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Calibration of the Keynes-Metzler-Goodwin Model to Stylized Facts of the German Business Cycle

by

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Abstract

The paper studies the quantitative dynamics of a deterministic disequilibrium macro model of the business cycle. The approach incorporates theoretical elements that, in particular, are connected with the names of Keynes, Metzler, and Goodwin. Based on regular exogenous oscillations of two central variables, 14 reaction coefficients are determined in such a way that the cyclical patterns of the endogenous variables are broadly compatible with stylized facts of German business cycles. This calibration procedure is organized in a hierarchical structure, so that subsets of the parameters can be established step by step. Subsequently, the exogenous variables are endogenized and the corresponding additional parameters in the investment function are chosen. The resulting dynamic system, which in its reduced form is of dimension six, generates persistent cyclical behaviour with similar time series properties of the variables as obtained before.

JEL classification: E 12, E 24, E 25, E 32.

Keywords: Disequilibrium theory, business cycles, calibration, stylized facts, German data.

Contents

1 Introduction

The paper takes up a deterministic macrodynamic modelling framework from the literature as it has been recently expounded by Chiarella and Flaschel (2000, Ch. 6), Chiarella et al. (2000, Chs 3 and 4), or Flaschel et al. (2002). Allowing for disequilibrium on the product and labour markets, which gives rise to sluggish price and quantity adjustments, the approach incorporates elements of economic theory that are, especially, connected with the names of Keynes, Metzler and Goodwin. Briefly, Keynesian elements are encountered in the treatment of aggregate demand (besides an LM-sector), Metzler has stimulated the modelling of production and inventory investment decisions, and Goodwinian ideas are reflected in the income distribution dynamics. Although each modelling block is quite simple, already the basic version of the model is of dimension six. Remarkably, it is still possible to carry out a mathematical analysis that provides meaningful conditions for local stability (see Köper, 2000). Given, however, that the main interest in the model lies in its potential for explaining persistent cyclical behaviour, an investigation of the system's global dynamics becomes necessary, so that one has to rely on computer simulations. This, in turn, raises the problem of assigning numerical values to the various coefficients in the model. After all, apart from the steady state ratios and similar magnitudes, and even without the investment function, we are confronted with 14 parameters that have to be determined.

One approach to numerical parameter setting is, of course, econometric estimation. On the basis of U.S. data and using single equation or subsystem estimations, it was employed in Flaschel et al. (2002; for a slightly different version of the model, to be precise). This study, however, cannot yet deemed to have settled the issue since the presentation is not always transparent and, more seriously, not all coefficients appear credible.[1] Supplementarily to this kind of work, we therefore choose an alternative approach. That is, referring to a business cycle context we seek to calibrate the model. Besides, we are now dealing with German cycles.

A few words may be in order to clarify the concept of calibration as we understand it here. The aim of calibrating a model economy is to conduct (computer) experiments in which its properties are derived and compared to those of an actual economy. In this respect calibration procedures can be regarded as a more elaborate version of the standard back-of-the-envelope calculations that theorists perform to judge the validity of a model. It is nevertheless important to point out the underlying notion that every model is known to be false. Accordingly, a model is not a null hypothesis to be tested, it is on the contrary an improper or simplified approximation of the true data generating process of the actual data; and a calibrator is not interested in verifying

[1]For example, the stock adjustment speed is implausibly low, or a discussion of the cyclical implications for the real wage dynamics is missing.

whether the model is true (the answer is already known from the outset), but in identifying which aspects of the data a false model can replicate.[2]

Our investigation of how well the model-generated trajectories match the data follows the usual practice. We select a set of stylized facts of the business cycle, simulate the model on the computer, and assess the corresponding cyclical properties of the resulting time series. Since a (false) model is chosen on the basis of the questions it allows to ask, and not on its being realistic or being able to best mimic the data, we share the point of view that rough reproduction of simple statistics for comovements and variability is all that is needed to evaluate the implications of a model. As opposed to econometrics, the discussion is here more or less informal. This may not be considered a lack of precision, it is rather a way to discuss where parameters are easy or difficult to find, and to elucidate the kind of compromises that we have to make in the multi-purpose endeavour with which we are concerned.[3] In sum, our philosophy of setting the numerical parameters is similar to that of the real business cycle school, though the methods will be different in detail.

Fortunately, it turns out that the model gives rise to a hierarchical structure in the calibration process. Some variables which are exogenous in one model building block are endogenous within another module at a higher level. Thus, the parameters need not all be chosen simultaneously, but fall into several subsets that can be successively determined. This handy feature makes the search for suitable parameters and the kind of compromises that we finally accept more intelligible.

The evaluation of the numerical parameters takes place at two main stages, where most of the work is done at the first stage. Here we suppose exogenous fluctuations of two exogenous variables that drive the rest of the model, namely, capacity utilization and, synchronously with it, the capital growth rate. Since random shocks are neglected in our framework, these exogenous motions may well be of a regular and strictly periodic nature, most conveniently specified as sine waves. It suffices to view this perhaps somewhat unusual approach as a heuristic device. That it is indeed a useful and appropriate device has to be corroborated by the results at the end of the paper.

The decisive test for the numerical parameters that are found so far is carried out at stage two, where we endogenize capacity utilization and propose an investment function. Setting the parameters thus newly introduced, the model becomes fully endogenous and we can study the

[2]See also the introductory discussion in Canova and Ortega (2000, pp. 400–403).

[3]It is also interesting to refer to Summers (1991, p. 145), who has expressed his skepticism about decisive formal econometric tests of hypotheses: "the empirical facts of which we are most confident and which provide the most secure basis for theory are those that require the least sophisticated statistical analysis to perceive."

properties of the time series it generates. The calibration will have passed this test if the model produces persistent cyclical behaviour with similar features as found before.

The remainder of the paper is organized as follows. Section 2 sets forth the stylized facts of German business cycles that will be used as guidelines, and gives some further details regarding the numerical simulations to follow. Section 3 presents the model at calibration levels $1-3$, whose major topic is the determination of the wage-price dynamics. Section 4 first deals with the interest rate, and then with aggregate demand and the quantity adjustments on the goods market, investigations that have their place at levels $4-6$. The complete endogenous model with the investment function to be added is examined in Section 5. Taking the final steps of finding suitable reaction coefficients in the investment function, it, in particular, turns out that we also better reset one or two of the parameter values that have been established before. At the end, we decide on a definite parameter scenario and give a detailed collection of the resulting cyclical features of the economy. Section 6 concludes.

2 Numerical preliminaries

2.1 Desirable cyclical features

At the beginning of a calibration procedure the stylized facts of the business cycle have to be established that the numerical model should attempt to match. To this end we put forward a list of desirable cyclical features. The collection covers the variability and the comovements of a number of macroeconomic key variables and is based on the investigation of German data (from DIW) in Franke (2002) over the sample period 1963 – 1982 (which unfortunately had to be rather short). Variability is represented by the standard deviation of the time series over this time span. For better comparison it is related to the standard deviation of the variable defining the business cycle. In our model this is capacity utilization u or, what amounts to the same, the deviations of the output-capital ratio $y = Y/K$ from a (in reality perhaps variable) 'trend' line y^o; i.e., $u = (y - y^o)/y^o$ (in the model itself, y^o will be a constant value). When referring to Franke (2002) it has, however, to be taken into account that there the business cycle is measured by the output gap, the deviations of gross value added from trend.

Since it would not be without problems to construct a quarterly series of the output-capital ratio from the German data available to us[4] and since, on the other hand, the output-capital ratio and the output gap are well known to be strongly correlated, we simply prefer to rescale the latter

[4]There are only annual data of a capital stock. The series also encompasses the housing sector, while we think of fixed capital as plant and equipment in the non-financial corporate business sector.

and take it as a proxy for the empirical output-capital ratio. Hints as for the scaling factor can be obtained from US data.[5] Computing the standard deviations of the GDP gap and the (detrended) output-capital ratio Y/K over the four major trough-to-trough cycles from 1961 to 1991, where Y and K are limited to the non-financial corporate business sector, the latter standard deviation is found to be 1.158 times higher than the former. Accordingly, while Franke (2002) reports the standard deviation of a variable x relative to the standard deviation of gross value added (GVA), we here refer to the ratios $\sigma_x/(1.158 \cdot \sigma_{GVA})$.[6]

The resulting guidelines for calibration are given in Table 1. On the whole, it comprises seven macro variables, namely, labour productivity $z = Y/L$; the employment rate $e = L/L^s$ (L being hours of employment and L^s hours of labour supply, proxied by the trend of L); the price level p; the real wage rate ω; the wage share $v = wL/pY$; excess demand (i.e., aggregate demand Y^d, or sales, minus output) relative to total output $\xi = (Y^d - Y)/Y$; and the consumption ratio C/Y. Here and in the following, x^o denotes the trend or, respectively, the steady state value of variable x.

2.2 The stylized motions of the exogenous variables

It has already been indicated in the introduction that most of the calibration is based on stylized sine wave motions of capacity utilization u and the capital growth rate g_k, which in that stage of the investigation are treated as exogenous. As the German economy runs through three major trough-to-trough cycles from 1963 to 1982, the period of these oscillations should be 6.67 years. Regarding utilization we assume an amplitude of $\pm 3\%$, so that

$$u(t) \quad = \quad 1 \; + \; 0.03 \cdot \sin(2\pi t/6.67) \tag{1}$$

The amplitude gives rise to a standard deviation of u over the cycle of 2.14%, which is slightly higher than the number $\sigma_u = 2.05\%$ reported in fn 6.

Although in the theoretical model inventories buffer the fluctuations of aggregate demand and so allow for a certain degree of production smoothing, it turns out that the model can still

[5] The US data are taken from the database that is made available by Ray Fair on his homepage (`http://fairmodel.econ.yale.edu`), with a description being given in Appendix A of his US Model Workbook. Detrending was always done by employing the Hodrick-Prescott filter with $\lambda = 1600$ as the standard value for the smoothing parameter.

[6] The standard deviations for the US are $\sigma_{GDP} = 1.77\%$ and $\sigma_u = 2.05\%$. The first statistic is in the same range as in Germany, where for gross value added we have $\sigma_{GVA} = 1.75\%$. Also the standard deviations of the American and German fixed capital growth rates and the relative timing of their turning points appear to be comparable (see below). These observations may be sufficient support for our short-cut as a reasonable approximation to the precise German ratios σ_x/σ_u.

variable x	σ_x/σ_u	Lag x
z/z^o	0.49	-0.50 – 0.00
$e - e^o$	0.65	0.00 – 0.75
$-(p/p^o)$	0.50	-0.75 – 0.00
ω/ω^o	0.59	0.25 – 1.25
$v - v^o$	0.31	$\approx 1/4$ cycle
$-(\xi - \xi^o)$	0.46	-0.75 – 0.00
$-[C/Y - (C/Y)^o]$	0.35	-0.25 – 0.50

Table 1: Desirable features of macrodynamic oscillations on the basis of German data.

