financial difficulties over substantial period... leads to a euphoric economy..." (Minsky, 1986);

**Perception of bankruptcy risk:** "... the economy wide level of bankruptcy risk plays a crucial role in the propagation of the recession." (Bernanke, 1981)

## 1 Introduction

There is a long tradition which maintains that liquidity and credit impact economic activity. Recent events have given fresh support of this line of reasoning. Economic theory attempts now to study mechanisms that explain how real or financial shocks are magnified through the financial sector possibly leading to large output losses. In the search for such mechanisms liquidity of economic agents and the credit channel have been viewed as central. There is, of course, a long tradition of older economic literature that shows that liquidity and credit accelerates downswings.<sup>1</sup> In recent times various lines of work have embellished this basic theme.

In the tradition of Keynesian theory the role of liquidity and credit has been studied in the context of IS-LM models.<sup>2</sup> Since in those models credit is difficult to consider there, usually, the link between liquidity and output is stressed. Shinasi (1981), for example, linked liquidity and output in a nonlinear IS-LM model. Important further recent contributions along this line of research are made in papers by Day and Shafer (1985) and Day and Lin (1991).<sup>3</sup> A model of a similar type can be found in Foley (1987) and Flaschel, Franke and Semmler (1997, ch. 12).

In the context of IS-LM models it has been studied how monetary policy has an impact on liquidity, credit conditions and spending of economic agents; see for example, the work by Eckstein et al. (1974, 1986). Often with respect to the financial history of the U.S it is shown that there are, for example, certain periods where monetary contractions have led to a credit crunch and thus have worsened the borrowing conditions for households and firms.

The propagation effect of shocks has also been studied in the recent theory of imperfect capital markets. Here, from a microeconomic perspective, the link between credit and output has been considered in explicit models of borrowing and lending. The propagation effect in this recent approach is described by the so called financial accelerator.<sup>4</sup> Central for this view are credit dependent agents (firms, households) where by the ease and tightness of credit as well as credit cost are linked to net worth of economic agents. In this model variant it is then described how swings in the balance sheet variables of economic agents, for example

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<sup>1</sup>An excellent survey of earlier theories on credit and output is given in Boyd and Blatt (1988). The work of Minsky (1975) has continued this theoretical tradition.

<sup>2</sup>Different variants of this type of models, however, for a growing economy are discussed in Flaschel, Franke and Semmler (1997), ch 4.

<sup>3</sup>In these papers transaction and speculative demand for liquidity is endogenous whereas money supply is exogenously given. An unstable accelerator effect destabilizes the system in the vicinity of the equilibrium and the instability is contained by a countercyclical variation in liquidity or interest rate

<sup>4</sup>The term financial accelerator has first been introduced by Bernanke and Gertler (1989).

net worth, affect credit flows and credit cost and thus magnify spending and output when shocks occur.<sup>5</sup>

Empirically, the link between liquidity, credit and output has been studied, for example, in papers by Eckstein et al. (1974, 1986); and also by Friedman (1983) and Blinder (1989). Empirical studies, from a micro perspective, can be found in Gertler, et al (1991) and from a macro perspective in Franke and Semmler (1995). Studying the link between liquidity, credit and output with econometric time series methods has turned out to be rather difficult.<sup>6</sup> There are complicated lead and lag patterns in the credit output interaction and thus it is not easily possible to identify this link in the data. There is, however, plenty of indirect empirical evidence that appears to support the covariation of liquidity and credit with the business cycle; for example, it is often found that the (marginal) cost of external funds moves counter-cyclically<sup>7</sup> and that availability of funds moves pro-cyclically.<sup>8</sup>

There are, of course, also numerous studies that consider not the short-run relationship between liquidity, credit and output but their long-run relationship. In models such as in Semmler and Sieveking (2000), credit constraints are not binding in the short-run but rather in the long-run when the intertemporal budget constraint of an agent becomes binding. Frequently, however, the temporary shortage of liquidity and credit are sufficient to disrupt economic activity and to lead to bankruptcy of the economic agents. It is this short-run relationship that the current study is concerned with.<sup>9</sup>

A motivation for such a study we employ, a nonlinear model of the interaction between liquidity, credit and output developed by Semmler and Sieveking (1993). This model gives some predictions of the behavior of liquidity and credit as output varies over the business cycle. The model allows for regime changes in the relationship between the variables as the variables pass through certain thresholds.

In order to undertake an empirical estimation of such a model we suggest some econometric methods that are suitable to empirically study regime changes.<sup>10</sup> Since our suggested model is written in continuous time we will first undertake a direct test of the model whereby

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<sup>5</sup>In Townsend (1976) and Gale and Hellwig (1985), for example, costly state verification of a borrower's investment project results in a greater marginal cost of funds. Costly state verification implies that a smaller collateral on the borrower's side increases the default risk and thus borrowing cost. Along those lines it is then posited that credit may set in motion a strong propagation mechanism.

<sup>6</sup>In earlier times the effects of monetary shocks were discussed in VAR type of money-output models with rather inconclusive results. A recent evaluation of the success and failure of those VAR studies is given in Bernanke and Blinder (1992).

<sup>7</sup>Those propositions have been tested, mostly in a linear setting, by Gertler et al. (1991), and Franke and Semmler (1995) where the cost of funds is measured by the spread between the commercial paper rate and the interest rate on treasury bonds. They demonstrate that a counter-cyclical spread can indeed be observed.

<sup>8</sup>Pro-cyclical credit flows are documented in Friedman (1983), and Blinder (1989). Blinder, by decomposing credit market debt, finds that private credit market debt, in particular trade credit - moves strongly pro-cyclically.

<sup>9</sup>This point is often stressed in recent work on the Asian financial crisis 1997-1998, see Chang and Velasco (1999).

<sup>10</sup>Such type of models have arisen in the literature due to (non-convex) lumpy adjustment costs. Typical examples are the S-s inventory, holding, money holding and price adjustment models (Blanchard and Fischer 1989, ch. 8), and employment models with lumpy adjustment costs (Caballero et al. 1995). For more details, see Flaschel, Franke and Semmler (1997).

the continuous time form is discretized through the Euler procedure and by a method recently proposed by Ozaki (1985, 1994). In both variants the lag structure of the model is constrained. Then we also employ an indirect method such as the recently developed Smooth Transition Regression (STR) model<sup>11</sup> that is well suited to capture regime changes. In the latter version, however, the lag structure will be unrestricted.

The remainder of the paper is organized as follows. In Section 2 we briefly lay out the model. Section 3 presents the empirical analysis and Section 4 concludes.

## 2 A Model with Regime Changes

The model by Semmler and Sieveking (1993) is grounded in an IS-LM version for a growing economy and links liquidity, credit and output. It assumes that liquidity of economic agents is enhanced and credit conditions improved when the variables pass through certain thresholds.<sup>12</sup> The agents in this model may simply be represented by their balance sheets. When balance sheets deteriorate (improve) creditworthiness deteriorates (improves). In accordance with the above cited literature, we presume then that credit conditions (creditworthiness), and thus spending of economic agents, depends on liquidity and income. As measure for liquidity we take liquid assets. At high level of economic activity liquidity rises, default risk falls and creditworthiness rises.<sup>13</sup> The reverse may be assumed to happen during a low level of economic activity. As liquidity shrinks, default risk rises and creditworthiness falls.<sup>14</sup> This in particular is posited to occur after the variables have passed certain thresholds. With respect to spending we may thus assume that spending accelerates (decelerates) when income and liquidity rises above (falls below) some threshold values.

The main features of a dynamic model in liquidity, credit and output for a growing economy which gives rise to regime changes through state-dependent reactions can be represented in a deterministic form as follows.<sup>15</sup> In presuming that economic agents respond to both financial variables (balance sheet variables) and real variables<sup>16</sup> the model might be written

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<sup>11</sup>For applications see Tong (1990), Granger and Teräsvirta (1993) and Granger, Teräsvirta and Anderson (1993), Ozaki (1985).

<sup>12</sup>We also want to note that liquidity and available credit may also have smoothing effects on production or consumption at least for small shocks. Thus, actual economies may exhibit corridor-stability, see Leijonhuvud (1973) and Semmler and Sieveking (1993). In this view small shocks do not give rise to deviation amplifying fluctuations but large shocks can lead to a different regime of propagation mechanism. Thus, only large shocks are predicted to result in magnified economic activities.

<sup>13</sup>Ideally, empirically one would like to employ net worth as collateral for borrowing, as referred to by the recent theory of the financial accelerator. Net worth should then be computed in terms of the net present value of the economic actions where net worth is the present value of the agents income flows reduced by the current and future debt payment commitments. Economic proxies for this variable are, however, hard to obtain. Alternatively one could take credit lines that agents obtain from banks as proxy for creditworthiness. Time series data of sufficient length also do not exist for this variable. We are therefore left with other balance sheet variables. Given the above mentioned role of liquidity for economic activity we take liquid assets as the balance sheet variable.

<sup>14</sup>It is thus only in this narrow sense that our model resembles the financial accelerator.

<sup>15</sup>For details of the model and its analytical and numerical study the reader is referred to Semmler and Sieveking (1993).

<sup>16</sup>Semmler and Koçkesen (1996) shows that one can also incorporate a monetary policy reaction function.

in the following generic form:

$$\dot{\lambda} = \lambda f_1(\lambda, \rho) \quad (1)$$

$$\dot{\rho} = \rho f_2(\lambda, \rho) \quad (2)$$

where  $\lambda = L/K$ ,  $\rho = Y/K$ , with  $L$  denoting liquid assets, a balance sheet variable,  $Y$ , income, a real variable, and  $K$  the capital stock. Since we want to undertake the empirical estimate with data on firms we interpret income,  $Y$ , as firms' income and  $\rho$  as firms' income relative to capital stock. Thus,  $\rho$  denotes the current rate of return on capital. A model of the type(1)-(2) can be derived from an aggregate model assuming that firms' income is linear in aggregate income. Roughly speaking, model (1)-(2) then says that swings in liquidity of firms is impacted by fluctuations in income (and liquidity) and swings in income is impacted by fluctuations in liquidity (and income). The responses of the variables on the left hand side to the variables on the right hand side are, however, state dependent. The local partial derivatives about the steady state are thus not necessarily informative.

As shown in the appendix the model can be thought of as being composed of two parts. First there is a basic part of the model which exhibits no thresholds and regime changes. It may be represented solely by linear coefficients and a sign structure of the coefficients such as follows

$$\dot{\lambda} = \lambda(\alpha - \beta\rho - \epsilon_1\lambda) \quad (3)$$

$$\dot{\rho} = \rho(-\gamma + \delta\lambda - \epsilon_2\rho) \quad (4)$$

A variant such as (3)-(4) can be derived from a conventional IS-LM approach for a growing economy, although, as pointed out in the appendix, the sign structure of the model may still be subject to empirical verification. A similar system is discussed in Ozaki (1987), there, however, for a nonlinear model in interest rate and output.

A second part of our model allows explicitly regime changes due to state-dependent reactions. Referring to the above discussion we may postulate regime changes to occur when the variables pass through certain thresholds. We posit that spending may accelerate (decelerate) when income and liquidity rise above (fall below) some threshold values. On the other hand, liquidity may also respond positively (negatively) when income or liquidity rise (fall) above (below) some thresholds. More formally, a model with regime changes in the cross effects between the variables can be written as follows

$$\dot{\lambda} = \lambda(\alpha - \beta\rho - \epsilon_1\lambda + g_1(\lambda, \rho)) \quad (5)$$

$$\dot{\rho} = \rho(-\gamma + \delta\lambda - \epsilon_2\rho + g_2(\lambda, \rho)) \quad (6)$$

where for  $i = 1, 2$

$$g_i = \begin{cases} g_i(\lambda, \rho) > 0 & \text{for } \left\{ \begin{array}{l} \lambda > \mu_1; \quad \mu_1 > \lambda^* \\ \text{and} \\ \rho > \nu_1; \quad \nu_1 > \rho^* \end{array} \right. \\ g_i(\lambda, \rho) < 0 & \text{for } \left\{ \begin{array}{l} \lambda < \mu_2; \quad \mu_2 < \lambda^* \\ \text{and} \\ \rho < \nu_2; \quad \nu_2 < \rho^* \end{array} \right. \end{cases} \quad (7)$$

and  $g_i = 0$  otherwise. The star, \*, denotes equilibrium values.