Note: Reference series u is capacity utilization, superscript 'o' denotes steady state or trend values. The lags are measured in years.

be viewed as being basically driven by investment demand, and that utilization and the capital growth rate move very much in line. For this reason we posit here that g_k is perfectly synchronized with u. The amplitude of g_k derives from two kinds of evidence. First, in the US the (detrended) growth rate of fixed capital in non-financial corporate business exhibits a standard deviation of 0.59%, which is a fraction 0.29 of the abovementioned value $\sigma_u = 2.05\%$. On the other hand, using simple interpolation we constructed a quarterly series from German annual data on the net capital stock in the industry and the trade and transport sectors and computed the standard deviation σ_g of the corresponding growth rates.[7] The result was $\sigma_g = 0.72\%$, that is, $\sigma_g = 0.36 \cdot (1.158 \cdot \sigma_{GVA})$. Because in other parts of the economy the variations of the capital stock are probably weaker, we decide to settle on

$$g_k(t) \quad = \quad g^o \;+\; 0.30 \cdot [u(t) - 1] \tag{2}$$

where g^o is the equilibrium growth rate around which g_k oscillates. Incidentally, the amplitude associated with (2) is $\pm\, 0.90$ percentage points.

[7] Because of the strong downward trend from the 60s to the 80s, detrending is absolutely essential here. The timing of the turning points of the US capital growth rate and our auxiliary series for Germany is similar, both variables being clearly procyclical with perhaps a short lag.

2.3 Steady state magnitudes

In this subsection the numerical steady state relationships and some constant parameters related to them are put forward. To begin with the long-run equilibrium values of the endogenous variables of the model, these are the output-capital ratio y; the wage share v; the corresponding markup rate μ on unit labour cost; the capital-labour ratio in efficiency units $k^s = K/z^oL^s$; the inflation climate π, which in this state is equal to the actual rate of inflation; the nominal bond rate of interest i; the real balance ratio $m = M/pK$; and the inventory-capital ratio $n = N/K$. Growth rates to be considered are those of the money supply g_m, trend labour productivity g_z, labour supply g_ℓ, and total output g^o (the first three of them will be supposed to be fixed). In addition, certain constant coefficients will enter the demand relationships, namely, the rate of capital depreciation δ; the ratio of real government expenditures to the capital stock $\gamma = G/K$; the ratio of desired inventories N^d to expected demand Y^e, $\beta_{ny} = N^d/Y^e$; and the tax parameter θ_c, according to which real taxes T^c on rental income are supposed to be in a fixed proportion to the capital stock after real interest receipts are deducted, $T^c - iB/p = \theta_c K$. Measuring the flows at annual rates, our numerical choice of all these magnitudes is as follows (some numbers are rounded; see below):

$$\begin{array}{rclrclrclrcl} g^o &=& 3\% & g_z &=& 2\% & g_\ell &=& 1\% & g_m &=& 6.0\% \\ y^o &=& 0.70 & v^o &=& 70\% & \mu^o &=& 0.429 & \delta &=& 8.0\% \\ (k^s)^o &=& 1.429 & m^o &=& 0.140 & \pi^o &=& 3\% & i^o &=& 7\% \\ \gamma &=& 0.133 & \theta_c &=& 0.026 & \beta_{ny} &=& 0.232 & n^o &=& 0.161 \end{array} \qquad (3)$$

The four growth rates (which of course are linked by $g^o = g_z + g_\ell$ and $g_m = g^o + \pi^o$), the wage share, the inflation rate and the bond rate are directly set freehand. Taking v^o for granted, the equilibrium markup rate μ^o derives from the definitional equation $p = (1+\mu^o)wL/Y$, or $(1+\mu^o)v^o = 1$.

Arguments for the choice of y^o and β_{ny} are given in Franke (2002, Section 4). Anticipating eq. (58) in Section 4.3 below, the equilibrium inventory-capital ratio is then residually determined as $n^o = y^o\beta_{ny}/(1 + \beta_{ny}g^o)$.

The value of the depreciation rate rests on two statistics. As a time average over 1960 – 89, the series of depreciation and total capital from DIW yield a ratio of roughly 5% per year. The capital series, however, includes the housing sector, whereas in our model fixed capital is plant and equipment of firms. Taking the housing sector out of the capital series raises the ratio to approximately 8.5%. Of course, housing restoration and repairs have also to be deleted from total depreciation, but this is certainly a much smaller fraction than the share of housing in total capital. So the number was lowered to 8%.[8] Note that the implied equilibrium (gross) rate of return on real capital is $(1 - v^o)\, y^o - \delta = 13\%$, which does not appear too implausible.

The equilibrium value of the capital-labour ratio is obtained from the decomposition of the employment rate as $e = L/L^s = z^o(L/Y)(Y/K)(K/z^o L^s) = (z/z^o) y k^s$. With $e = e^o = 1$ and $Y/L = z = z^o$ in the steady state, this gives $(k^s)^o = 1/y^o = 1.42857$.

Regarding the real balances ratio $m = M/pK$ we assumed $M/pY = 0.20$, so that $m = (M/pY)(pY/pK) = 0.20 \cdot y^o$. In a similar manner, the government spending coefficient γ can be written as as $\gamma = G/K = (G/Y)(Y/K)$. Here $G/Y = 0.19$ is employed as a reasonable ratio of government consumption in Germany.

To get an idea of the order of magnitude of the tax parameter θ_c, view capital income taxes net of interest receipts, $pT^c - iB = \theta_c\, pK$, as a fraction τ_c of the profits of firms, which in the model are entirely distributed among their shareholders. Dividing the equation $\tau_c\,(pY - wL - \delta pK) = \theta_c\, pK$ by pK allows us to express the parameter as $\theta_c = \tau_c\,[(1 - v^o) y^o - \delta]$. The above value $\theta_c = 0.026$ comes about if $\tau_c = 20\%$ is assumed.

3 Wage-price dynamics

3.1 Productivity and employment rate

Wage and price adjustments will below be represented by two Phillips curves. Besides the standard arguments, which are the employment rate e for the wage Phillips curve and capacity utilization u for the price Phillips curve, both curves will also include the wage share v as an additional factor. As is shortly made explicit, e as well as v are connected with capacity utilization through average labour productivity $z = Y/L$. The evolution of z has therefore to be treated first.

While we wish to account for the procyclicality of productivity in Table 1, for a small macrodynamic model to be analytically tractable this should be done in a simplified way. To this end we neglect the possible lead of z in the comovements with u and postulate a direct positive effect of u on the percentage deviations of z from its trend value z^o.[9] Like the functional specifications to follow, we assume linearity in this relationship,

$$z/z^o \;=\; f_z(u) \;:=\; 1 + \beta_{zu}\,(u - 1) \tag{4}$$

β_{zu} and all other β-coefficients later on are positive constants.

[8] If this reduction appears too little, it might also be taken into account that the ratio of depreciation to total capital has steadily increased over the sample period.

[9] Leaving aside (suitably scaled and autocorrelated) random shocks to the technology, an immediate explanation of the comovements of z and u may be overhead labour and labour hoarding.

Trend productivity is assumed to grow at an exogenous constant rate g_z. To deal with dynamic relationships, it is convenient to work in continuous time (where for a dynamic variable $x = x(t)$, $\dot{x}$ is its time derivative and $\hat{x}$ its growth rate; $\dot{x} = dx/dt$, $\hat{x} = \dot{x}/x$). Thus,

$$\hat{z}^o \;=\; g_z \tag{5}$$

In the theoretical model, trend productivity also serves to deflate real wages, or to express them in efficiency units. We correspondingly define

$$\omega \;=\; w\,/\,p\,z^o \tag{6}$$

For short, ω itself may be referred to as the real wage rate. Obviously, if w/p grows steadily at the rate of technical progress, ω remains fixed over time. Since $v = wL/pY = (w/pz^o)\,(z^oL/Y) = (w/pz^o)\,(z^o/z)$, the wage share and the real wage rate are linked together by

$$v \;=\; \omega\,/\,f_z(u) \tag{7}$$

To express the employment rate by variables which in a full model would constitute some of the dynamic state variables, we make use of a similar decomposition as in Section 2.3. Before, the relationship between capacity utilization and the output-capital ratio has to be spelled out. Here we employ the notion of a fixed output-capital ratio y^n that would prevail under 'normal' conditions. Of course, y^n will also be the long-run equilibrium value. With respect to a given stock of fixed capital K, productive capacity is correspondingly defined as $Y^n = y^n K$, and current output Y utilizes this capacity at rate $u = Y/Y^n = y/y^n$. With these variables we get $e = L/L^s = z^o\,(L/Y)\,(Y/Y^n)\,(Y^n/K)\,(K/z^oL^s)$. Hence, with capital per head in efficiency units $k^s = K/z^oL^s$, the employment rate can be written as

$$e \;=\; y^n\,u\,k^s\,/\,f_z(u) \tag{8}$$

Apart from the growth rates of trend productivity, g_z, and labour supply, g_ℓ, where the latter is likewise supposed to be constant,

$$\hat{L}^s \;=\; g_\ell \tag{9}$$

the motions of the capital-labour ratio are basically governed by the fluctuations of the capital growth rate g_k. Upon logarithmic differentiation of k^s, the differential equation reads

$$\dot{k}^s \;=\; k^s\,(g_k - g_z - g_\ell) \tag{10}$$

Once the time paths $u = u(t)$ and $g_k = g_k(t)$ are given, the time path of the employment rate is determined as well, *via* (10) and (8) — independently of the rest of the economy. The only parameter here involved is β_{zu} from the hypothesis on labour productivity in eq. (4). This constitutes the first level in the hierarchy of calibration steps. In sum:

<u>*Level 1:*</u> employment rate e (parameter β_{zu})

$$\dot{k}^s = k^s\,(g_k - g_z - g_\ell) \tag{10}$$

$$e = y^n\,u\,k^s\,/\,[1+\beta_{zu}\,(u-1)] \tag{8}$$

By virtue of $g^o - g_z - g_\ell = 0$ from (3), the variations of k^s resulting from the sine waves (1), (2) and from eq. (10) are stationary, so that a suitable choice of the initial value of k^s at $t=0$ can make the oscillations symmetrical around $(k^s)^o$. The corresponding amplitude of k^s is $\pm\,0.013585$. This motion will be underlying all of the investigations with the exogenous oscillations of u and g_k.

Table 1 suggests setting β_{zu} at 0.49. This value, however, gives rise to a variability of the employment rate that is lower than desired, the relative standard deviation being $\sigma_e/\sigma_u = 0.58$. The lag behind u, in contrast, is fully satisfactory: lag e = 0.67 years or 8 months.[10] σ_e can be increased by reducing the coefficient β_{zu}. Thus, $\sigma_e/\sigma_u = 0.62$ and $\sigma_e/\sigma_u = 0.67$ is obtained for β_{zu} at 0.45 and 0.40, respectively. The lag is in both cases 0.58 years. Since the model's β_{zu} should not be too different from the ratio in Table 1, we choose

$$\beta_{zu} = 0.45 \tag{11}$$

Incidentally, owing to the nonlinearity in (4) the induced oscillations of the employment rate are slighly asymmetrical around $e^o = 1$. The precise value of the time average of e over a full cycle is 99.99%, while the amplitude is ±1.875%.

3.2 The two Phillips curves and the inflation climate

The model employs a wage as well as a price Phillips curve. Both formulations will, however, be more flexible than the standard versions. Beginning with the adjustments of nominal wages, the main point is that the usual positive feedback from the employment rate is augmented by a negative feedback from the wage share, an effect that will turn out to be essential in the calibration of the real wage dynamics. The theoretical content of this extension is discussed in Franke (2001).[11]

[10] As the full model will be simulated as a monthly economy, we may already here slice time into monthly intervals.

[11] It may directly be argued that at relatively low levels of the wage share, workers seek to catch up to what is considered a normal, or 'fair', level, and that this is to some degree taken up in a wage bargaining process. More rigorously, the negative wage share effect can also be derived from the wage setting model by Blanchard and Katz (1999, p. 6), which makes reference to the workers' reservation wage and interpret it as depending on labour productivity and lagged wages.

Apart from that, the changes in the wage rate w are measured against the changes in prices and trend labour productivity, which brings us to a second point. Regarding benchmark inflation here invoked, workers may be conceded to have full knowledge of the short-term evolution of prices. This makes clear that myopic perfect foresight is no problem at all for Keynesian macroeconomics. Nevertheless, besides current inflation $\hat{p}$, the wage bargaining process may refer to expectations about inflation that are related to the medium term. Our notion is that the latter represents a general inflation climate, designated π. Combining these elements we have

$$\hat{w} \;=\; \hat{z}^o + \kappa_{wp}\hat{p} + (1-\kappa_{wp})\pi \;+\; f_w(e,v) \tag{12}$$
$$f_w \;=\; f_w(e,v) := \;\beta_{we}\,(e-1) \;-\; \beta_{wv}\,(v-v^o)/v^o \tag{13}$$

where the abbreviation f_w will simplify the presentation below, κ_{wp} is a weighting parameter between 0 and 1, unity is the normal rate of employment, and v^o serves as a reference value for the wage share (to simplify, we here employ the steady state value directly).

Turning to the price adjustments, the following version of a price Phillips curve is put forward,

$$\hat{p} \;=\; \kappa_{pw}(\hat{w}-\hat{z}^o) + (1-\kappa_{pw})\pi \;+\; f_p(u,v) \tag{14}$$
$$f_p \;=\; f_p(u,v) := \;\beta_{pu}\,(u-1) \;+\; \beta_{pv}\,[(1+\mu^o)v-1] \tag{15}$$

The parameter κ_{pw} $(0 \le \kappa_{pw} \le 1)$ weighs the influence of current wage inflation (corrected for technical progress) and the inflation climate, which provides a benchmark for the price adjustments. As utilization u reflects the pressure of demand, the term $\beta_{pu}(u-1)$ signifies a demand-pull term. The last component, $\beta_{pv}\,[(1+\mu^o)v-1]$, can be viewed as a cost-push term proper, which goes beyond taking the present inflationary situation into account. Devising μ^o as a target markup rate, we mean by this that prices tend to rise by more than what is captured by the other terms if labour costs are so high that, at current prices, $p < (1+\mu^o)wL/Y$, which is equivalent to $0 < (1+\mu^o)wL/pY - 1 = (1+\mu^o)v - 1$. For reasons of consistency it is assumed that the target markup is compatible with the normal level v^o of the wage share in (13), i.e., $(1+\mu^o)v^o = 1$.[12]

Since in (12) and (14) current wage and price inflation $\hat{w}$ and $\hat{p}$ are mutually dependent on each other, in the next step the two equations have to be solved for $\hat{w}$ and $\hat{p}$. This presupposes that the weights κ_{pw} and κ_{wp} are not both unity. Obviously, in the resulting reduced-form expressions for $\hat{w}$ and $\hat{p}$, wage inflation also depends on the core terms in the price Phillips curve, and price inflation on the core terms in the wage Phillips curve. In detail,

$$\hat{w} \;=\; \hat{z}^o \;+\; \pi \;+\; \kappa\,[\kappa_{wp}f_p(u,v) + f_w(e,v)] \tag{16}$$

[12] Empirical support for a positive impact of v on $\hat{p}$ can be inferred from Brayton et al. (1999, pp. 22–27). The connection is more clearly explained in Franke (2001).

$$\hat{p} \;=\; \pi \,+\, \kappa\,[f_p(u,v) + \kappa_{pw} f_w(e,v)] \qquad (17)$$

$$\kappa \;=\; 1\,/\,(1 - \kappa_{pw}\kappa_{wp}) \qquad (18)$$

It is then seen that in the growth rate of the real wage, $\hat{\omega} = \hat{w} - \hat{p} - \hat{z}^o$, not only trend productivity growth but also the inflation climate π cancels out. This independence of the income distribution dynamics from inflationary expectations may be considered a particularly attractive feature of the approach with two Phillips curves. On the other hand, as emphasized by the notation of the functional expressions f_p and f_w, six new parameters are entering at this level:

<u>*Level 2:*</u> real wage ω, wage share v (κ_{pw}, κ_{wp}, β_{pu}, β_{pv}, β_{we}, β_{wv})

$$\dot{\omega} \;=\; \omega\,\kappa\,[\,(1-\kappa_{pw}) f_w(e,v;\beta_{we},\beta_{wv}) \;-\; (1-\kappa_{wp}) f_p(u,v;\beta_{pu},\beta_{pv})\,] \qquad (19)$$

$$v \;=\; \omega\,/\,f_z(u) \qquad (7)$$

$$\kappa \;=\; 1\,/\,(1 - \kappa_{pw}\kappa_{wp}) \qquad (18)$$

The inflation climate does have a bearing on the rate of inflation. The law governing the variations of π is specified as a mix of two simple mechanisms. One of them, adaptive expectations, often proves destabilizing if the speed of adjustment is high enough. The other rule, regressive expectations, constitutes a negative feedback. Introducing the weight $\kappa_{\pi p}$ and adopting π^o as a 'normal' value of inflation (or directly the steady state value in a full model), we posit with β_π as the general adjustment speed,

$$\dot{\pi} \;=\; \beta_\pi\,[\kappa_{\pi p}(\hat{p} - \pi) \,+\, (1-\kappa_{\pi p})(\pi^o - \pi)] \qquad (20)$$

Though after the intellectual triumph of the rational expectations hypothesis, working with adaptive expectations has become something of a heresy, in a disequilibrium context there are a number of theoretical and empirical arguments which demonstrate that adaptive expectations make more sense than is usually attributed to them (see Flaschel et al., 1997, pp. 149–162; or more extensively, Franke, 1999). This is all the more true if π is not inflation expected for the next quarter, but if it represents a general climate that is employed as a benchmark value in wage and price decisions, complementarily to current inflation. Contenting ourselves with univariate mechanisms, gradual adjustments in the direction of $\hat{p}$ are thus a meaningful device. The regressive mechanism in (20), by contrast, expresses a 'fundamentalist' view, in the sense that the public perceives a certain tendency of inflation to return to normal after some time.[13]

[13] The general idea that an inflation expectations mechanism, which includes past observed rates of inflation only (rather than observed increases in the money supply), may contain an adaptive and a regressive element is not new and can, for example, already been found in Mussa (1975). The specific functional form of eq. (20) is borrowed from Groth (1988, p. 254).

Taken on their own, both principles ($\kappa_{\pi p} = 1$ or $\kappa_{\pi p} = 0$) are of course rather mechanical. They are, however, easy to integrate into an existing macrodynamic framework and, in their combination of stabilizing and destabilizing forces, already allow for some flexibility in modelling the continuous revision of benchmark rates of inflation.

The time paths of $\pi(\cdot)$ from (20) will evidently lag behind actual inflation $\hat{p}(\cdot)$. This, as such, is no reason for concern; it is even consistent with inflationary expectations themselves, as they are made in the real world. Here forecast errors are found to be very persistent, and forecasts of inflation often appear to be biased (see, e.g., Evans and Wachtel, 1993, fig. 1 on p. 477, and pp. 481ff).

The time paths of $e(\cdot)$ and $v(\cdot)$ being computed at level 1 and 2, eq. (17) with $f_p = f_p(u, v)$ and $f_w = f_w(e, v)$ for $\hat{p}$ can be plugged in the dynamic equation (20) for the adjustments of π. The solution of $\pi(\cdot)$ here computed can also be used in (17) to record the time path of the inflation rate itself. Apart from the two parameters β_π, $\kappa_{\pi p}$, all parameters have already been set before. These operations may be reviewed in one step at calibration level 3:

Level 3: price inflation $\hat{p}$, inflation climate π (parameters β_π, $\kappa_{\pi p}$)

$$\dot{\pi} = \beta_\pi \left[\kappa_{\pi p}(\hat{p} - \pi) + (1-\kappa_{\pi p})(\pi^o - \pi)\right] \tag{20}$$

$$\hat{p} = \pi + \kappa \left[f_p(u, v) + \kappa_{pw} f_w(e, v)\right] \tag{17}$$

In the introduction, the hierarchical structure of the calibration procedure has been stressed, which allows us to concentrate on a few paramters at each calibration level. The six Phillips curve parameters at Level 2 are, however, already quite an unwieldy number to treat; at least the transparency of our reasons for settling on a particular combination would suffer. We therefore decompose the task of the parameter search into two smaller subproblems. First, ideal counter-cyclical motions of the price level are assumed, and we focus on the wage Phillips curve and its implications for the distributional dynamics. Subsequently these exogenous motions of the price level, or the inflation rate, for that matter, are dropped. The parameters β_{we}, β_{wu}, κ_{wp} found at the first step are taken over and we look for parameters β_{pu}, β_{pv}, κ_{pw} in the price Phillips curve that largely leave the cyclical features of the motions of v and ω intact, as they have been previously established. In addition, the now endogenously generated motions of the price level through eq. (17) should match the desirable features of Table 1.