Thus, in the upper regime there is a positive perturbation of liquidity and/or spending whereas in the lower regime there is a negative perturbation of liquidity and/or spending. Assuming the above sign structure of the model and the perturbation terms we can state the following propositions.<sup>17</sup>

**Proposition 1** *System (3)-(4) is asymptotically stable.*

**Proposition 2** *If the perturbation terms  $g_1(\lambda, \rho)$ ,  $g_2(\lambda, \rho) \neq 0$  are small enough the system (5), (6) is asymptotically stable.*

**Proposition 3** *For any  $g_1(\lambda, \rho)$ ,  $g_2(\lambda, \rho) \neq 0$  system (5), (6) becomes unstable for  $\epsilon_1 = 0$ ,  $\epsilon_2 = 0$ . The trajectories, however, remain in a positively invariant set for any  $\epsilon_1, \epsilon_2 > 0$  even for large  $g_1(\lambda, \rho)$ ,  $g_2(\lambda, \rho)$ .*

We also want to note that system (5), (6) may exhibit, when proposition 3 holds, corridor stability in the sense of Leijonhufvud (1973).

For the purpose of our study we assume the perturbation terms  $g_i$  to be concave in  $\lambda$  and  $y$ ; for example,  $g_1 = k[\min(0, \rho - \nu_2) * \min(0, \lambda - \nu_1)]$  where  $k > 0$ . A sampling of computer simulations illustrates the effects of perturbations of the dynamics in the different regions of the state space. Parameter employed are given in the appendix. As shown in Semmler und Sieveking (1993) all perturbations of the basic part of the model lead to bounded fluctuations (limit cycles).<sup>18</sup>

### 3 The Empirical Analysis

We will follow two quite different but related methodologies to test for the above proposed liquidity-output interaction. The first method, which we shall refer to as the *direct method*, consists of discretizing the model given by (5), (6) and then directly estimating its parameters.<sup>19</sup> The second method, which we will call the *indirect method*, is less model dependent and based upon approximating the continuous time model by a discrete time state-dependent dynamics leaving the lag structure to be determined by the data. Therefore, the direct method is actually a test of the model whereas the indirect method could be thought of as a test of the presence of nonlinearities in the liquidity-output interaction without imposing the restrictions implied by the model.

A model with state depending reactions is, for example, the van der Pol equation

$$\ddot{x} - \alpha(1 - x^2)\dot{x} + \beta x = \varepsilon$$

where  $\varepsilon$  is a white noise shock and  $\alpha, \beta$  constant coefficients. Such second order differential equation in  $x$  can be estimated through direct procedures by employing, for example, the

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<sup>17</sup>For details of the following results, see Semmler and Sieveking (1993) and the numerical extensions in Semmler and Koçkesen (1996). Note that the subsequent statements hold when the above sign structure holds. To what extent this is empirically confirmed will be studied below.

<sup>18</sup>The existence of corridor stability which gives rise to two limit cycles, a repelling and an attracting one is studied in Semmler and Sieveking (1993).

<sup>19</sup>For a survey and comparison of the numerical accuracy of different discretization methods, see Kloeden, Platen and Schurz (1991) and for the local linearization procedure, see Ozaki (1985, 1994).

*Euler procedure* (see Ozaki 1986, 1987). Although the Euler procedure is a very convenient method of estimating a continuous time model, it is not the most precise one. In fact, it is possible that instability can arise in the discretized equation, although the corresponding differential equation is stable (see Kloeden, Platen and Schurz 1991 and Ozaki 1994)

In various papers Ozaki (1986, 1987, 1989, 1994) proposes another method, a *local linearization procedure*, that overcomes the shortcomings of methods such as the Euler scheme. He suggests a local linearization of the nonlinear stochastic differential equations by computing the Jacobian at each point in the state space. Given, for example, a nonlinear differential equation

$$\dot{z} = f(z|\theta) + \varepsilon$$

where  $\theta$  is the parameter set and  $\varepsilon$  is a white noise process, one can transform it into a discrete time model through local linearization using

$$z_{t+\Delta t} = A(z_t)z_t + B(z_t)\varepsilon_{t+\Delta t}$$

where

$$\begin{aligned} \varepsilon_{t+\Delta t} &: \text{ discrete time white noise} \\ A(z_t) &= \exp(L(z_t)\Delta t) \\ L(z_t) &= \frac{1}{\Delta t} \log \{ I + J_t^{-1} (e^{J_t \Delta t} - I) F_t \} \\ J_t &= \left\{ \frac{\partial f(z)}{\partial z} \right\}_{z=z_t} \\ F_t &: \text{ derived from } F_t z_t = f(z_t) \\ B(z_t) &: \text{ a function of the eigenvalues of } L(z_t) \end{aligned}$$

This local linearization is consistent since as  $\Delta t \rightarrow 0$  the original differential equation is obtained. The estimation procedure can be undertaken by nonlinear least squares or maximum likelihood procedures (see Ozaki 1994).<sup>20</sup>

On the other hand, following the indirect approach, an equation such as the van der Pol equation can be approximated by a discrete time locally self-exciting but globally bounded system of the following type (see Ozaki 1985).

$$x_t = (\phi_1 + \pi e^{-x_{t-1}^2})x_{t-1} + (\phi_2 + \pi_2 e^{-x_{t-1}^2})x_{t-2} + \varepsilon_t$$

or equivalently by a (piecewise linear or nonlinear) threshold model such as

$$x_t = \begin{cases} \pi(\mu_1)x_{t-1} + \varepsilon_t & \text{for } x_{t-1} < \mu_1 \\ \pi(x_{t-1})x_{t-1} + \varepsilon_t & \text{for } \mu_1 \leq x_{t-1} < \mu_2 \\ \pi(\mu_2)x_{t-1} + \varepsilon_t & \text{for } x_{t-1} \geq \mu_2 \end{cases} .$$

Discrete time threshold autoregressive models have become popular since they offer a rich array of dynamic behavior, including regime changes, asymmetries and state-dependent

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<sup>20</sup>The local linearization method of Ozaki subsequently to be used is written in GAUSS and is available from the authors upon request.



reactions. Tong (1990), Potter (1994), Mittnik & Niu (1994), DeGooijer & Kumar (1992), Pesaran & Potter (1993), and Sayers (1994) provide extensive review of the literature on this framework.

A generalization of this framework, called *Smooth Transition Regression* (STR) model, is elaborated by Luukkonen et al.(1988a, 1988b), Granger and Teräsvirta (1993), Teräsvirta (1994). It is based upon the idea that, in contrast to threshold autoregressive models, the transitions between regimes may take place smoothly.

A single equation STR can be written as

$$y_t = \beta' \mathbf{x}_t + (\theta' \mathbf{x}_t)F(z_t) + \varepsilon_t \quad (8)$$

$$\mathbf{x}_t = (1, y_{t-1}, \dots, y_{t-p}; x_{1t}, \dots, x_{kt})'$$

where  $z_t$  is any variable which is postulated to be governing the transition between regimes, and  $F$  some continuous function.

One widely used form of STR model is *logistic STR (LSTR) model* characterized by a logistic function  $F$ , i.e.,

$$F(z_t - c) = [1 + \exp(-\gamma(z_t - c))]^{-1}, \quad \gamma > 0 \quad (9)$$

with  $F(-\infty) = 0, F(+\infty) = 1, F(0) = 1/2$ .

Another form of is called *exponential STR (ESTR) model* where function  $F$  is exponential, i.e.,

$$F(z_t - c) = 1 - \exp[-\gamma(z_t - c)^2], \quad \gamma > 0 \quad (10)$$

with  $F(\pm\infty) = 1, F(0) = 0$ .

Granger and Teräsvirta (1993), Teräsvirta (1994), and Eitrheim and Teräsvirta (1995) present a full-fledge testing, specification, estimation and evaluation procedure for STR models and show that STR family contains threshold autoregressive and exponential autoregressive models as special cases. Teräsvirta & Anderson (1992), Granger, Teräsvirta & Anderson (1993), and Teräsvirta (1993) applied this procedure to various economic series.

### 3.1 Estimation of the Continuous Time Regime Change Model

Before we apply the first method to identify the model directly using actual data we will first test the method by employing a data set from a simulated model for which the parameter values are known in advance. We simulate the following stochastic differential equation system, which is the corresponding stochastic version of system (5) and (6) using the Euler method as discretization procedure. Randomizing (5)-(6) we would have a two dimensional system with Brownian motions such as:

$$d\lambda = \lambda(\alpha - \beta\rho - \theta_1\lambda)dt + \sigma_1dB^1 \quad (11)$$

$$d\rho = \rho(-\gamma + \delta\lambda - \theta_2\rho)dt + \sigma_2dB^2 \quad (12)$$

where  $\alpha, \beta, \theta_1, \sigma, \gamma, \delta, \theta_2$  are parameters and  $B^1$  and  $B^2$  are independent Brownian motions. Let  $(\lambda^*, \rho^*)$  be the steady state for the deterministic part of (11)- (12) satisfying

$$\begin{aligned}(\alpha - \beta\rho - \theta_1\lambda) &= 0 \\ (-\gamma + \delta\lambda - \theta_2\rho) &= 0\end{aligned}$$

To represent the acceleration term in our model we assume a state dependent reaction  $g(\lambda, \rho)$  with:

$$g(\lambda, \rho) = \left\{ \begin{array}{ll} k((\lambda - \mu)(\rho - \nu)) & \text{if } 0 < \lambda < \mu < \lambda^* \text{ and } 0 < \rho < \nu < \rho^* \\ 0 & \text{otherwise} \end{array} \right\}$$

For our experiment it is sufficient to solely consider a  $g(\lambda, \rho)$  in equ. (11). We thus have

$$d\lambda = \lambda(\alpha - \beta\rho - \theta_1\lambda - g(\lambda, \rho))dt + \sigma_1dB^1 \quad (11')$$

$$d\rho = \rho(-\gamma + \delta\lambda - \theta_2\rho)dt + \sigma_2dB^2 \quad (12')$$

where and  $k, \mu, \nu$ , are parameters.

For the simulation we choose the parameters

$$\begin{aligned}\alpha &= 0.1, \beta = 0.6, \theta_1 = 0.045 \\ \gamma &= 0.07, \delta = 0.7, \theta_2 = 0.078 \\ \sigma_1 &= 0.002, \sigma_2 = 0.002\end{aligned}$$

with a steady state at

$$(\lambda^*, \rho^*) = (0.082, 0.161)$$

For  $g(\lambda, \rho)$  we choose  $\mu = 0.06 (< 0.082)$ ,  $\nu = 0.08 (< 0.161)$ .  $k = 0.4$ .

With initial values  $(\lambda_0, \rho_0) = (0.11, 0.16)$  data were generated for  $\Delta t = 0.05$  and  $n = \{0, 1, \dots, 9999\}$ .

The data generating process with the Euler approximation is obtained from:

$$\begin{aligned}\lambda_{n+1} - \lambda_n &= \lambda_n(\alpha - \beta\rho_n - \theta_1\lambda_n - g(\lambda_n, \rho_n))\Delta t + \sigma_1(B_{\Delta t(n+1)}^1 - B_{\Delta tn}^1) \\ \rho_{n+1} - \rho_n &= \rho_n(-\gamma + \delta\lambda_n - \theta_2\rho_n)\Delta t + \sigma_2(B_{\Delta t(n+1)}^2 - B_{\Delta tn}^2)\end{aligned}$$

*Figure 1 about here*

Trajectories from the stochastic simulations, exhibiting bounded fluctuations, are shown in Figure 1.