In order to limit the discussion, these investigations are based on an *a priori* decision about the coefficients β_π and $\kappa_{\pi p}$ for the gradual changes in the inflation climate π in eq. (20). Given the benchmark character of π, which is *not* just expected inflation for the next quarter, the adjustments toward current inflation should not be too fast. Likewise, agents will not expect inflation to return to normal too quickly. We therefore choose a moderate size of the adjustment speed β_π. As for the role of adaptive and regressive expectations, simply equal weights are assumed. Hence, we posit

$$\beta_\pi \;=\; 1.00 \qquad\qquad \kappa_{\pi p} \;=\; 0.50 \tag{21}$$

Similar motions of π, by the way, can also originate with quite different parameter combinations of β_π and $\kappa_{\pi p}$. A more ambitious study could proxy π by empirical inflation forecasts and try to obtain meaningful estimates of β_π and $\kappa_{\pi p}$ on this basis. The ensuing calibration of the model components could then proceed along the same lines as with eq. (21).

3.3 Setting the wage Phillips curve

A straightforward method for obtaining countercyclical motions of the price level is an accelerationist price Phillips curve.[14] β_p being a positive coefficient, the continuous-time relationship reads in our context,

$$d\hat{p}/dt \;=\; \beta_p\,(u-1) \tag{22}$$

It is easily checked that (22) indeed gives rise to a countercyclical price level. Furthermore, with the exogenous sine waves of utilization, there is neither a lag nor a lead. The oscillations of the inflation rate itself, which lag u by a quarter of a cycle, are symmetrical around the equilibrium value π^o if $\hat{p}$ is suitably initialized.

In detail, after solving eq. (22) for the rate of inflation, which in our discrete-time approximation gives us a monthly series, we reconstruct the time path of the log of the price level, extract a quarterly series, detrend it by Hodrick-Prescott with smoothing parameter $\lambda = 1600$, and interpolate these trend deviations to get the same number of monthly data points as we have available for $u(\cdot)$. It is the standard deviation of the thus resulting time series to which we refer and that is designated σ_p. For completeness, it should be remarked that the HP 1600 trend is not a straight line, so that these trend deviations are (slightly) different from the theoretically appropriate expressions $\ln p(t) - \ln p^o(t)$, where $\ln p^o(t) = \pi^o\, t$ + const. are the steady state equilibrium prices that would rise at the constant equilibrium rate of inflation π^o.[15]

[14] In one version or another, this type of Phillips curve is presently widely employed in research on monetary policy rules (mostly together with just a dynamic IS relationship). So the results of this subsection for the parameters of the wage Phillips curve may also have some interest of their own; for example, if the models just mentioned were to be moderately enriched by incorporating income distribution dynamics.

[15] The simulated log series of the price level may be viewed as arising from a first-order integrated process. On

According to Table 1, the required standard deviation of the price level is $\sigma_p/\sigma_u = 0.50$. This variability is achieved by a coefficient $\beta_p = 0.54$. The amplitude of inflation thus brought about is $\pm 1.72\%$.

On this basis we can now reconsider the changes in real wages, as they are obtained from $\hat{\omega} = \hat{w} - \hat{p} - \hat{z}^o$. Using (12), this yields

$$\dot{\omega} \;=\; \omega\,[\,-\,(1-\kappa_{wp})(\hat{p} - \pi) \;+\; f_w(e, v; \beta_{we}, \beta_{wv})\,] \tag{23}$$

Given that the time paths of $\hat{p}$ and π can be computed from (22) and (20), respectively, the motions of the real wage rate in (23) are fully determined. So we have all information available to study the impact of the three parameters β_{we}, β_{wu}, κ_{wp} on the cyclical properties of the income distribution dynamics.

To this end we lay a grid over the three-dimensional parameter space (serviceable boundaries are found after a few explorations). For each triple β_{we}, β_{wu}, κ_{wp} a simulation is run and the standard deviations as well as the lags of ω and v are computed (recall that the wage share is given by (7)). Each parameter combination is recorded whose statistics fall within a range that may be called "admissible". In view of Table 1 and the underlying cycle period of 6.67 years, we define it by the intervals

$$\begin{array}{rclcrcl} 0.58 \le & \sigma_\omega/\sigma_u & \le 0.60 & \qquad & 0.00 \le & \text{lag}\,\omega & \le 1.25 \\ 0.27 \le & \sigma_v/\sigma_u & \le 0.35 & \qquad & 1.50 \le & \text{lag}\,v & \le 2.00 \end{array} \tag{24}$$

To conform to the standard deviation in Table 1, regarding σ_ω it is here understood that the real wage series has been normalized before by division through its steady state value ω^o (the exact notation σ_{ω/ω^o} looks too cumbersome; note that this normalization does not apply to the wage share).

Figure 1 shows the projection of the set of the thus specified parameter values onto the (β_{we}, β_{wv})-plane (CCP in the caption abbreviates the present scenario of a countercyclical price level). That is, to each pair (β_{we}, β_{wv}) in the dotted area corresponds one (or more) value(s) of the weighting parameter κ_{wp} between zero and unity such that the trajectories generated by (23) and (7) satisfy the conditions stated in (24). Observe that not only the slope coefficient of the employment rate β_{we} is markedly bounded away from zero, but also the coefficient β_{wv} that measures the negative influence of the wage share on the nominal wage changes. This is a remarkable result beyond the context of the model here discussed: if one aims at (basically)

the other hand, it is well-known that the HP filter is an optimal signal extractor for univariate time series x_t in an uncorrelated components model which implies that x_t would be an I(2) process. Hence, by construction, the HP filter removes too much as trend from the price series.

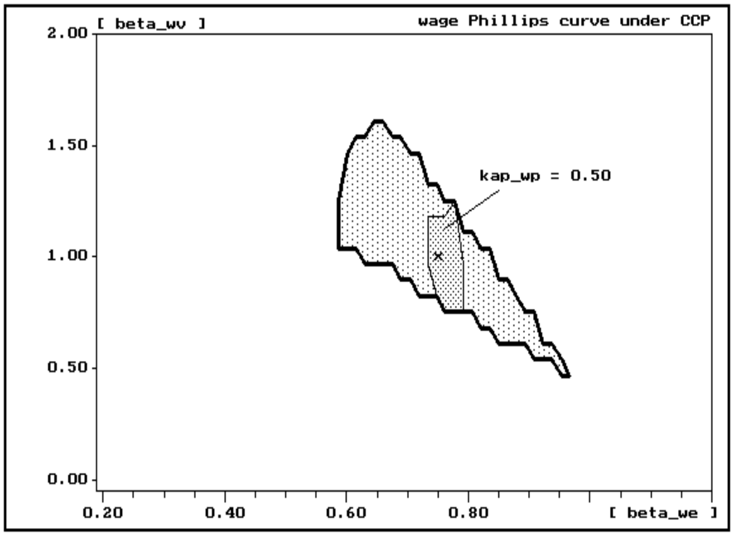

Figure 1: Admissible parameters in the wage Phillips curve under CCP.

procyclical real wages together with a countercyclical pricel level, then the standard wage Phillips curve with the utilization of the labour force as its only core element is not a sufficient device.

The hatched area in the set of admissible β_{we}, β_{wv}, represents pairs that are associated with an equal weight of current inflation and the inflation climate in the specification of reference inflation in (12), $\kappa_{wp} = 0.50$. The cross in this subset indicates the combination of the wage Phillips curve parameters on which we choose to settle down for the following investigations,

$$\beta_{we} = 0.75 \qquad \beta_{wv} = 1.00 \qquad \kappa_{wp} = 0.50 \tag{25}$$

It gives rise to the cyclical statistics

$$\sigma_\omega/\sigma_u = 0.58 \qquad \mathrm{lag}\,\omega = 0.92 \qquad \sigma_v/\sigma_u = 0.30 \qquad \mathrm{lag}\,v = 1.83 \tag{26}$$

3.4 Completing the calibration of the income distribution dynamics

We can now drop the exogenous price movements and turn to calibration level 2, with the wage Phillips curve parameters already being given by (25). Since also the parameters β_π and $\kappa_{\pi p}$ entering the adjustments of the inflation climate at level 3 have been set before, we should look for the three parameters β_{pu}, β_{pv}, κ_{pw} in the price Phillips curve that not only bring about acceptable income distribution dynamics, but at the same time also take care of countercyclical prices with sufficient variability. It turns out, however, that the admissible range defined in (24) cannot be fully maintained and that we have here to cut back a little. Thus, let us be satisfied with the following cyclical features,

$$
\begin{array}{rcccrcccl}
0.56 & \le & \sigma_\omega/\sigma_u & \le & 0.60 & \qquad 0.00 & \le & \text{lag}\,\omega & \le & 1.25 \\
0.27 & \le & \sigma_v/\sigma_u & \le & 0.35 & \qquad 1.50 & \le & \text{lag}\,v & \le & 2.00 \\
0.47 & \le & \sigma_p/\sigma_u & \le & 0.53 & \qquad -0.75 & \le & \text{lag}(-p) & \le & 0.00
\end{array}
\tag{27}
$$

σ_p in the third row stands for $\sigma_{dev\,p}$, where dev p denotes the deviations from the HP 1600 trend of prices when processed as a quarterly series; lag$(-p)$ abbreviates lag$(-\text{dev}\,p)$.

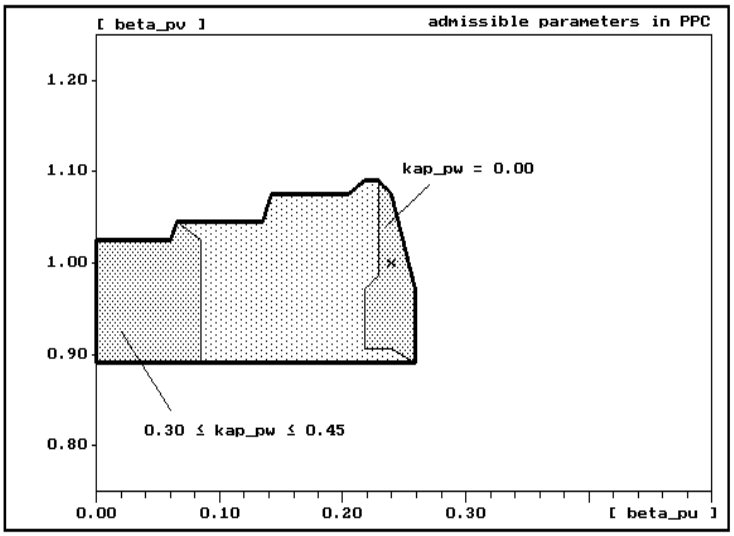

Figure 2: Admissible parameters in the price Phillips given (25).

Figure 2 shows the outcome of a three-dimensional grid search across the parameters κ_{pw}, β_{pu}, β_{pv} ('PPC' in the title refers to 'price Phillips curve'). Similarly as before, the diagram shows the set of pairs (β_{pu}, β_{pv}) for which at least one value of κ_{pw} exists such that these coefficients, together with the parameter values of (25), meet the six conditions for dev ω, dev v, dev p in (27). For each pair (β_{pu}, β_{pv}) in the dotted area there is mostly a wider range of κ_{pw} with that property. The diagram additionally indicates the sets where the accompanying κ_{pw} can lie between 0.30 and 0.45, or be equal to 0.00, respectively. $\kappa_{pw} = 0.45$ is also the maximum value of all admissible κ_{pw} we find.