For the estimation we choose data at every 20th-unit then we have 500 points for our estimation. There are two steps in the estimation: first we approximate the "continuous" time by a discrete time system. The discretization of the continuous time system exhibits a very small step size in contrast with the discrete time system with larger step size which is used for the estimation. Second it is to decide the estimation method. For our case we in fact know that a parametric model with normal distribution and constant variance of the noise term the nonlinear least square (NLLS) estimation method is equivalent to the maximum likelihood (ML) method. We undertake NLLS estimations. To approximate and estimate the continuous time two dimensional system (11')-(12') we use two methods: (1) the Euler approximation (first order approximation of a stochastic differential equation) and (2) the Ozaki local linearization method. The numerical results are as follows whereby we take the true values as start values and estimate three simulated series, a, b, and c.

Table 1a: Estimation with the Euler approximation (simulated data)

	a	b	c
$\alpha = 0.1$	0.109*	0.109*	0.106*
$\beta = 0.6$	0.584*	0.6*	0.644*
$\theta_1 = 0.045$	0.136	0.116*	0.029*
$\gamma = 0.07$	0.078	0.074	0.070*
$\delta = 0.7$	0.757	0.763	0.687*
$\theta_2 = 0.078$	0.067	0.102*	0.073
$\sigma_1 = 0.002$	0.002	0.002	0.002
$\sigma_1 = 0.002$	0.002	0.002	0.002
$\mu = 0.06$	0.06	0.06	0.06
$\nu = 0.08$	0.08	0.08	0.08
$k = 0.4$	0.4	0.4	0.4

Table 1b: Estimation with the Ozaki approximation (simulated data)

	a	b	c
$\alpha = 0.1$	0.113	0.113	0.110
$\beta = 0.6$	0.617	0.619	0.664
$\theta_1 = 0.045$	0.123*	0.124	0.040
$\gamma = 0.07$	0.073*	0.068*	0.065
$\delta = 0.7$	0.710*	0.708*	0.646
$\theta_2 = 0.078$	0.067	0.104	0.074*
$\sigma_1 = 0.002$	0.002	0.002	0.002
$\sigma_1 = 0.002$	0.002	0.002	0.002
$\mu = 0.06$	0.06	0.06	0.06
$\nu = 0.08$	0.08	0.08	0.08
$k = 0.4$	0.4	0.4	0.4

\* means a smaller bias

As can be observed from Tables 1a and 1b the estimated parameters do not depart too much from the parameters used for the simulation of the system and moreover, the parameters from both of our estimation methods seem to be similar. We want to note, however, that the computation by using the Ozaki method is more cumbersome and needs much more time than the Euler method.<sup>21</sup>

Next, we gain undertake simulations with the estimated parameter sets. Simulations for our estimated parameter sets from the Euler approximation showed similar results, a typical picture is given in Figure 2.

*Figure 2 about here*

A typical picture from simulations using the parameters from the local linearization method of Ozaki is shown in Figure 3.

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<sup>21</sup>The estimation with the Ozaki method often required 48 hours computation time on a PC (and not always succeeding) whereas the Euler method succeeds in a few minutes.

*Figure 3 about here*

Next, we apply the Euler scheme and the Ozaki method to an actual data set.<sup>22</sup> The time period of the analysis is 1960.1-1988.4 using quarterly data of the U.S. economy. The liquidity variable is the ratio of liquid assets of non-financial corporate business sector from the Flow of Funds Accounts (1994) to the capital stock for the same sector as estimated by Fair (1992). The resulting series has been seasonally adjusted before the analysis has been conducted. As real variable we take the rate of return on capital. It is the flow of profits of non-financial corporate business sector from the Flow of Funds Accounts divided by the capital stock. Although the data were available for 1952.1-1991.4, the test and estimation period has been chosen as 1960.1-1988.4 for two reasons. First, before 1960.1 the series contain outliers which has been argued to favor the hypothesis of nonlinearities. Second, the period between 1988.4-1991.4 will later be used for out-of-sample forecasts.

The estimates using both NLLS and ML gave us the results as reported in Table 2. We report for the Euler approximation both the estimates for NLLS and ML. For the estimation we employ a similar acceleration term as used in (11')-(12'). However, for reasons of simplicity, we employ the mean of the variables  $\lambda$  and  $\rho$  as thresholds below which the acceleration term,  $g(\lambda, \rho)$ , is activated.

*Table 2: Parameter estimates for actual data*

	<b>Euler (NLLS)</b>	<b>Euler (ML)</b>	<b>Ozaki</b>
$\alpha$	0.023	0.023	0.158
$\beta$	0.170	0.175	0.577
$\theta_1$	0.015	0.011	0.325
$k$	0.05	0.005	0.002
$\gamma$	-0.015	-0.0167	0.156
$\delta$	0.29	0.279	0.648
$\theta_2$	0.489	0.472	-0.230

As can be observed from Table 2, overall our theoretical model appears to match closely what we find in the data. The signs of the parameters obtained by the Euler method are as expected except for  $\gamma$ . Also, the parameters from NLLS and the ML estimations are similar. The simulation with the parameters from the Euler NLLS estimates, when the system is simulated with the diffusion terms as in (11')-(12') generates qualitative the same picture as in figure 2 (not included here).

As Table 2 also shows the estimates of the parameters from the Ozaki methods are slightly different. Using the estimated parameters from the local linearization method of Ozaki we

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<sup>22</sup>Note that for our theoretical model it would be better to use for the liquidity variable a measure of bank credit potentially available to firms, for example credit lines. Since, however, time series data on credit lines are not available for our time period we simply employ liquid assets for firms as our measure of liquidity. This is equivalent to positing, as our theoretical exposition suggests, a positive covariation between liquid assets, creditworthiness and credit flows. We do not take actual credit flows as our financial variable as the actual credit flows are not a good measure for creditworthiness.

have obtained from a simulation with the diffusion terms as in (11')-(12') similar trajectories as in Figure 3 (not included here).

Overall, we found that the Ozaki method captures better than the Euler method the fluctuations in the data. Although it might be interesting to further study the forecast properties of the estimated nonlinear models this is here left aside. A detailed diagnostic study of the forecast properties of an estimated nonlinear model is pursued in the next section where we are also able to compare it to an appropriate linear model.

We conclude this subsection by noting that the direct estimation of the continuous time model seems to be a fruitful strategy. In particular we want to note that there does not seem to be great differences in estimation results using an Euler approximation and more refined methods such as the Ozaki method. Both methods pick up the accelerator term. We also want to note that a further refinements of the step size in the Euler procedure does not seem to improve the estimation results. We have undertaken experiments of such kind and found not much difference in the parameter estimates.<sup>23</sup> On the other hand our direct estimations of the above stochastic differential equations restrict the lag structure. This might be a misspecification of the model driving the actual data. To investigate this further we next undertake a more data based methodology (Smooth Transition Regression) and let the data determine the type of nonlinearity (if any exists) and the appropriate lag structure.

### 3.2 Estimation with the Smooth Transition Regression Methodology

The empirical methodology of STR is composed of the following steps first specifying a linear model as the null hypothesis against which the linearity is tested. A test with power against both LSTR and ESTR involves testing  $H_0 : \phi_1 = \phi_2 = \phi_3 = 0$  in the following auxiliary regression:

$$y_t = \beta' \mathbf{x}_t + \phi_1' \mathbf{x}_t z_{t-d} + \phi_2' \mathbf{x}_t z_{t-d}^2 + \phi_3' \mathbf{x}_t z_{t-d}^3 + \eta_t$$

The above regression can also be used to select the transition variable  $z_t$  by conducting the test for different variables. If linearity is rejected for several choices of  $z_t$ , then select the one with the smallest probability value as the transition variable. Given that the linearity hypothesis is rejected in favor of an STR type of nonlinearity one determines whether an LSTR or ESTR model is more appropriate by using the following sequence of nested hypotheses:

$$\begin{aligned} H_{03} : \phi_3 &= 0 \\ H_{02} : \phi_2 &= 0 \mid \phi_3 = 0 \\ H_{01} : \phi_1 &= 0 \mid \phi_3 = \phi_2 = 0 \end{aligned}$$

If the test of  $H_{02}$  has the smallest probability value one chooses the ESTR, and otherwise chooses the LSTR family. Estimation of a specified STR model can then be undertaken by nonlinear least squares.

For the following analysis we employ the same data on firms as described above. Both the liquidity and the rate of return series have one unit root as indicated by the aug-

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<sup>23</sup>The reason is presumably that there is no independent information added if one employs a finer step size.

mented Dickey-Fuller tests. The data have been detrended and rendered stationary using the Hodrick-Prescott (HP) filter (with smoothness parameter 1600).<sup>24</sup>

### 3.2.1 The Linear Model:

We use the following linear model as the basis model:

$$\begin{aligned}\lambda_t &= a_1 + B_1(L)\lambda_t + C_1(L)\rho_t + \varepsilon_{1t} \\ \rho_t &= a_2 + B_2(L)\lambda_t + C_2(L)\rho_t + \varepsilon_{2t}\end{aligned}\quad (13)$$

where  $B_1(L)$  and  $C_1(L)$  are polynomials in lag operator  $L$  of degree 6 whereas  $B_2(L)$  and  $C_2(L)$  are of degree 8.<sup>25</sup> The estimated linear equation for liquidity equation is<sup>26</sup>

$$\begin{aligned}\lambda_t &= \underbrace{-3 \times 10^{-5}}_{(.3 \times 10^{-4})} + \underbrace{.96}_{(.10)} \lambda_{t-1} - \underbrace{.15}_{(.13)} \lambda_{t-2} - \underbrace{.13}_{(.12)} \lambda_{t-3} + \underbrace{.53}_{(.12)} \lambda_{t-4} \\ &\quad - \underbrace{.53}_{(.13)} \lambda_{t-5} + \underbrace{.18}_{(.10)} \lambda_{t-6} + \underbrace{.02}_{(.04)} \rho_{t-1} - \underbrace{.06}_{(.05)} \rho_{t-2} - \underbrace{.001}_{(.05)} \rho_{t-3} \\ &\quad - \underbrace{.06}_{(.05)} \rho_{t-4} - \underbrace{.04}_{(.05)} \rho_{t-5} + \underbrace{.01}_{(.04)} \rho_{t-6}\end{aligned}\quad (14)$$

$$\begin{aligned}R^2 &= 0.81 & SE &= 0.003 & LM(7) &= 1.64(0.13) \\ ARCH(1) &= 3.66(0.06) & BJ &= 0.93(0.63) & RESET(2) &= 1.73(0.18)\end{aligned}$$

For the rate of return equation we obtained:

$$\begin{aligned}\rho_t &= \underbrace{-1 \times 10^{-4}}_{(.7 \times 10^{-4})} + \underbrace{.81}_{(.24)} \lambda_{t-1} - \underbrace{.30}_{(.32)} \lambda_{t-2} + \underbrace{.13}_{(.32)} \lambda_{t-3} - \underbrace{.56}_{(.29)} \lambda_{t-4} \\ &\quad - \underbrace{.06}_{(.30)} \lambda_{t-5} + \underbrace{.27}_{(.32)} \lambda_{t-6} - \underbrace{.30}_{(.32)} \lambda_{t-7} + \underbrace{.60}_{(.23)} \lambda_{t-8} + \underbrace{.72}_{(.10)} \rho_{t-1} \\ &\quad - \underbrace{.12}_{(.12)} \rho_{t-2} + \underbrace{.12}_{(.11)} \rho_{t-3} - \underbrace{.05}_{(.11)} \rho_{t-4} - \underbrace{.001}_{(.11)} \rho_{t-5} + \underbrace{.06}_{(.11)} \rho_{t-6} \\ &\quad + \underbrace{.04}_{(.11)} \rho_{t-7} - \underbrace{.27}_{(.09)} \rho_{t-8}\end{aligned}\quad (15)$$

$$\begin{aligned}R^2 &= 0.73 & SE &= 0.008 & LM(8) &= 1.32(0.26) \\ ARCH(1) &= 0.44(0.50) & BJ &= 2.36(0.31) & RESET(2) &= 0.64(0.53)\end{aligned}$$

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<sup>24</sup>The model described by equations (5)-(6) are cast in terms of normalized variables, i.e., liquidity and income normalized by the capital stock. In that sense their steady state values are bounded: actually at the steady state the growth rate is zero. Thus, the cyclical variations implied by the model are cycles around a zero (or more generally a constant) growth rate. Clearly, this is not the case in the data. Furthermore, the theory behind testing for and estimating STR models have been developed for stationary data. Since the theoretical models which inspired our empirical implementation explicitly refer to business cycles we considered Hodrick-Prescott filter as an appropriate detrending technique.

<sup>25</sup>Appropriate lag lengths were determined by using Akaike Information Criterion and diagnostic tests on residuals.

<sup>26</sup>Standard errors are in parantheses. *SE* stands for standard error of regression; *LM* is Breusch-Godfrey *LM* test for serial correlation (*F* version); *ARCH* is the *LM* test for *ARCH* effects; *BJ* is Bera-Jaque test for normality; *RESET* is Ramsey's *RESET* test. Numbers in parantheses are the order of the tests and next to the diagnostic statistics the probability values associated with them.

The standard diagnostic tests do not indicate any misspecification for any of the equations. When simulated as a system the linear model generates, as one would expect, convergent cycles and loses its forecasting ability soon after the iterations are started.

### 3.2.2 Linearity Tests and Model Specification

We postulate the following nonlinear equations as our alternatives to the linear ones above:

$$\lambda_t = \alpha_1 + \Phi_1(L)\lambda_t + \Theta_1(L)\rho_t + (\beta_1 + \Psi_1(L)\lambda_t + \Omega_1(L)\rho_t) F_1(z_{i(t-d_1)}; \gamma_1, c_1) + u_{1t} \quad (16)$$

$$\rho_t = \alpha_2 + \Phi_2(L)\lambda_t + \Theta_2(L)\rho_t + (\beta_2 + \Psi_2(L)\lambda_t + \Omega_2(L)\rho_t) F_2(z_{i(t-d_2)}; \gamma_2, c_2) + u_{2t} \quad (17)$$

which would result in either a logistic smooth transition regression (LSTR) model characterized by a logistic function  $F$ ,

$$F(z_{i(t-d_j)} - c_j) = [1 + \exp(-\gamma_j(z_{i(t-d_j)} - c_j))]^{-1}, \quad \gamma_j > 0; \quad i = 1, 2; \quad j = 1, 2 \quad (18)$$

or an exponential smooth transition regression (ESTR) model where function  $F$  is given by,

$$F(z_{i(t-d_j)} - c_j) = 1 - \exp[-\gamma_j(z_{i(t-d_j)} - c_j)^2], \quad \gamma_j > 0; \quad i = 1, 2; \quad j = 1, 2 \quad (19)$$

The auxiliary equation to test for linearity is the same in both forms of the function  $F$ . We run the tests for each equation and for various lags of both variables as potential transition variables.

Table 3 gives the results for the cases where the overall linearity tests have probability values ( $p$ ) smaller than 0.10. The probability value for  $H_{03}$  is  $p3$ , for  $H_{02}$  it is  $p2$ , and for  $H_{01}$  it is  $p1$ .<sup>27</sup>

Table 3: Linearity Tests: Liquidity Equation

Equation	Variable	p	p3	p2	p1
Liquidity					
	$\rho_{t-6}$	0.06	0.43	0.33	0.008
Rate of Return					
	$\lambda_{t-7}$	0.001	0.005	0.31	0.01
	$\lambda_{t-8}$	0.02	0.10	0.15	0.04
	$\rho_{t-4}$	0.08	0.50	0.08	0.06
	$\rho_{t-8}$	0.03	0.16	0.10	0.11

The linearity hypothesis is rejected at a 6% level of significance for the liquidity equation when the transition variable is  $\rho_{t-6}$ . It is rejected for several transition variables in the case of the rate of return equation, the most significant one arising when  $\lambda_{t-7}$  is the transition variable.<sup>28</sup> Also, for both equations LSTR type models are suggested. Hence, we have the

<sup>27</sup>See section 1.2 for a description of these hypotheses.

<sup>28</sup>We also explored the possibility of cointegration between the variables and of building an error correction STR model. Although the long term relationship between the variables would have been forced to be linear, the existence of an error correction mechanism would constitute an interesting case to study. However, Johansen cointegration tests, when applied to the whole period (1960.1-1988.4), did not indicate cointegration. However, there is some indication (particularly for the seasonally adjusted data) of cointegration when the test period is restricted to that of 1960.1-1979.4. However, in this case, because of numerical problems, we failed even to implement the linearity tests.

following general specifications,

$$\lambda_t = \alpha_1 + \Phi_1(L)\lambda_t + \Theta_1(L)\rho_t + (\beta_1 + \Psi_1(L)\lambda_t + \Omega_1(L)\rho_t) F_1(\rho_{t-6}; \gamma_1, c_1) + u_{1t} \quad (20)$$

$$\rho_t = \alpha_2 + \Phi_2(L)\lambda_t + \Theta_2(L)\rho_t + (\beta_2 + \Psi_2(L)\lambda_t + \Omega_2(L)\rho_t) F_2(\lambda_{t-7}; \gamma_2, c_2) + u_{2t} \quad (21)$$

where  $F_1$  and  $F_2$  are both logistic functions.<sup>29</sup>

### 3.2.3 Estimation and Diagnostics

The STR models can be estimated using nonlinear least squares. Certain lags of both variables which proved to be insignificant were excluded to arrive at more parsimonious specifications. The estimated LSTR equation for liquidity is

$$\begin{aligned} \lambda_t = & \underset{(.0004)}{.0008} + \underset{(.06)}{.75}\lambda_{t-1} + \underset{(.09)}{.55}\lambda_{t-4} - \underset{(.08)}{.36}\lambda_{t-5} - \underset{(.05)}{.22}\rho_{t-5} \\ & + \underset{(.05)}{.18}\rho_{t-6} + \left( \underset{(.14)}{-.56}\lambda_{t-3} - \underset{(.07)}{.18}\rho_{t-4} + \underset{(.12)}{.36}\rho_{t-5} - \underset{(.10)}{.39}\rho_{t-6} \right) \\ & \times \left( 1 + \exp \left[ \underset{(1.42)}{-2.85} \times 72.59 \left( \rho_{t-6} - \underset{(.003)}{.005} \right) \right] \right)^{-1} \end{aligned} \quad (22)$$

$$\begin{aligned} R^2 = 0.85 & \quad SE = 0.0029 & \quad LM(7) = 1.81(0.09) \\ ARCH(1) = 0.06(0.81) & \quad BJ = 1.45(0.48) \end{aligned}$$

In terms of static in-sample forecasting the LSTR model for liquidity performs better than its linear counter-part. The standard diagnostics do not indicate any misspecification. The test for remaining nonlinearity indicates that the estimated model captures the nonlinearity well.

The transition parameter is 207.21 which indicates a fast transition between what can be regarded as two regimes. The lower regime corresponds to that part of the state-space where the rate of return is below 0.0049 (which is slightly greater than the steady-state value of the rate of return) and the upper regime to the part where the rate of return is above that level. The dynamic properties of the model at different regions of the state space will be discussed later once the rate of return equation is also estimated.

The LSTR model for rate of return is given by

$$\begin{aligned} \rho_t = & \underset{(.001)}{-.004} + \underset{(.14)}{.76}\lambda_{t-1} - \underset{(.20)}{.58}\lambda_{t-5} + \underset{(.06)}{.63}\rho_{t-1} - \underset{(.11)}{.34}\rho_{t-5} \\ & + \underset{(.11)}{.36}\rho_{t-6} - \underset{(.06)}{.21}\rho_{t-8} + \left( \underset{(.003)}{.006} - \underset{(.27)}{1.02}\lambda_{t-4} + \underset{(.38)}{1.18}\lambda_{t-5} \right. \\ & \left. - \underset{(.39)}{.82}\lambda_{t-7} + \underset{(.26)}{1.10}\lambda_{t-8} + \underset{(.18)}{.57}\rho_{t-5} - \underset{(.19)}{.53}\rho_{t-6} \right) \\ & \times \left( 1 + \exp \left[ \underset{(20.38)}{-18.81} \times 138.50 \left( \lambda_{t-7} - \underset{(.0004)}{.0002} \right) \right] \right)^{-1} \end{aligned} \quad (23)$$

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<sup>29</sup>To check the robustness of the test results we conducted the test for different sample periods and for the first and fourth differences of the data (both seasonally adjusted and unadjusted). In almost every case the hypothesis of linearity was rejected though with different suggested models.



$$\begin{array}{lll}
R^2 = 0.79 & SE = 0.007 & LM(9) = 0.93(0.51) \\
ARCH(1) = 7.17(0.01) & BJ = 0.093(0.95) & LIN(\rho_{t-8}) = 2.26(0.01)
\end{array}$$

where  $LIN(\rho_{t-8})$  denotes the linearity test with  $\rho_{t-8}$  as the transition variable. With respect to the static fit the LSTR equation for the rate of return again performs better than the linear one. Only the ARCH test indicates misspecification in the equation.<sup>30</sup> However, the estimated model still exhibits uncaptured nonlinearity at variable  $\rho_{t-8}$  of an ESTR type. An appropriate strategy would be to reestimate the equation using two transition variables ( $\lambda_{t-7}$  and  $\rho_{t-8}$ ) one entering with a logistic and the other as exponential function.<sup>31</sup>

The transition parameter is much bigger (2605.77) for this equation indicating an even faster transition between the lower (liquidity less than 0.0002 - a value less than the steady-state of the liquidity) and upper regimes (liquidity greater than 0.0002). Hence, the transition between regimes takes place much more quickly as a response to changes in the liquidity (the financial variable) than to the rate of return (the real variable). This could be seen as an indication of the fact that firms adjust their behavior very fast as their liquidity gets closer and crosses over a critical level. The adjustment to the changes in rate of return, however, takes place more smoothly.

*Figure 4 and Figure 5 about here*

When simulated as a system, the LSTR model exhibits a periodic motion which captures the overall motion of the actual data much better than the linear model does (see Figures 4 and 5). Several in-sample and out-of-sample simulation statistics are reported in Table 4. To evaluate dynamic forecast performance we first used the model estimated for the period 1960.1-1988.4 and calculated both the in-sample (1960.1-1988.4) and multi-step ahead forecasts (1989.1-1991.4). We also repeated this exercise using a Monte-Carlo simulation, rather than a deterministic one, where we shocked the system each period with a normally distributed random term (whose standard deviation is taken to be the standard error of the regression in respective models). We repeated this 1000 times and took the mean of the resulting simulations as our point forecasts.

Using the deterministic simulation method we found that the nonlinear model does a better job in simulating the rate of return within the sample whereas the converse is true for the liquidity variable (see row 1960.1-1988.4 under Deterministic Simulations in Table 4). However, the nonlinear model fares slightly better when we use the Monte-Carlo simulations (row 1960.1-1988.4 under Monte-Carlo Simulations). Out-of-sample forecast performance of the linear model is slightly better using the deterministic simulations. However, when we used Monte-Carlo simulations nonlinear model performed better in forecasting the rate of return (row 1989.1-1991.4).

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<sup>30</sup>We have not yet been able to jointly model the nonlinearity in the conditional mean and ARCH effects.

<sup>31</sup>Our attempts to do so run into numerical difficulties in the estimation procedure.

Table 4: Forecast Mean Squared Errors( $\times 1,000$ )

	Linear Model		Nonlinear Model	
	liquidity	rate of return	liquidity	rate of return
<b>Deterministic Simulations</b>				
1960.1-1988.4	7.45	13.82	7.82	13.00
1986.1-1991.4	8.08	11.50	5.99	12.26
1986.1-1988.4	9.85	14.86	6.98	13.23
1989.1-1991.4	4.16	4.52	6.96	4.78
<b>Monte-Carlo Simulations</b>				
1960.1-1988.4	7.42	13.76	7.08	13.31
1989.1-1991.4	4.23	4.43	6.94	4.20

We also estimated the equations for the period 1960.1-1985.4 and conducted deterministic forecasts for the periods 1986.1-1991.4 and 1986.1-1988.4 (see rows 1986.1-1991.4 and 1986.1-1988.4 in Table 4). In terms of the shorter period we found that the nonlinear model has significantly better forecasting performance for both variables. However, for the longer period nonlinear model performs better only for the liquidity variable. Although the overall evidence seems to be inconclusive, on average we can conclude that the linear model has a slightly better forecasting performance for the liquidity variable whereas the nonlinear model has a significantly better forecast performance for the rate of return variable.