The striking feature to note in Figure 2 is that the typical Phillips curve coefficient β_{pu} is even allowed to vanish. Moreover, the maximum admissible value of β_{pu} does not appear too high. The influence of utilization in the price Phillips curve is in fact inferior to the wage share. That is, β_{pv} is larger than $0.87 > 0.26/0.30 > \beta_{pu}/0.30 \approx \beta_{pu}/(\sigma_v/\sigma_u)$; which means $\sigma_u\,\beta_{pu} < \sigma_v\,\beta_{pv}$. Actually, it might be asked whether a Phillips curve with a dominant influence of the wage share could any longer be considered a Phillips curve proper.

The relaxation of conditions (27) versus (24) concerns the variability of real wages, as all admissible parameters in Figure 2 entail $\sigma_\omega/\sigma_u < 0.575$. On the other hand, there are no problems with the lag of ω, which is uniformly 0.92 years in the entire region. The relative standard deviation of the wage share ranges narrowly between 0.287 and 0.298, with lags between 1.83 and 1.92 years. The admissible range for σ_p is nearly exhausted, while the price level leads utilization in all these cases by 0.17 years, i.e. two months.

When after this summary of the results we finally have to choose a specific parameter combination for the price Phillips curve, we do not wish to depart too much from the standard formulations and so decide on a triple with a relatively high utilization coefficient β_{pu}. Concretely, we adopt the combination indicated by the cross in Figure 2,

$$\beta_{pu} \;=\; 0.24 \qquad \beta_{pv} \;=\; 1.00 \qquad \kappa_{pw} \;=\; 0.00 \tag{28}$$

Rounded to two digits, the cyclical statistics it gives rise to are

$$\begin{array}{rclrcl} \sigma_\omega/\sigma_u & = & 0.57 & \text{lag}\,\omega & = & 0.92 \\ \sigma_v/\sigma_u & = & 0.29 & \text{lag}\,v & = & 1.83 \\ \sigma_p/\sigma_u & = & 0.50 & \text{lag}(-p) & = & -0.17 \end{array} \tag{29}$$

Regarding ω and v it is seen that the differences to (26) are relatively minor.

4 Supply and demand on money and goods markets

4.1 The LM sector

Financial markets are treated at a textbook level. Only two assets are relevant: money and government bonds.[16] Given the money supply M, the bond rate of interest i is determined by a quasi-linear LM equation,

$$M \;=\; pY\,(\beta_{mo} \;-\; \beta_{mi}\,i) \tag{30}$$

In intensive form with output-capital ratio $y = Y/K = (Y/Y^n)(Y^n/K)$ and real balances normalized by the capital stock, $m = M/pK$, eq. (30) is readily solved as

$$i \;=\; (\beta_{mo} \;-\; m/y)\,/\,\beta_{mi} \tag{31}$$

$$y \;=\; u\,y^n \tag{32}$$

[16] For reasons of consistency, equities may be present to finance fixed investment of firms. Their price, however, can remain in the background.

The responsiveness of money demand is best measured by the interest elasticity $\eta_{m,i}$, which may be conceived as a positive number. Referring to an equilibrium position with output-capital ratio $y^o = y^n$, a real balances ratio m^o and bond rate i^o, the elasticity is defined as $\eta_{m,i} = -(\partial M/\partial i) \cdot (i/M) = +\beta_{mi}\, i^o/(\beta_{mo} - \beta_{mi}\, i^o) = \beta_{mi}\, i^o/(m^o/y^n)$. Hence, if for the calibration we choose a value of the interest elasticity, the two coefficients β_{mo} and β_{mi} are residually computed as

$$\beta_{mi} = \eta_{m,i}\, m^o \,/\, y^n\, i^o \tag{33}$$

$$\beta_{mo} = \beta_{mi}\, i^o + m^o/y^n \tag{34}$$

Monetary policy is supposed to be completely neutral, in order to concentrate on the properties inherent in the private sector. Correspondingly, the money supply grows at a constant rate g_m,

$$\hat{M} = g_m \tag{35}$$

By logarithmic differentiation of $m = M/pK$, real balances therefore evolve according to the differential equation

$$\dot{m} = m\,(g_m - \hat{p} - g_k) \tag{36}$$

Since $g_k(\cdot)$ is exogenous and the time path of $\hat{p}(\cdot)$ has been obtained at the previous level of calibration, no further parameter is needed to determine the solution of (36). On this basis, we can then study the implications of different values of the interest elasticity $\eta_{m,i}$ for the motions of the interest rate i. To summarize:

Level 4: interest rate i (parameter $\eta_{m,i}$)

$$\dot{m} = m\,(g_m - \hat{p} - g_k) \tag{36}$$

$$i = (\beta_{mo} - m/y)\,/\,\beta_{mi} \tag{31}$$

$$y = u\, y^n \tag{32}$$

$$\beta_{mi} = \eta_{m,i}\, m^o \,/\, y^n\, i^o \tag{33}$$

$$\beta_{mo} = \beta_{mi}\, i^o + m^o/y^n \tag{34}$$

With $g_m = 6\%$ and the equilibrium growth rates of capital and prices being $g^o = 3\%$ and $\pi^o = 3\%$, the time path of real money balances oscillates symmetrically around its steady state value m^o if m starts from a suitable initial value. The same then holds (approximately) true for the time path of the bond rate.

The timing of the latter's turning points is independent of the interest elasticity, as $\eta_{m,i}$ does not affect the evolution of m/y but only the coefficients β_{mo} and β_{mi} in eq. (31). Since $m(\cdot)$

leads $y(\cdot)$ shortly by half a year and the sign of the time derivative of i is given by the expression $m\dot{y} - y\dot{m}$, it follows that i still increases when y is already on the downturn (di/dt being still positive when $\dot{y}$ is already negative but so small that $|m\dot{y}| < -y\dot{m}$). Numerically, it turns out that the bond rate peaks 0.83 years after u or y, respectively. Variations of interest rates are in reality certainly more complex than this. On the other hand, given the extreme simplicity of the financial sector as well as the chosen specification of neutral monetary policy, the present pattern may even be considered rather acceptable.[17]

As the only effect of the interest elasticity is on the amplitude of the bond rate oscillations, $\eta_{m,i}$ may be set at any level desired. The outcome of a selected number of elasticities is reported in Table 2.

$\eta_{m,i}$	0.08	0.10	0.12	0.14	0.16	0.20
Amplitude	± 1.32	± 1.07	± 0.89	± 0.76	± 0.67	± 0.53

Table 2: Amplitude of the bond rate (in per cent) at calibration level 4.

A familiar order of magnitude of the elasticity is perhaps $\eta_{m,i} \approx 0.20$. This value is not so attractive here since it brings about a fairly low variation of the bond rate. In order to achieve oscillations with an amplitude of about one per cent, $\eta_{m,i}$ has to be reduced as much as $\eta_{m,i} \approx 0.11$. The reason for this phenomenon is, of course, the relatively low variation in the real balances ratio M/pK, which is due to the constant growth rate of M (in fact, we obtain motions with turning points $100 \cdot m = 14.00 \pm 0.30$). For the model itself, we will adopt

$$\eta_{m,i} = 0.10 \tag{37}$$

In the formulation of excess demand in the next subsection, the bond rate plays no role as long as investment demand is treated as an exogenous variable. In this respect, the interest elasticity might also have been determined at a lower level of the calibration procedure later on. Nevertheless, if we take into account that other versions of the model may, in particular, make consumption demand dependent on the bond market, then the present level 4 would be the appropriate one for the interest rate.

[17] Which does not rule out that a more elaborate financial sector, and the incorporation of monetary policy rules (interest rate reaction functions), is an important task for future modelling.

4.2 The excess demand for goods

In modelling disequilibrium on the goods market, it is assumed that demand for final goods is always realized. This demand is satisfied from current production and the existing stocks of inventories, while any excess of production over sales replenishes inventories. The thus implied motions of inventories are discussed below. Let us first consider aggregate demand Y^d, which is made up of consumption C, net investment in fixed capital I, replacement investment δK (δ the constant rate of depreciation), and real government spending G,

$$Y^d \quad = \quad C + I + \delta K + G \tag{38}$$

Among the components of demand that are presently treated as endogenous, the most important feedback effects are contained in consumption demand of private households. Here we differentiate between workers and asset owners, or more precisely, between consumption financed out of wage income and consumption financed out of rental income. As for the former, it is assumed for simplicity that disposable wage income is exclusively spent on consumption. With respect to a tax rate τ_w and hours worked L, this component of (nominal) consumption expenditures is given by $(1-\tau_w)wL$.

Next, let B be variable-interest bonds outstanding, whose price is fixed at unity. Disposable income of asset owners consists of interest payments iB plus dividends from firms, minus taxes pT^c. A fraction s_c of this income is saved, the remainder is consumed. Regarding dividends, firms are supposed to pay out all net earnings to the shareholders, where the earnings concept may be based on expected sales, Y^e. Another assumption is that equities are the only external source of financing fixed investment, so that firms incur no interest on debt. Hence dividends are given by $pY^e - wL - \delta pK$, and (nominal) consumption spending out of total rental income is $(1-s_c)\,(pY^e - wL - \delta pK + iB - pT^c)$.

In addition to consumption out of wage and rental income, we identify consumption by that part of the population who do not earn income from economic activities, like people living on welfare or unemployment benefits, or retired people drawing on a pension. Since these expenditures are not too closely linked to the business cycle, they may be assumed to grow with the capital stock pK, as governed by a coefficient $c_p > 0$.[18] We expect this type of consumption to help account for the observed countercyclical consumption ratio C/Y.

[18] $c_p pK$ can be thought of as being financed by taxes. In a full model this expression would also have to show up in the government budget restraint, which lends $c_p pK$ the same formal status as government expenditures. A part of the tax collections could be conceived of as payments into a pension fund, which are directly passed on to retired people. Admittedly, this interpretation neglects the fact that pension funds accumulate financial assets and actively operate on the financial markets, which might be an issue for a more elaborated financial sector.

Collecting the terms of the three consumption components, total consumption expenditures sum up to

$$pC \;=\; c_p\,pK \,+\, (1-\tau_w)wL \,+\, (1-s_c)\,(pY^e - wL - \delta\,pK + iB - pT^c) \tag{39}$$

Fiscal policy, too, should presently play a neutral role, with minimal feedbacks on the private sector. This most conveniently means that taxes T^c, which are conceived as net of real interest receipts, and government spending G are postulated to remain in a fixed proportion to the capital stock even outside the steady state:

$$G \;=\; \gamma\,K \tag{40}$$

$$T^c \;=\; \theta_c\,K + iB/p \tag{41}$$

On this basis basis aggregate demand, normalized by the capital stock, is now fully determined. Defining the constant term a_y,

$$a_y \;:=\; c_p + \gamma + s_c\delta - (1-s_c)\theta_c \tag{42}$$

dividing (38) by K, using (39) – (41), and denoting $y^e = Y^e/K$, $y^d = Y^d/K$, we arrive at

$$y^d \;=\; (1-s_c)y^e \,+\, (s_c-\tau_w)vy \,+\, g_k \,+\, a_y \tag{43}$$

The coefficients entering (42) and (43), however, cannot all be freely chosen, given that the parameters δ, γ and θ_c as well as the equilibrium values for the real growth rate g^o, the wage share v^o, and the output-capital ratio $y^o = y^n$ have already been set before. When discussing the production decisions of firms in the next subsection, it will also be shown that equilibrium demand $(y^d)^o$ is slightly less than output y^o. We here anticipate that, besides g^o and y^o, the two are connected through one further further parameter set in (3), namely, the desired ratio β_{ny} of firms' inventories to expected sales; the relationship itself reading $(y^d)^o = y^o/(1+\beta_{ny}g^o)$ (cf. eqs (49) and (57) below). Considering (43) in the steady state with $y^e = y^d$ and solving this equation for a_y, we have therefore only two 'free' parameters left on which this magnitude depends: the tax rate on wages τ_w and the propensity s_c to save out of rental income. In explicit terms, a_y and subsequently c_p result like

$$a_y = a_y(s_c,\tau_w) \;=\; s_c\,y^o/(1+\beta_{ny}g^o) \,-\, (s_c-\tau_w)v^o y^n \,-\, g^o \tag{44}$$

$$c_p = c_p(s_c,\tau_w) \;=\; a_y(s_c,\tau_w) \,-\, \gamma \,-\, s_c\delta \,+\, (1-s_c)\theta_c \tag{45}$$

As we are concerned with the motions of relative excess demand $\xi = (Y^d - Y)/Y$, it remains to put

$$\xi \;=\; y^d/y \,-\, 1 \tag{46}$$

Especially in models where the rigid rule (40) for government expenditures is relaxed, one might also be interested in the cyclical pattern of the consumption ratio C/Y. Using (39) and (41), it is given by

$$C/Y \;=\; (s_c - \tau_w)\, v \;+\; [\,(1-s_c)\, y^e + c_p - (1-s_c)(\delta + \theta_c)\,]\,/\,y \tag{47}$$

4.3 Production and inventory decisions

The modelling of stock management and production of firms follows the production-smoothing / buffer-stock approach, which was initiated by Metzler (1941). Although in recent times its economic significance has been questioned (cf. the survey article by Blinder and Maccini, 1991), it was demonstrated in Franke (1996) that it can be made compatible with the main stylized facts of the inventory cycle.

The approach distinguishes between actual and desired changes in inventories, which are denoted by N. The actual change is just the difference between production Y and sales = demand Y^d,

$$\dot{N} \;=\; Y - Y^d \tag{48}$$

Desired inventory changes are based on the abovementioned ratio β_{ny} of inventories to expected sales. Correspondingly, the desired level N^d of inventories is given by

$$N^d \;=\; \beta_{ny}\, Y^e \tag{49}$$

N^d generally differs from N, and firms seek to close this gap gradually with speed β_{nn}. That is, if everything else remained fixed, the stock of inventories would reach its target level in $1/\beta_{nn}$ years. In addition, firms have to account for the overall growth of the economy, for which they employ the long-run equilibrium growth rate g^o. The desired change in inventories, designated I_N^d, thus reads

$$I_N^d \;=\; g^o N^d \;+\; \beta_{nn}\,(N^d - N) \tag{50}$$

Eq. (50) is the basis of the so-called production-smoothing model; see, e.g., Blinder and Maccini (1991, p. 81).

Production of firms takes care of these desired inventory changes. Otherwise, of course, firms produce to meet expected demand,

$$Y \;=\; Y^e \;+\; I_N^d \tag{51}$$

Eq. (51) represents the buffer-stock aspect. In fact, by inserting (51) into (48), which yields $\dot{N} = I_N^d + (Y^e - Y^d)$, it is seen that sales surprises are completely buffered by inventories.

In specifying the formation of sales expectations, we assume adaptive expectations as a straightforward device. Invoking growth similarly as in (50), they take the form[19]

$$\dot{Y}^e \;=\; g^o Y^e + \beta_y\,(Y^d - Y^e) \tag{52}$$

The time rate of change of the expected sales ratio $y^e = Y^e/K$ is then obtained from $\hat{y}^e = \hat{Y}^e - \hat{K} = g^o + \beta_y[(Y^d - Y^e)/K]\cdot(K/Y^e) - g_k$. The implied evolution of inventories, equally studied in the intensive form of the inventory ratio $n = N/K$, derives from (48) and $\hat{n} = \hat{N} - \hat{K} = (\dot{N}/K)\cdot(K/N) - g_k = [(Y - Y^d)/K]/n - g_k$.

On the whole, the goods market dynamics is represented by the following set of equations. Since, as has been pointed out, g^o, y^n, δ, γ, θ_c and β_{ny} have been determined in advance of the cyclical calibration, we only have to deal with the savings propensity s_c, the tax rate on wages τ_w, and the adjustment speed of the adaptive sales expectations β_y.

Level 5: excess demand ξ, consumption ratio C/Y, inventory ratio n
(parameters s_c, τ_w, β_y)

$$\xi \;=\; y^d/y - 1 \tag{46}$$

$$C/Y \;=\; (s_c - \tau_w)\,v \;+\; [\,(1-s_c)\,y^e + c_p - (1-s_c)(\delta+\theta_c)\,]\,/\,y \tag{47}$$

$$y^d \;=\; (1-s_c)y^e + (s_c - \tau_w)vy + g_k + a_y \tag{43}$$

$$\dot{y}^e \;=\; (g^o - g_k)\,y^e + \beta_y\,(y^d - y^e) \tag{53}$$

$$\dot{n} \;=\; y - y^d - n g_k \tag{54}$$

$$a_y = a_y(s_c,\tau_w) \;=\; s_c y^n/(1+\beta_{ny}g^o) - (s_c - \tau_w)v^o y^n - g^o \tag{44}$$

$$c_p = c_p(s_c,\tau_w) \;=\; a_y(s_c,\tau_w) - \gamma - s_c\delta + (1-s_c)\theta_c \tag{45}$$

It may have been noticed that one behavioural parameter has not yet been made use of, namely, the stock-adjustment speed β_{nn} from eq. (50). Equally important, the theory of production put forward has not yet been fully exploited. The point is that the output level in (51) implies an endogenous determination of the rate of utilization. So we face the following situation: the exogenous variations of utilization u and the capital growth rate g_k give rise to variations in income distribution (and inflation), which in turn determine aggregate demand, which in turn determines sales expectations and the motions of inventories, from which then firms derive their production decisions and, thus, the utilization of their present productive capacity.

[19] As an alternative to the usual interpretation of partial adjustments of expected sales Y^e towards realized sales Y^d, (52) can also be viewed as an approximation to the results of (univariate) extrapolative forecasts on the basis of a rolling sample period. If the latter has length T, the speed of adjustment β_y is related to T by $\beta_y = 4/T$ (Franke, 1992). Such extrapolative predictions are in the same spirit as the simple extrapolative forecasts that Irvine (1981, p. 635) reports to be common practice in real-world retailer forecasting.

Denoting the endogenously determined value of utilization by u^{endo}, the crucial problem is how such an endogenous time path of $u^{endo}(\cdot)$ compares to the exogenous time path $u(\cdot)$ from which it has been ultimately generated. Ideally, we would like the two trajectories $u^{endo}(\cdot)$ and $u(\cdot)$ to coincide. That is, we are looking for a set of parameters that not only produce acceptable cyclical patterns for the variables already discussed, but which also imply that the underlying motions of utilization exhibit a 'fixed-point property'. It goes without saying that we will be content if the time paths of $u^{endo}(\cdot)$ and $u(\cdot)$ are close, while too large discrepancies between the two might indicate difficulties when we subsequently turn to the fully endogenous model.

In detail, using (51), (50), (49), u^{endo} is determined from $Y = Y^e + I_N^d = Y^e + (g^o + \beta_{nn})\beta_{ny} Y^e - \beta_{nn} N$. Division by K gives the endogenous output-capital ratio y^{endo} as a function of y^e and n,

$$y^{endo} \quad = \quad f_y(y^e, n) := [\,1 + (g^o + \beta_{nn})\beta_{ny}\,]\, y^e \; - \; \beta_{nn} n \tag{55}$$

where now also the parameter β_{nn} comes in. β_{nn} can therefore be set at level 6 of the calibration procedure.

<u>*Level 6:*</u> endogenous utilization u^{endo} (parameter β_{nn})

$$u^{endo} \quad = \quad f_y(y^e, n) \,/\, y^n \tag{56}$$

$$f_y(y^e, n) \quad = \quad [\,1 + (g^o + \beta_{nn})\beta_{ny}\,]\, y^e \; - \; \beta_{nn} n \tag{55}$$

At the end of the section, we may provide the argument determining the steady state value of y^d, which entered the coefficient a_y in (44) above. In the same step, the equilibrium value for the inventory ratio n can be derived. Note first that $\dot{y}^e = 0$ and $g_k = g^o$ in (53) gives $y^d = y^e$ in the steady state. Then, putting $y = f_y(y^e, n)$ and, in eq. (54), $\dot{n} = 0$, we obtain $0 = y - y^d - ng_k = [1 + (g^o + \beta_{nn})\beta_{ny}]\, y^e - \beta_{nn} n - y^e - ng^o = (g^o + \beta_{nn})\beta_{ny}\, y^e - (g^o + \beta_{nn})\, n$; hence $n = \beta_{ny} y^e$. Inserting this in $y = f_y(y^e, n)$ leads to $y = (1 + \beta_{ny} g^o) y^e$. In sum,

$$(y^d)^o \; = \; (y^e)^o \; = \; y^n \,/\, (1 + \beta_{ny} g^o) \tag{57}$$

$$n^o \; = \; \beta_{ny}\, y^n \,/\, (1 + \beta_{ny} g^o) \tag{58}$$

4.4 Calibration of the goods market dynamics

As argued in the previous subsection, we treat calibration levels 5 and 6 simultaneously. Accordingly, we now have to study the implications of the four parameters s_c (the savings propensity), τ_w (the tax rate on wages), β_y (the speed of adaptive expectations for sales), and β_{nn} (the stock adjustment speed for inventories). The dynamic variables involved at this stage are relative excess demand ξ, the consumption ratio C/Y, and endogenous utilization u^{endo}. Since their cyclical characteristics turn out to change in a monotonic and regular way in response to *ceteris paribus* changes in the above parameters (except for β_{nn}, perhaps), we only need to report the results for a number of selected values. It will become sufficiently clear in this way how we arrive at the parameter combination that we finally choose to employ.