### 3.2.4 Evaluation of the Dynamic Properties

The theoretical model given by equations (5) and (6) implies two thresholds and four regimes in which three types of dynamics are activated, namely dynamics with  $g_i > 0$ ,  $g_i < 0$ , and  $g_i = 0$ . Note also that  $g_1$  and  $g_2$  take always the same signs, i.e., when  $g_1$  is positive so is  $g_2$  and when  $g_1$  is zero so is  $g_2$  etc. However, the empirical model has  $F_1$  and  $F_2$  functions which in the extreme take values of zero and one but also take potentially many intermediate values. Due to this fact and due to the existence of many lags in the empirical model it is very difficult to establish a one-to-one correspondence between the regimes which arise in the theoretical and empirical models. However, we can roughly say that when both  $F_1$  and  $F_2$  are equal to zero we have the case which corresponds to  $g_1, g_2 < 0$  in the theoretical model, and when  $F_1$  and  $F_2$  are equal to 1 we have the case of  $g_1, g_2 > 0$ . For some intermediate  $F$  values we have the case where  $g_1, g_2 = 0$ . Thus, specification of the empirical model, keeping in mind the limitations mentioned above, is similar to the theoretical model.

The steady-state values of the liquidity and the rate of return have been calculated numerically and found to be equal to 0.0013 for liquidity and 0.0035 for rate of return.<sup>32</sup>

The system at the steady state has a pair of complex eigenvalues with a maximum modulus greater than unity indicating local instability. Furthermore, the fact that there exists one pair of complex eigenvalues whose real part is greater than unity indicate the possibility of a Hopf bifurcation taking place as some parameters change. If this is the case then one would expect the emergence of a stable limit cycle.<sup>33</sup> The system was simulated

<sup>32</sup>Mean trend plus steady-state is 0.15 for liquidity and 0.13 for rate of return which are both slightly above the means of the series before detrending.

<sup>33</sup>Assuming that further conditions hold [see Guckenheimer and Holmes(1983)].

over extended periods (up to 15,000 iterations) and it was found that this is indeed the case (see Figure 6). The simulations with different initial values inside and outside the limit cycle showed that this periodic motion was stable. The cycles have a period around 26 quarters (or 6.5 years) which conforms well to the definition of the business cycle by NBER<sup>34</sup>.

*Figure 6 about here*

Thus we can conclude that the dynamic interaction between liquidity and the rate of return on capital contains essential nonlinearities which lead to sustained fluctuations in both variables even in the absence of exogenous shocks to the system. However, we could not detect corridor stability (as apparent from the fact that the steady-state is locally unstable) which would require a more refined methodology, i.e., a model with multiple threshold variables in equations.

Furthermore, we found that the response of the system to shocks in the variables is asymmetric. Following Potter (1994) we define a nonlinear impulse response function (NLIRF) as

$$NLIRF_n(v; y_t) = E[Y_{t+n} | Y_t = y_t + v, Y_{t-1} = y_{t-1}, \dots] - E[Y_{t+n} | Y_t = y_t, Y_{t-1} = y_{t-1}, \dots] \quad (24)$$

which is, in practical terms, simply the difference between the series at time  $t + n$ , when  $y_t$  is perturbed by  $v$  (size of the shock) at time  $t$ , and the series without the perturbation. Similarly, one can define  $NLIRF_n(-v; y_t)$  to denote the impulse response when the shock is negative. In a linear system the impulse response function is symmetric and thus

$$ASY_n(v; y_t) = NLIRF_n(v; y_t) + NLIRF_n(-v; y_t) \quad (25)$$

will be zero.  $ASY_n(v; y_t)$  can be regarded as a measure of asymmetry: the extent to which it deviates from zero indicates the extent of asymmetric response to shocks.

We calculated (by simulation) the nonlinear response functions and the measures of asymmetry for both variables (for up to 12 periods) when the system is perturbed by a single shock to one of the variables (where the size of the shock is one standard deviation of the variable in question). We also calculated the mean asymmetry measures ( $MASY$ ) defined as:

$$MASY = \frac{1}{T} \sum_{n=1}^T \frac{ASY_n(v; y_t)}{\sigma_y} \quad (26)$$

where  $\sigma_y$  is the standard deviation of the corresponding series. Thus, a negative  $MASY$  will indicate stronger response to negative shocks and vice versa.

The initial states at which the system is perturbed are given by Table 5.

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<sup>34</sup>See however Cogley and Nason (1995) who claim that Hodrick- Prescott filter implants business cycle periodicity to the filtered data even if the original series is a random walk. This also raises the concern that HP detrended liquidity and rate of return series covary because of this property of the HP filter. To check whether this is the case we estimated linear VAR models with both first-differenced and HP detrended data and did not detect any spurious cross-correlations as a result of HP detrending.

Table 5: *Impulse Response Analysis Initial States*

State I	$\rho$ at max,	$\rho \downarrow, \lambda \downarrow$
State II	$\lambda$ at min,	$\rho \downarrow, \lambda \uparrow$
State III	$\rho$ at min,	$\rho \uparrow, \lambda \uparrow$
State IV	$\lambda$ at max,	$\rho \uparrow, \lambda \downarrow$

If one uses the movements of rate of return as an index of business cycle, then State I corresponds to the peak, State II to the lower phase of the downswing, State III to the trough, and State IV to the upper phase of the upswing. Table 6 reports the mean asymmetry measures at different states.

Table 6: *Mean Asymmetry Measures*

Responses of	Shocks to liquidity		Shocks to rate of return	
	liquidity	rate of return	liquidity	rate of return
State I	-0.0002	-0.0026	-0.0001	-0.0001
State II	-0.1086	-0.0992	-0.0778	-0.0651
State III	-0.2194	-0.4222	-0.1813	-0.0291
State IV	-0.0617	-0.0593	0.0044	-0.0242

Although the response is symmetric for initial periods (5 to 7 periods) it ceases to be so afterwards. In general, the extent of the asymmetric response to shocks to liquidity is bigger. In every state of the system negative shocks to liquidity generates bigger response by both liquidity and rate of return. The same is true for shocks to rate of return except in State IV where the response of liquidity to a positive shock is bigger.<sup>35</sup> The largest asymmetry occurs in State III (trough) whereas the smallest in State I (peak). In other words, although the agents seem to react more strongly to negative shocks, the extent to which they do so is larger at the trough as compared to the peak.<sup>36</sup>

These exercises point out that the pattern of asymmetric response to shocks in a model of financial-real interaction is very rich. In particular one should distinguish between shocks of different signs, shocks to real and financial variables, and shocks during the different phases of the business cycles.

Due to the high dimensionality of the dynamical system it is impossible to analytically study its global stability properties. However, if we can imagine that the model has basically four distinct regimes (other than the neighborhood around the steady-state) which can be approximated by four linear models, the eigenvalues in those regimes could give some information.<sup>37</sup>

The regimes A and D, in Table 5, correspond to the upper and lower regimes, respectively, which have been discussed before, i.e., the upper regime corresponds to that region of the state-space where liquidity and rate of return are greater than their steady-state values and the lower regime corresponds to the region where they are smaller. We can see from Table 7 that the eigenvalues at these two regimes are complex with moduli smaller than

<sup>35</sup>Even at State IV, the first 8-period negative response is bigger than the positive response.

<sup>36</sup>Although not reported, we found that the time pattern of impulse responses and asymmetry measures also differ among different states and shocks to different variables.

<sup>37</sup>See Granger and Teräsvirta (1993) for a similar analysis.

one indicating convergent cycles. The cycles in the lower regime have a period around 15 quarters whereas those in the upper regime exhibit periods around 24 quarters.

*Table 7: Maximum Moduli and Periods at Different States*

States	Maximum Modulus	Periods
A. ( $F_1 = 1, F_2 = 1$ )	0.99	24
B. ( $F_1 = 1, F_2 = 0$ )	0.97	24
C. ( $F_1 = 0, F_2 = 1$ )	1.03	13
D. ( $F_1 = 0, F_2 = 0$ )	0.95	15

If we linearize the system at different states of the system we can obtain more insight into the nature of interaction between the liquidity and rate of return. Table 8 reports the long-term multipliers for both equations in five different states.

*Table 8: Multipliers at Different States of the System*

States	Liquidity Equation	Rate of return Equation
Steady-State	-0.433*	1.128**
A. ( $F_1 = 1, F_2 = 1$ )	-0.399	1.188
B. ( $F_1 = 1, F_2 = 0$ )	-0.399	0.307
C. ( $F_1 = 0, F_2 = 1$ )	-0.706	1.188
D. ( $F_1 = 0, F_2 = 0$ )	-0.706	0.307
* $F_1 = 0.424$	** $F_2 = 0.937$	

As one would expect, the impact of the rate of return on liquidity is negative whereas that of liquidity on the rate of return is positive in every state. Furthermore, the effect of rate of return (which is the transition variable in liquidity equation) is larger in the lower regime (i.e., regime D) than in the upper regime (i.e., regime A), which lends some support to the idea in Semmler and Sieveking (1993). However, the converse is true for the effect of liquidity on rate of return, i.e., liquidity exerts a stronger impact on rate of return in the upper regime than in the lower regime. These results which lend support to the model given in equations (5) and (6) imply that firms will increase their investment expenditures more as a response to an increase in their liquidity ratios if the rate of return on capital and the rate of investment is already higher than their steady-state levels.

On the basis of these results we can conclude that a model of liquidity-output interaction should take into account the differences in the system's response to real and financial disturbances as well as the state (or history) dependent responses.

Before we close this section we should also note that we attempted to apply the same methodology to household income (both for wage and salary income and for aggregate household income) and household liquidity series. In this case we detected nonlinearity in income equation whereas no nonlinearity was detected for the liquidity equation. Analysis of the dynamic properties of the estimated systems revealed, however, asymptotically convergent cyclical behavior.<sup>38</sup> It seems, therefore, that the theoretical models of liquidity-output interaction which imply complex dynamics are most suitable to the firm sector of the economy.

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<sup>38</sup>Details of the results are available from the authors on request.

## 4 Concluding Remarks

Our empirical analysis has shown that state-dependent reactions and regime changes in cross effects between liquidity and output variables can, with some success, be studied by a continuous time regime change model, the direct method (Euler scheme and Ozaki's local linearization method), as well as a discrete time version, by the indirect method (STR methodology). Encouraging are both the application of the Euler method as well as the local linearization procedure of Ozaki. Although the first is easier to apply it may generate instability. The latter is more difficult to employ since for a higher dimensional state space the number of observations to be used are limited and the computation is very time consuming. Although the direct method did reveal some expected dynamic behavior, such as regime change behavior, when estimated, yet lag-effects are neglected.

Particularly interesting was the application of the STR model. As shown, a linear model fits the data well and exhibits a convergent behavior, indicating a stable steady-state.<sup>39</sup> In other words, if one were to assume that the data had a linear representation, the system would be regarded as being stable around the steady-state and the cause of fluctuations would have to be attributed to exogenous shocks. However, a regime change model which we claim is a better representation of the data, reveals that the actual dynamics of the system is characterized by a locally unstable steady-state which is contained by stable outer regions.<sup>40</sup>

Regime change models are capable of asking and answering more interesting questions which have been prevalent in the theoretical literature but could not find a way to be examined by empirical analysis. The results of this paper lend support to a certain class of credit and output interaction models characterized by an unstable steady-state, regime shifts, and asymmetric response to shocks. This encourages further work to examine regime changes in other macroeconomic relationships, for example, in relations between stock market data and output [see Chiarella, Semmler and Koçkesen (1996)] or exchange rate data and output. Threshold principle time series models seem to be particularly useful in this endeavor.