To start with, let us consider the values $s_c = 0.80$, $\tau_w = 0.35$, $\beta_y = 8$, $\beta_{nn} = 3$, which do not appear too implausible. Regarding the first two parameters, eq. (45) yields $c_p = 0.114$ for the constant 'trend' term c_p in the consumption function (39); this, too, looks acceptable.[20] The parameter quadruple itself gives rise to the cyclical statistics shown in the first row in Table 3.

				ξ		C/Y		u^{endo}	
s_c	τ_w	β_y	β_{nn}	$\tilde{\sigma}$	lag	$\tilde{\sigma}$	lag	$\tilde{\sigma}$	lag
0.80	0.35	8.00	3.00	0.21	2.25	0.26	2.50	0.87	0.50
0.60	—	—	—	0.20	2.00	0.23	2.33	1.02	0.58
—	0.40	—	—	0.22	2.42	0.28	2.67	0.82	0.25
—	—	12.0	—	0.20	2.25	0.25	2.50	0.93	0.42
—	—	—	1.00	0.21	2.25	0.26	2.50	0.87	0.42
0.75	0.38	12.0	3.00	0.20	2.33	0.25	2.58	0.94	0.33
0.75	0.38	12.0	1.00	0.20	2.33	0.25	2.58	0.90	0.33
0.80	0.38	12.0	3.00	0.21	2.33	0.26	2.67	0.90	0.25
0.75	0.35	12.0	3.00	0.20	2.17	0.24	2.50	0.97	0.42

Table 3: Cyclical features of variables at calibration level 5 and 6.

Note: $\tilde{\sigma}$ is the standard deviation of the respective variable in relation to σ_u; the hyphens indicate that the values in the first row are taken over. The underlying cycle period is 6.67 years.

The first thing one will notice is that the standard deviations of both ξ and C/Y are considerably below the desirable variation given in Table 1. Furthermore, in contrast to that table, C/Y has here a higher variation than ξ. Similar order of magnitudes are obtained in all other parameter scenarios that we have tried, not only those exemplified in Table 3. Clearly, the limited possibilities that we have to exert influence on σ_ξ and $\sigma_{C/Y}$ are due to the fixed

[20] Note that because of the limited compatibility that our still relatively simple modelling framework exhibits with empirical data on the income flows of groups like 'workers' and 'rentiers', we have some freedom in choosing the numerical values for s_c and τ_w, anyway.

proportions assumed in the formulation of aggregate demand. Since we presently do not wish to extend this part of the model, the statistics in Table 3 have to be accepted as a price to be paid for the otherwise convenient specification of consumption and government demand. Also, the formulation of demand is at least flexible enough that ξ and C/Y display a certain tendency for countercyclical motions, though this feature is still far from perfect, and it is weaker for excess demand.

Taking the features of ξ and C/Y for granted, the initial parameter scenario is obviously unable to generate satisfactory fluctuations of the third dynamic variable, endogenous utilization. To overcome this deficiency, we examine the effects on u^{endo} of variations in each of the four parameters in the ensuing four rows of Table 3. Reducing the propensity to save to 0.60 succeeds in raising the standard deviation of u^{endo} to the desired level. On the other hand, the lag of u^{endo} behind the exogenous motions of u increases, and the statistics of ξ and C/Y change in the wrong direction. The latter are improved by an increase in the tax rate τ_w, but this time the standard deviation of u^{endo} reacts adversely, while, as another positive aspect, its lag shortens. An increase in the adjustment speed of adaptive sales expectations up to $\beta_y = 12$ has a positive effect on both statistics of u^{endo}: it increases its standard deviation and reduces the lag. Also, the negative effect on ξ and C/Y is relatively minor. However, since in the numerical simulations of the model the discrete-time version is a monthly economy, β_y should not exceed the value 12. Lastly, ξ and C/Y must remain unaffected by a change in the stock adjustment speed β_{nn}. Unfortunately, even a large diminuition of β_{nn} from 3 down to 1 induces only a small reduction of the lag of u^{endo}, the standard deviation does not change at all.

On the basis of these observations we look for the outcome if the increase of β_y to 12 (to raise the variation of u^{endo} and reduce its lag) is combined with a moderate decrease in s_c to 0.75 (for a further increase in the standard deviation of u^{endo}) and a moderate increase in τ_w to 0.38 (to 'undo' the negative effects on ξ and C/Y and the rise in the lag of u^{endo} caused by s_c). There is no problem with the constant term c_p in this case; it just increases to $c_p = 0.124$. The cyclical statistics of the dynamic variables are given in the sixth row of Table 3. While, as compared to the first row, the standard deviations of ξ and C/Y have fallen a little, their lag have slightly increased and the 'performance' of u^{endo} has noticeably improved.

This advancement notwithstanding, a still higher standard deviation of u^{endo} would certainly be desirable. So in the last three rows of the table we once again reduce β_{nn} to 1 and, on the other hand, put the values of s_c and τ_w at their original level. Whereas changing the stock adjustment speed in the initial scenario to $\beta_{nn} = 1$ reduced the lag of u^{endo} and had no impact on its standard deviation, it is here the other way round. Obviously the effects of β_{nn} on u^{endo} are not too systematic but depend on the combination of the other parameters. The result in row 7 of the table suggests changing β_{nn} in the opposite direction. Indeed, $\beta_{nn} = 5$ produces a standard deviation of u^{endo} of 0.97 and even reduces the lag to 0.25 years. We nevertheless abstain from employing this value since it appears somewhat too high to us, and because of the sensitivity just observed we also cannot be sure that $\beta_{nn} = 5$ would be similarly 'successful' in the fully endogenous model.

Setting the savings propensity back to 0.80 brings down the standard deviation of endogenous utilization; $s_c = 0.75$ should hence be maintained. Setting the tax rate back to 0.35 has a positive effect on the standard deviation of u^{endo}, but increases its lag, decreases the lags of ξ and C/Y, and decreases the standard deviation of C/Y. On account of the latter deterioration we prefer to work with $\tau_w = 0.38$. Thus, on the whole, we settle down on

$$s_c = 0.75 \qquad \tau_w = 0.38 \qquad \beta_y = 12.0 \qquad \beta_{nn} = 3.0 \tag{59}$$

4.5 Summary of calibration results

For a better overview of what has been done and achieved, we collect the numerical parameter values put forward so far in an extra box. From eqs (11), (21), (25), (28), (37), (59), these are 14 in total. Subsequently, Table 4 lists the statistics of the cyclical features to which they give rise (it goes without saying that 'dev p' stands for the percentage trend deviations of the price level). A comparison with Table 1 shows where a fairly good reproduction of the stylized of the German business cycle has been obtained and where we have to lower our sights.

Synopsis of numerical parameters

Level 1:	$\beta_{zu} = 0.45$		
Level 2:	$\beta_{pu} = 0.24$	$\beta_{pv} = 1.00$	$\kappa_{pw} = 0.00$
	$\beta_{we} = 0.75$	$\beta_{wv} = 1.00$	$\kappa_{wp} = 0.50$
Level 3:	$\beta_{\pi} = 1.00$	$\kappa_{\pi p} = 0.50$	
Level 4:	$\eta_{m,i} = 0.10$		
Level 5:	$s_c = 0.75$	$\tau_w = 0.38$	$\beta_y = 12.0$
Level 6:	$\beta_{nn} = 3.00$		

We repeat that it could not have been our goal to obtain a perfect match of the cyclical statistics of the empirical series. And even if we were to came close to full success in this respect, we would not yet know what it would be worth since admittedly the exogenous sine wave motions of utilization u are very stylized indeed. The results in Table 4 and the way we arrived at them are more of a heuristic value, and we will have to see if the present set of numerical parameters

variable x	σ_x/σ_u	lag x
g_k	0.30	0.00
z/z^o	0.45	0.00
e	0.62	0.58
ω/ω^o	0.57	0.92
v	0.29	1.83
$-\ \mathrm{dev}\, p$	0.50	−0.17
i	0.35	0.83
$-\xi$	0.20	−1.00
$-C/Y$	0.25	−0.75
u^{endo}	0.94	0.33

Table 4: Cyclical statistics of variables under exogenous sine wave oscillations of utilization u and the capital growth rate g_k.

can survive in the full model when capacity utilization and investment are endogenized.

5 The fully endogenous model and its calibrated dynamics

5.1 The investment function

The modelling equations so far provided can already be viewed as constituting a fully endogenous macrodynamic model if the exogenous motions of utilization are dropped and $u = u^{endo}$ is obtained from (56). Eq. (2) for the capital growth rate g_k would then have the status of an investment function. In this respect, however, we want to be more flexible, both on the grounds of greater conceptual richness as it is common in modern Keynesian-oriented modelling, and in order to gain some control over the stability of the system. We therefore bring another variable into play.

Two motives for investment in fixed capital are considered. First, for reasons not explicitly taken into account in the model formulation, firms not only seek to avoid excess capacity but also desire no permanent overutilization of productive capacity. Hence investment increases with utilization u, as in eq. (2). The second motive refers to the profitability of firms. We may measure it by the rate $r := (pY - wL - \delta pK)/pK = (1{-}v)uy^n - \delta$. Since investment is exclusively financed by equities, this operative rate of profit is seen in relation to the yields from holding bonds, which is the alternative of financial investment that shareholders have, the relevant rate of return being the real (*ex-post*) interest rate $i - \pi$. In sum, besides utilization, fixed investment is additionally a positive function of the differential returns q, defined as $q := r - (i - \pi)$, or

$$q \;=\; (1-v)\, uy^n \;-\; \delta \;-\; (i-\pi) \tag{60}$$

Our methodological approach to persistent cyclical behaviour in this paper is a deterministic one. We do not, however, wish to rely on periodic orbits arising from a Hopf bifurcation (presupposing its existence).[21] Within a vicinity of the steady state position, the dynamics may rather be (mildly) destabilizing. Though there are a number of intrinsic nonlinearities in the model, they are only weak and 'dominated' by the many linear specifications in the behavioural functions. It thus turns out that the destabilizing forces are also globally operative. This means we have to build in some additional, extrinsic nonlinearities, which take effect in the outer regions of the state space and prevent the dynamics from totally diverging. For our present purpose, we can content ourselves with just one such nonlinearity, which we introduce into the investment function.