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<sup>39</sup>The linear system for firm data has complex eigenvalues with a maximum modulus of 0.95 and periods of 25 quarters and the STR variants for the household data show convergence when simulated.

<sup>40</sup>A similar comparison of linear and nonlinear systems is undertaken in Blatt (1978).

## 5 Appendix: Derivation of the Basic Model of Section 2.

The basic part of our model is consistent with a monetary growth model with an explicit LM schedule (for details, see Flaschel, Franke and Semmler (1997, ch.4)).

We take the ratio  $\lambda = L/K$  where  $L$  is liquidity and presume that the growth rate of liquidity in our basic model is equal to the growth rate of money. Then, with  $K$ , the capital stock, we obtain in terms of growth rates

$$\dot{\lambda} = g_L - g_k(i, y) \quad (\text{A1})$$

where  $g_L$  denotes the exogenous growth of money supply and whereby it is posited that the growth rate of the capital stock depends on the interest rate,  $g_{k_i} < 0$ , and income,  $g_{k_y} > 0$ , with  $y$  the ratio of income,  $Y$ , to capital stock,  $K$ . Note that “ $\dot{\phantom{x}}$ ” denotes growth rates. From  $y = Y/K$  we can obtain in terms of growth rates

$$\hat{y} = g_y(y, i) - g_k(i, y)$$

or, by positing that  $g_Y$  depends linearly on income and the interest rate, with  $g_{Y_y} > 0$ ,  $g_{Y_i} < 0$ , we get

$$\hat{y} = \alpha y - \beta i - g_k(i, y) \quad (\text{A2})$$

The money market equilibrium with a linear money demand function can be written as:

$$\begin{aligned} M &= L && (LM - equilibrium) \\ L &= h_1 Y - h_2 K (i - i^*), \text{ divided by } K \text{ gives} \\ \lambda &= h_1 y - h_2 (i - i^*) \end{aligned}$$

which yields for a fixed long-run natural interest rate  $i^*$

$$i = \frac{h_1 y - \lambda}{h_2} + i^*$$

Substituting the interest rate determination into (A2) gives:

$$\begin{aligned} \hat{y} &= \alpha y - \beta \left( \frac{h_1 y - \lambda}{h_2} + i^* \right) - g_k(\lambda, y) \\ \hat{y} &= -\beta i^* + \frac{(\alpha - \beta h_1) y}{h_2} + \beta \frac{\lambda}{h_2} - g_k(\lambda, y) \end{aligned} \quad (\text{A3})$$

Note that now the growth of income depends on income,  $y$ , and liquidity,  $\lambda$ . The system (A1), (A3) can be written in compact form as

$$\dot{\lambda} = \lambda (g_L - g_k(\lambda, y)) \quad (\text{A4})$$

$$\dot{y} = y(g_Y(\lambda, y) - g_k(\lambda, y)) \quad (\text{A5})$$

with  $g_{Y_m} > 0$ . Note that by collecting constant terms in (A2) and (A3) we may, with appropriate assumptions on the size of the constants, obtain the functional forms as in (A6), (A7) by additionally presuming linear functions for the growth of capital stock

$$\hat{\lambda} = g_L - (\beta_1 + \gamma_1 y + \gamma_2 \lambda) \quad (\text{A6})$$

$$\hat{y} = \beta_2 + \mu_1 y + \mu_2 \lambda - (\beta_1 + \gamma_1 y + \gamma_2 \lambda) \quad (\text{A7})$$

The linear specification of our functions will give rise to nonlinear differential equations though of the simplest type. In eqs. (A6) and (A7) the second term in brackets represents the growth rate of the capital stock with  $\beta_1$  the growth rate of the autonomous part of capital investment and  $\gamma_1 y, \gamma_2 \lambda$  the response of  $g_k$  to income and liquidity (both measured relative to capital stock).

Eq. (A6) can be simplified by using  $\alpha = g_L - \beta_1$ . We expect a positive sign for  $\alpha$ . We drop unnecessary terms in (A7) by denoting  $\gamma = \beta_2 - \beta_1$ ,  $\epsilon_2 = \mu_1 - \gamma_1$ , and  $\delta = \mu_2 - \gamma_2$ . If we assume that  $\beta_2 < \beta_1$ ,  $\mu_1 < \gamma_1$  (which presumes a strong accelerator effect for investment) and  $\mu_2 > \gamma_2$  we then can write our system of differential equations as

$$\hat{\lambda} = \alpha - \beta y - \epsilon_1 \lambda \quad (\text{A8})$$

$$\hat{y} = -\gamma + \delta \lambda - \epsilon_2 y \quad (\text{A9})$$

Equation system (A8), (A9) which we take as our bench mark model, is a nonlinear system of differential equations.<sup>41</sup> Note that it does not yet include perturbation terms arising from thresholds as discussed in Section 2. As there shown those perturbation terms arise from the creditors' response to liquidity and income — and thus creditworthiness of the economic agents— and, possibly, from monetary policy reactions.

As is demonstrated in Semmler and Sieveking (1993) the system (A8), (A9) has three equilibria ( $\lambda^* = 0, y^* = 0$ ), ( $\lambda^* > 0, y^* = 0$ ) and ( $\lambda^* > 0, y^* > 0$ ). The first two are saddle points and the last one is an attracting point. With the exception of those which start on one of the axes all of the trajectories converge to the unique attracting point  $\lambda^* > 0, y^* > 0$ .

With linear profit function, where the rate of return on capital is  $\rho = \pi/k$  with  $\pi$ , cash flows, we have

$$\dot{\lambda} = \lambda(g_L - g_k(\lambda, \rho)) \quad (\text{A10})$$

$$\dot{\rho} = \rho(g_\pi(\lambda, \rho) - g_k(\lambda, \rho)) \quad (\text{A11})$$

with  $g_\pi$  the growth rate of profit flows.

Using the linear form as in (A8), (A9) for the parameters an:  $\alpha = 0.1$ ,  $\gamma = 0.07$ ,  $\epsilon_1 = 0.045$ ,  $\beta = 0.6$ ,  $\delta = 0.7$ ,  $\epsilon_2 = 0.078$  create economically relevant equilibrium  $\lambda^* = 0.082$ ,  $\rho^* = 0.161$ .

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<sup>41</sup>Note that we here only have assumed a certain sign structure of the coefficients. Empirical estimations as undertaken in Section 3 will be needed to find out whether the sign structure is confirmed.



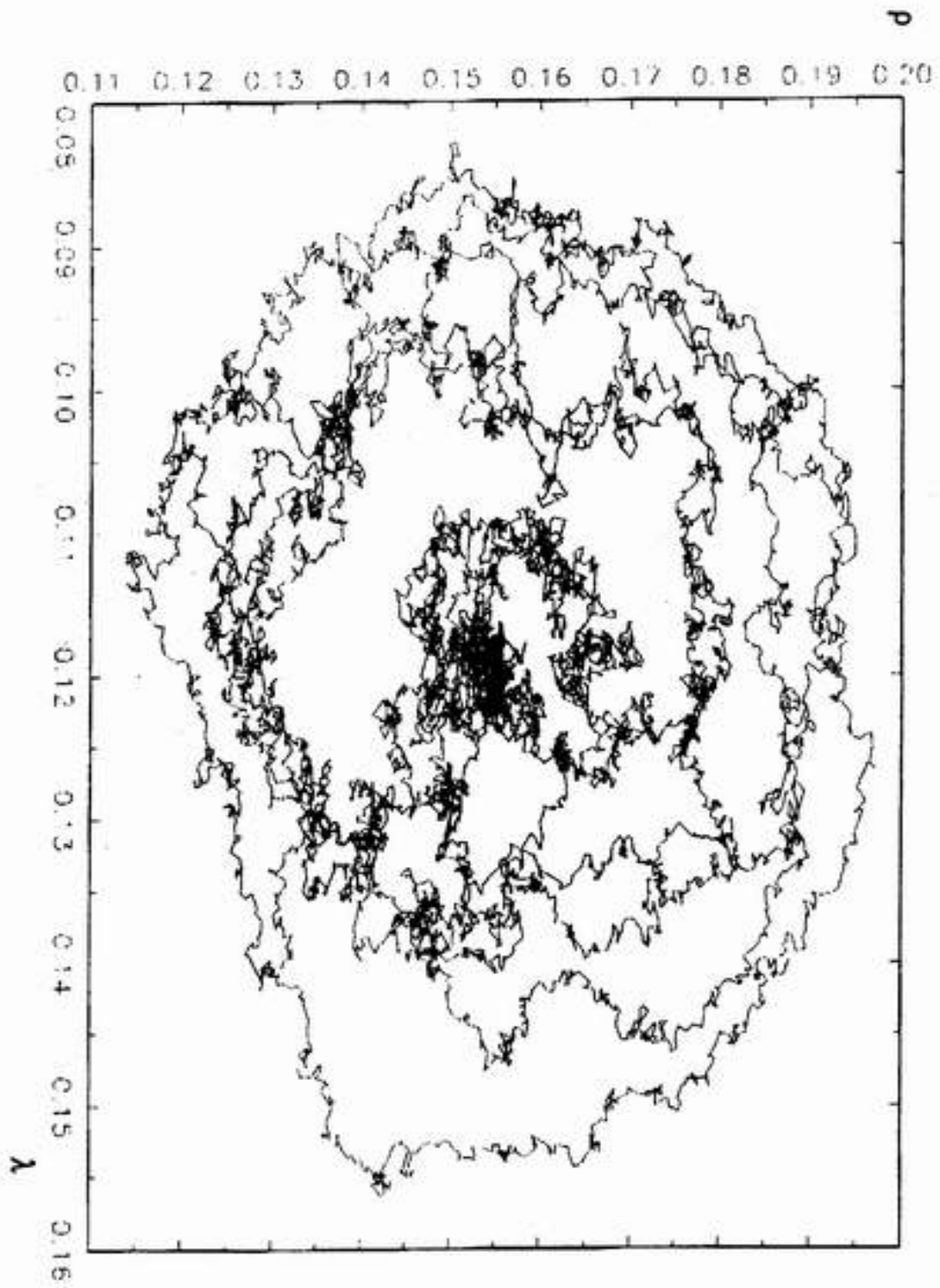


Figure 1:

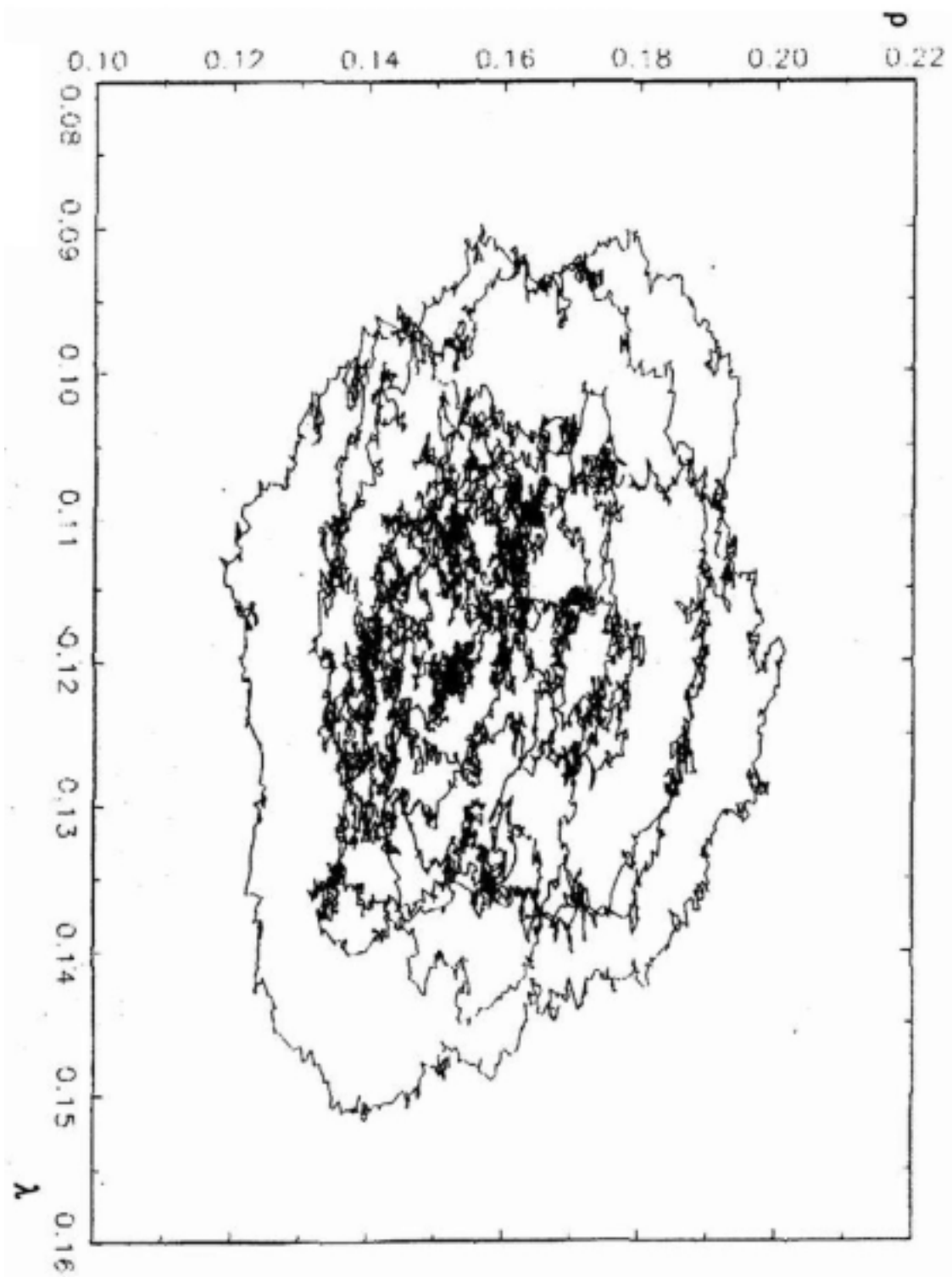


Figure 2:

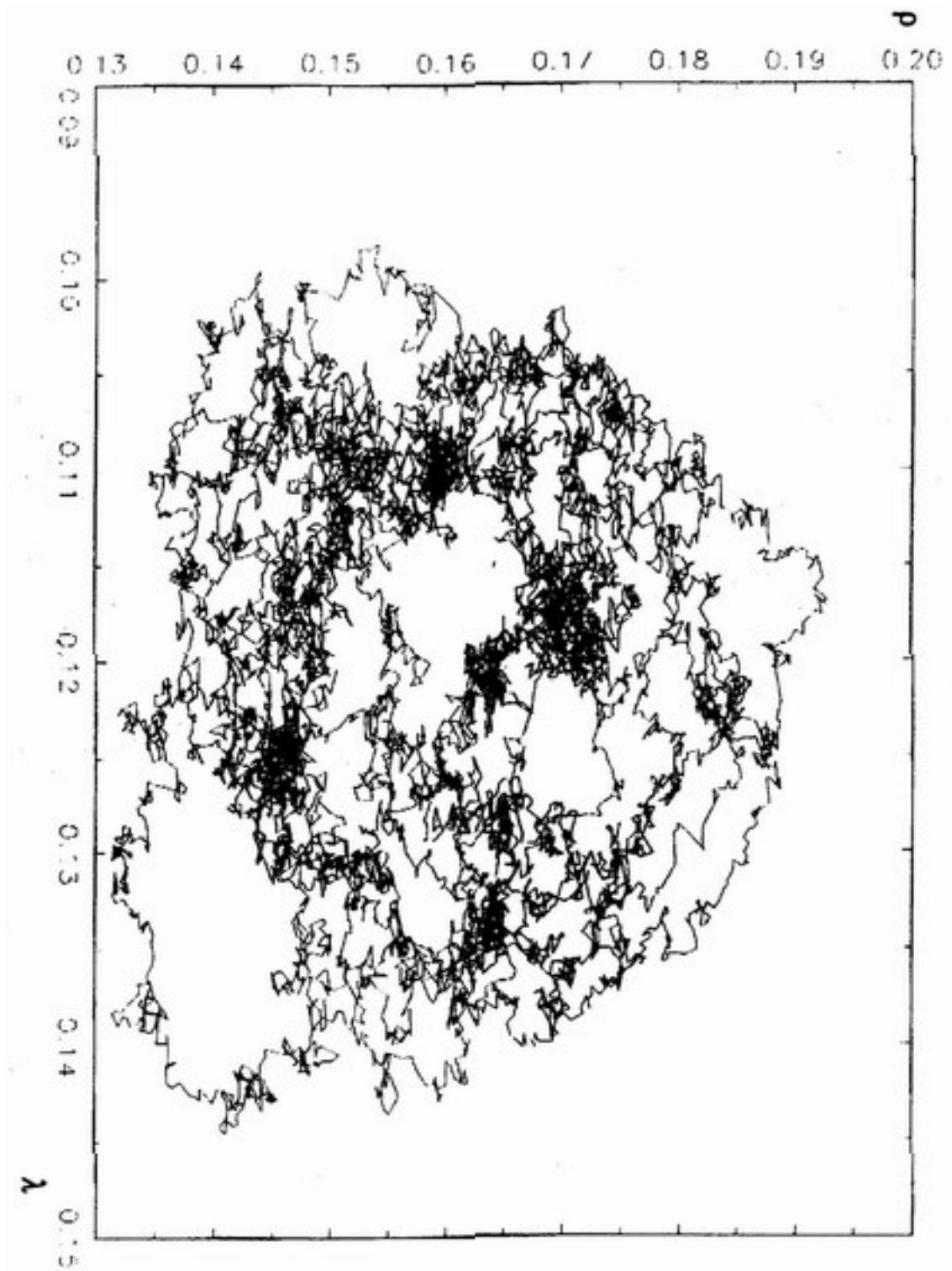


Figure 3:

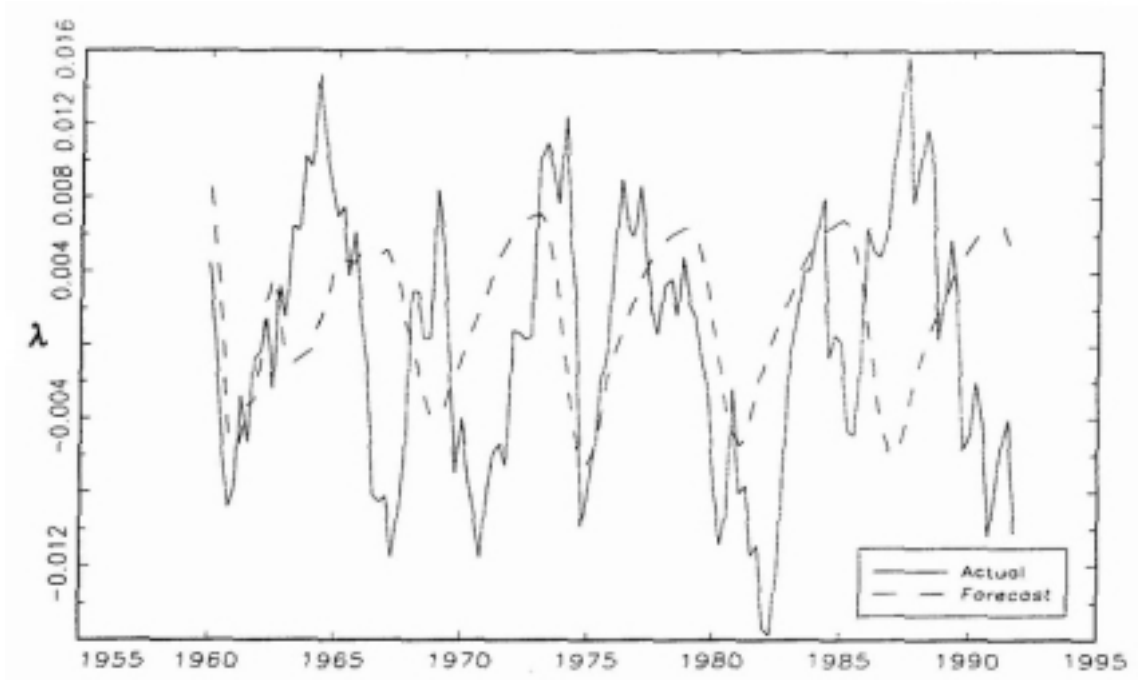


Figure 4:

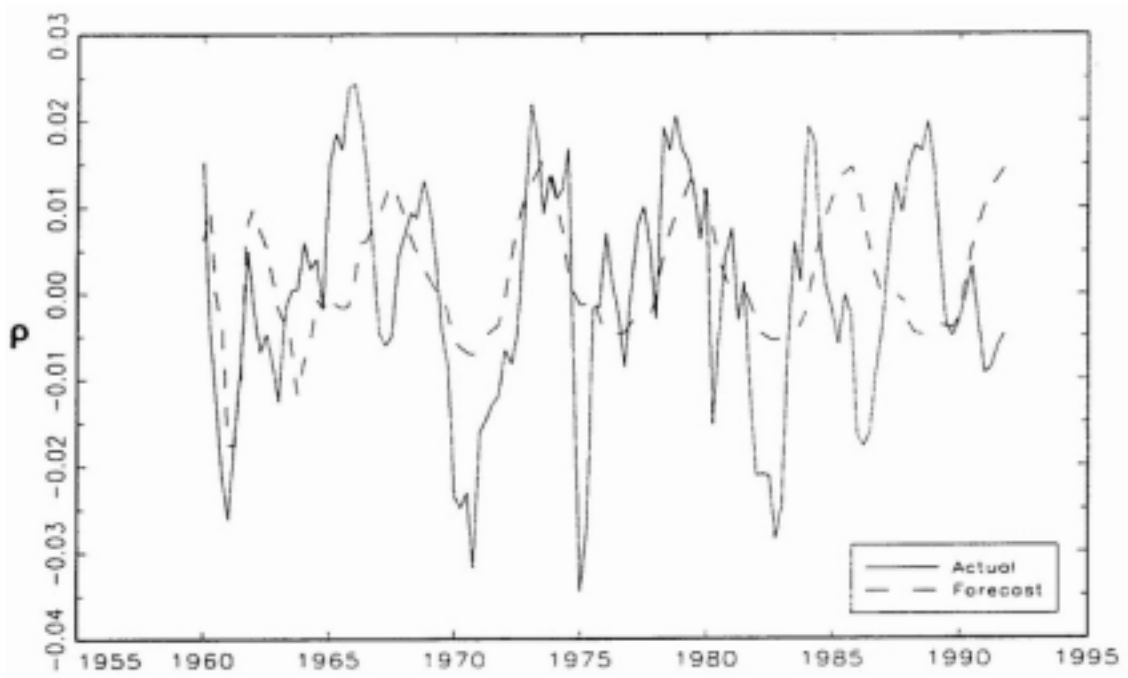


Figure 5:

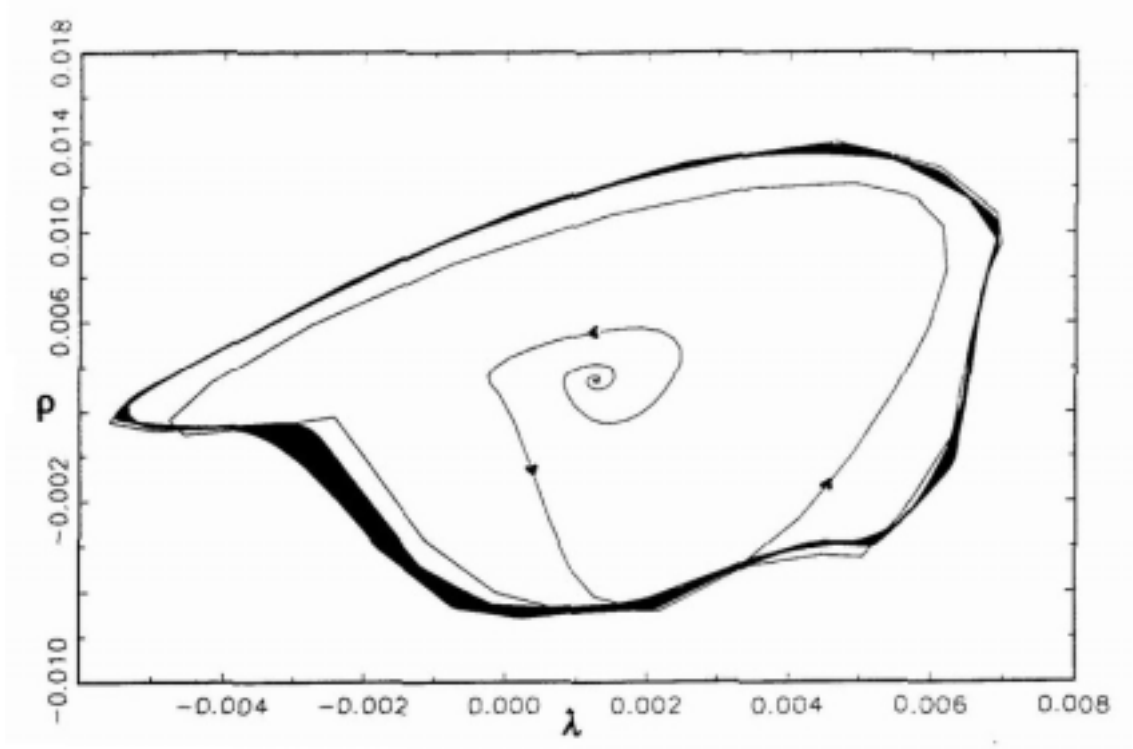


Figure 6:

## References

- [1] Bernanke, B. S. and M. Gertler (1989), "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79, no. 1, 14-32.
- [2] Bernanke, B. S. and A.S.Blinder (1988), "Credit, Money, and Aggregate Demand", *American Economic Review*, 78 (P.P.), 435–439.
- [3] Blanchard, O. and S. Fischer (1989), *Lectures in Macroeconomics*, Cambridge, MIT Press.
- [4] Blatt, J. M. (1978), "On the Econometric Approach to Business-Cycle Analysis," *Oxford Economic Papers*, 30, 292-300.
- [5] Blinder, A. S. (1989), *Macroeconomics Under Debate*, Harvester, New York.
- [6] Boyd, L and G. Blatt (1988), *Investment, Confidence and the Business Cycle*, Springer Verlag, Heidelberg and New York.
- [7] Caballero, R.J., E. M. Engel, and J. Haltiwanger (1995), "Aggregate Employment Dynamics: Building from Microeconomic Evidence," working paper no. 5642, NBER.
- [8] Campbell, J. Y. and N.G. Mankiw (1989), "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence", in O.J. Blanchard and S. Fischer (eds), *NBER Macroeconomic Annual 1989*, Cambridge:MIT-Press, 185-216.
- [9] Chang, R. and A. Velasco (1999), "Liquidity Crisis in Emerging Markets: Theory and Policies", NEBER Working paper no. 7272, Cambridge.
- [10] Chiarella, C., W. Semmler and L. Kockesen (1996). *The Specification and Estimation of a Nonlinear Model of Real and Stock Market Interaction*, New School for Social Research, mimeo.
- [11] Cogley, T. and J. M. Nason (1995), "Effects of Hodrick-Prescott Filter on Trend and Difference Stationary Time Series: Implications for Business Cycle Research," *Journal of Economic Dynamics and Control*, 19, 253-278.
- [12] Christiano, L. and M. Eichenbaum (1992), *Liquidity Effects and Monetary Transmission Mechanisms*, Federal Reserve Bank of Minneapolis, staff report, 150.
- [13] Day, R. H. and T. Y. Lin, (1991) "A Keynesian Business Cycle," in Nell, E. and W. Semmler, eds., *Nicholas Kaldor and Mainstream Economics*, MacMillan, London, New York.
- [14] Day, R. H. and W. Shafer, (1985), "Keynesian Chaos," *Journal of Macroeconomics*, 7, no. 3, 277-295.
- [15] Eckstein, O. , E. Green and A. Sinai (1974), "The Data Resources Model: Uses, Structure, and Analysis of the U.S. Economy", *International Economic Review*, vol. 15, no.3:595:615.

- [16] Eckstein., O. and A. Sinai (1986), "The Mechanisms of the Business Cycle in the Postwar Era", in R. Gordon (ed), *The American Business Cycle*, Chicago:university of Chicago Press.
- [17] Eitrheim, Ø. and T. Teräsvirta (1995), "Testing the Adequacy of Smooth Transition Autoregressive Models," unpublished paper.
- [18] Fair, F. C. (1992), *Fairmodel, Fairmodel Plus, Fairmodel Monthly Service*, Macro, Inc.
- [19] Flaschel, P., R. Franke, and W. Semmler (1997), *Nonlinear Macrodynamics: Instability, Fluctuations and Growth in Monetary Economies*, forthcoming, MIT Press, Cambridge, MA.
- [20] Foley, D. (1987), "Liquidity-Profit Rate Cycles in a Capitalist Economy," *Journal of Economic Behavior and Organization*, vol. 8, Sept., no. 3, 363-376.
- [21] Franke R. and W. Semmler (1995), "Bond Rate, Loan Rate and Tobin's q in a Temporary Equilibrium Model of Financial Sector," forthcoming *Metroeconomica* 1999.
- [22] Friedman, B. M. (1983), "The Roles of Money and Credit in Macroeconomic Activities," in J. Tobin, ed., *Macroeconomics, Prices and Quantities*, The Brookings Institution, Washington, DC.
- [23] Gale, D. and M. Hellwig (1985), "Incentive-Compatible Debt Contracts: The One-Period Problem," *Review of Economic Studies*, 52, 647-663.
- [24] Gertler, M., R. G. Hubbard and A. Kashyap (1991), "Interest Rate Spread, Credit Constraints and Investment," in R. G. Hubbard, ed., *Financial Markets and Financial Crises*, Chicago University Press, Chicago.
- [25] DeGooijer, J. G. and K. Kumar (1992), "Some Recent Developments in Nonlinear Time Series Modelling, Testing, and Forecasting," *International Journal of Forecasting*, 8, 135-156.
- [26] Granger, C. W. J., T. Teräsvirta and H. M. Anderson (1993), "Modeling nonlinearity over the business cycle," in Stoch, J. H. and M. W. Watson, Eds., *New Research on Business Cycles, Indicators and Forecasting*, Chicago University Press, Chicago.
- [27] Granger, C. W. J. and T. Teräsvirta (1993), *Modelling Nonlinear Economic Relationships*, Oxford University Press, Oxford
- [28] Greenwald, B. and J. Stiglitz (1993), "Financial Markets, Imperfections and Business Cycles," *Quarterly Journal of Economics*, 108, 77-114.
- [29] Guckenheimer J. and P. Holmes (1983), *Nonlinear Oscillations, Dynamical Systems, and Bifurcation of Vector Fields*, Springer-Verlag, Heidelberg, New York.
- [30] Kloeden, P.E., E. Platen and H. Schurz (1991), *Numerical Solutions of SNDE Through Computer Experiments*, Springer Verlag, Heidelberg/New York.

- [31] Leijonhufvud, A.(1973), “Effective Demand Failures,” *Sweedish Journal of Economics*, 75, 28-48.
- [32] Luukkonen, R., P. Saikkonen, and T. Teräsvirta, (1988a), “Testing Linearity Against Smooth Transition Autoregression Models,” *Biometrika*, 75, 3, 491-9.
- [33] Luukkonen, R., P. Saikkonen, and T. Teräsvirta, (1988b), “Testing Linearity In Univariate Time Series Models,” *Scandinavian Journal of Statistics*, 15, 161-175.
- [34] Minsky, H. (1975), *John Maynard Keynes*, Columbia University Press, New York.
- [35] Mishkin, F. (1978), “The Households Balance Sheets in the Great Depression,” *Journal of Economic History*,” no. 38, Dec., 918-937.
- [36] Mittnik, S. and Z. Niu, (1994), “Asymmetries in business cycles: Econometric techniques and empirical evidence,” in Semmler, W., ed., *Business Cycles: Theory and Empirical Methods*, Kluwer Academic Publishers, Boston.
- [37] Ozaki, T. (1985), “Non-linear Time Series Models and Dynamical Systems” in Hannan, E.J., P.R. Krishnaiah and M. M. Rao, Eds., *Handbook of Statistics*, Vol.5, North-Holland, Elsevier Science Publishers B.V.
- [38] Ozaki, T. (1986), “Local Gaussian Modeling of Stochastic Dynamical Systems in the Analysis of Non-Linear Random Vibrations,” in J. Gani and M. B. Priestly (eds.), *Essays in Time Series and Allied Processes: Papers in Honour of E.-J. Hannon*, Applied Probability Trust Sheffield: 241–255.
- [39] Ozaki, T. (1987), “Statistical Identification of Nonlinear Dynamics in Macroeconomics Using Nonlinear Time Series Models.”
- [40] Ozaki, T. (1989), “Statistical Identification of Nonlinear Random Vibration Systems,” *Journal of Applied Mechanics*, vol. 56, no. 1, March: 186–191.
- [41] Ozaki, T. (1994), “The Local Linearization Filter with Application to Nonlinear System Identification,” in H. Bozdogan (ed.), *Proceedings of the First US/Japan Conference on the Frontiers of Statistical Modeling: An Informational Approach*, Boston, Kluwer Academic Publishers: 217–240.
- [42] Pesaran, M. H. and S. M. Potter, eds., (1993), *Nonlinear Dynamics, Chaos and Econometrics*, John Wiley and Sons.
- [43] Potter, S. M. (1993), “A Nonlinear Approach to U.S. GNP,” mimeo, University of Wisconsin-Madison, Department of Economics.
- [44] Potter, S. M. (1994) “Asymmetric Economic Propagation Mechanisms,” in Semmler, W., ed., *Business Cycles: Theory and Empirical Methods*, Kluwer Academic Publishers, Boston.



- [45] Sayers, C. L. (1994), "Testing for Chaos and Nonlinearities in Macroeconomic Time Series," in Semmler, W., ed., *Business Cycles: Theory and Empirical Methods*, Kluwer Academic Publishers, Boston.
- [46] Semmler, W. and M. Sieveking (1993), "Nonlinear Liquidity-Growth Dynamics with Corridor-Stability," *Journal of Economic Behavior and Organization*, 22, 189-208.
- [47] Semmler, W. and M. Sieveking (2000), "Critical Debt and Debt Dynamics," *Journal of Economic Dynamics and Control*, vol. 24: 1121-44.
- [48] Semmler, W. and L. Koçkesen (1996), "Testing for Nonlinear Financial-Real Interaction," mimeo, New School for Social Research, New York.
- [49] Teräsvirta, T. and H. M. Anderson, (1992), "Characterizing Nonlinearities in Business Cycles Using Smooth Transition Autoregressive Models," *Journal of Applied Econometrics*, Vol.7, S119-S136.
- [50] Teräsvirta, T. (1993), "Modelling Nonlinearity in U.S. Gross National Product 1889-1987," Working Paper, Bank of Norway, Research Department.
- [51] Teräsvirta, T. (1994), "Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models," *Journal of the American Statistical Association*, March 1994, Vol. 89, No. 425, Theory and Methods.
- [52] Tong, H. (1990), *Non-linear Time Series, A Dynamical System Approach*, Oxford University Press, New York.
- [53] Torre, V. (1977), "Existence of Limit Cycles and Control in Complete Keynesian Systems by Theory of Bifurcations," *Econometrica*, 45, Sept., 1456-1466.
- [54] Townsend, R. (1976), "Optimal Contracts and Competitive Markets with Costly State Verification," *Journal of Economic Theory*, 21, 417-425.