A simple idea will prove sufficient. Suppose utilization is steadily rising in an expansionary phase. The corresponding positive influence on the flow of investment may be reinforced or curbed by the differential returns q. If, however, utilization has become relatively high, firms will not expect the economy to grow at the same speed for too long. If moreover q is relatively low in that stage, so that this influence on investment is already negative, then the positive utilization motive may be further weakened. That is, we assume that in such a phase of the cycle the negative effect from q is stronger than it would be at lower levels of capacity utilization. With signs reversed, the same type of mechanism applies when the economy is on the downturn. Introducing two positive reaction coefficients β_{Iu} and β_{Iq} and referring for simplicity directly to the growth rate g^o and the differential returns q^o in a long-run equilibrium, we specify this concept for g_k, the capital growth rate, as follows:

$$g_k \;=\; g_k(u,q) \;=\; g^o \;+\; \beta_{Iu}\,(u-1) \;+\; \alpha(u,q)\,\beta_{Iq}\,(q-q^o) \tag{61}$$

where with respect to given values d_1 and d_2, $0 < d_1 < d_2$, the flexibility function $\alpha = \alpha(u,q)$ is defined as[22]

$$\alpha \;=\; \alpha(u,q) \;=\; \begin{cases} 1 \;+\; [u-(1+d_1)]/(d_2-d_1) & \text{if } u \geq 1+d_1 \text{ and } q \leq q^o \\ 1 \;+\; [(1-d_1)-u]/(d_2-d_1) & \text{if } u \leq 1-d_1 \text{ and } q \geq q^o \\ 1 & \text{else} \end{cases} \tag{62}$$

[21] One reason is that, *a priori*, we can by no means be sure that the periodic orbits of the Hopf bifurcation are attractive. But even then, meaningful cyclical trajectories might only exist over a very small range of parameter values.

[22] It may be noted that though the function α is not continuous in q, the multiplicative term $\alpha\,(q-q^o)$ in g_k is.

Evidently, for this mechanism to work out it is required that the q-series peaks considerably before utilization, a property we have already checked in the sine wave experiments. Being essentially dependent on the relative amplitude of the bond rate and the rate of inflation in (60), the mechanism cannot necessarily be expected to be effective under all circumstances. In this sense, (62) represents only a minimal nonlinearity to tame the centrifugal forces in the economy.

On the whole, we have now a self-contained differential equations system of dimension six. The state variables are $k^s = K/z^oL^s$, capital per head (measured in efficiency units); $\omega = w/pz^o$, the real wage rate (deflated by trend labour productivity); π, the inflation climate; $m = M/pK$, the real balances ratio; $y^e = Y^e/K$, the expected sales ratio; and $n = N/K$, the inventory ratio (where clearly the term 'ratio' refers to the stock of fixed capital). The differential equations governing their motions were presented at calibration levels 1 – 6 and are, in the same order, (10), (19), (20), (36), (53), (54). Renumbering them anew, we therefore have to study the following system:

$$\dot{k}^s = k^s\,(g_k - g_z - g_\ell) \tag{63}$$

$$\dot{\omega} = \omega\,\kappa\,[\,(1-\kappa_{pw})f_w(e,v) \;-\; (1-\kappa_{wp})f_p(u,v)\,] \tag{64}$$

$$\dot{\pi} = \beta_\pi\,[\kappa_{\pi p}(\hat{p}-\pi) \;+\; (1-\kappa_{\pi p})(\pi^o-\pi)] \tag{65}$$

$$\dot{m} = m\,(g_m - \hat{p} - g_k) \tag{66}$$

$$\dot{y}^e = (g^o - g_k)\,y^e \;+\; \beta_y\,(y^d - y^e) \tag{67}$$

$$\dot{n} = y - y^d - n g_k \tag{68}$$

To see that actually no more than these six dynamic variables are involved, note that κ is defined in (18) and $g_k = g_k(u,q)$ is determined in (61) and (62), $u = f_y(y^e,n)/y^n$ is determined in (56), $q = q(u,v,i,\pi)$ in (60), $v = v(u,\omega)$ in (7), $e = e(u,k^s)$ in (8), $\hat{p} = \hat{p}(u,e,v,\pi)$ in (17), $i = i(m,y)$ in (31), $y = uy^n$ in (32), $y^d = y^d(y^e,v,y,g_k)$ in (43).

5.2 Difficulties obtaining the desired cycle period

To simulate the economy (63) – (68) on the computer, it remains to find suitable values for the four investment parameters in (61) and (62). A necessary condition for the system to generate persistent oscillatory behaviour in our deterministic setting is the instability of the steady state position. Responsible for this, given the other parameters as in the synopsis of section 4.5, are β_{Iu} and β_{Iq}. Figure 3 shows that such combinations exist; we only have to look for them outside the dotted region whose pairs (β_{Iu},β_{Iq}) induce local asymptotic stability. It can actually be broadly said that a stronger impact of capacity utilization on investment (a higher coefficient β_{Iu}) tends to destabilize the equilibrium, whereas a stronger impact of the return differential $q = r - (i - \pi)$ (a higher coefficient β_{Iq}) tends to stabilize it.[23]

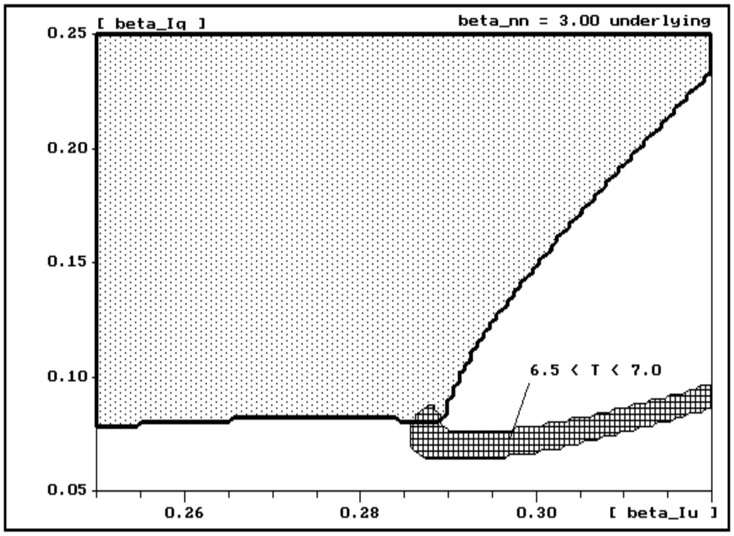

Figure 3: Parameter diagram of (β_{Iu}, β_{Iq}) with $\beta_{nn} = 3.0$ underlying.

Note: Dotted area indicates cyclical stability, points in hatched area induce eigen-values with a period between 6.5 and 7 years.

It can in addition be mathematically proven (similar as in Köper, 2000) that for (β_{Iu}, β_{Iq}) on the stability frontier (the bold line dividing the stability from the instability region), the Jacobian matrix of system (63) – (68) has always a positive determinant, so that here indeed a Hopf bifurcation occurs (as presupposed above). The relevant consequence for us is that all combinations (β_{Iu}, β_{Iq}) near the Hopf locus give rise to conjugate complex eigen-values as the leading eigen-values of the Jacobian (those with maximal real parts), implying that, as required, the system is characterized by cyclical behaviour. The numerical investigation makes sure that this is even true for the entire rectangle displayed in Figure 3.

Regarding the simulations of the dynamic system, to set it in motion we start out from a steady state growth path and disturb it by a strong temporary shock. Concretely, we do this by raising the growth rate of the money supply over one year from 6% up to 8%. Afterwards it is set back to its original level, from when on the economy is left to its own. Some 15 or 20 years after

[23] Note, however, that there is a small interval of values of β_{Iq} where, if they are fixed, the steady state is unstable for lower values of β_{Iu}, it is stable over some intermediate range of β_{Iu}'s, and it becomes unstable again as β_{Iu} rises further. This 'reswitching' phenomenon, which is otherwise of no minor significance since it warns us against premature conclusions, may here be left aside.

the shock, the fluctuations become quite regular and tend to repeat themselves. The statistics discussed below are computed for the limit cycle on which, by all appearances, the system has finally settled down.

While the values for β_{Iu} and β_{Iq} have to be sought in the instability region, we should not move too far away from the stability frontier. A few explorations are enough to learn that the destabilizing tendencies soon become very strong, so that the nonlinearity (62) in the investment function has difficulty taming the centrifugal forces, or the cycles thus arising look somewhat artificial.

We furthermore have to pay attention to the period of the oscillations we obtain, which must not deviate too much from the 6.67-year period of the exogenous utilization sine waves in the previous sections. The hatched area in the parameter diagram represents those pairs (β_{Iu}, β_{Iq}) whose associated Jacobian has leading (complex) eigen-values with a period between 6.5 and 7.0 years. To sum up, we should try a combination (β_{Iu}, β_{Iq}) in that area and near the Hopf locus (but outside the stability region, of course).

Since the computer simulations use a discrete-time version of the differential equations (63) – (68) (a monthly economy, in fact, as has already been indicated), the simulation results may be slightly different from the continuous-time system. We nevertheless soon find that setting $\beta_{Iu} = 0.293$ and $\beta_{Iq} = 0.080$ gives rise to cycles with a period in the desired range. Setting the nonlinearity parameters d_1 and d_2 at 0.015 and 0.050, respectively, keeps the system within reasonable bounds and produces fairly regular oscillations of the economic variables. We can therefore compute their cyclical features and compare them to those from the sine wave experiments, as they are summarized in Table 4. Besides the cycle period and the standard deviation of capacity utilization, it here suffices to concentrate on the wage-price dynamics, i.e., on the variability of the real wage, the wage share, and the price level.

Table 5 repeats in the first row the cyclical statistics of the sine wave calibration as our present reference scenario. The second row gives the results of our first trial with the fully endogenous system, which has just been described. It is thus seen that this combination of β_{Iu}, β_{Iq}, d_1, d_2 is only partially successful: while the variability of utilization, the real wage and the wage share are completely satisfactory, the standard deviation of the price level comes out unpleasantly high.

As a first attempt to improve on this outcome, we slightly destabilize the local dynamics by lowering the coefficient β_{Iq} to 0.070. At the same time, the nonlinearity coefficients d_1, d_2 are adjusted such that the amplitude of the utilization oscillations remains about the same. The third row in Table 5 shows that this does not help; the variability of the price level even increases. Observe that the cycle period T increases, too.

The variability of the price level can be reduced from 0.62 to 0.53 if in this situation β_{Iu} is increased from 0.293 to 0.294. The simultaneous decrease in the real wage variability may be tolerated, but unfortunately the cycle period falls below 6 years; see the fourth row of the table. The period can be increased back to $T = 6.75$ years by a certain weakening of the nonlinearity effect, a change in d_1 which is actually amazingly small. However, then again σ_p/σ_u rises above 0.60.

	β_{Iu}	β_{Iq}	d_1	d_2	T	σ_u	σ_ω/σ_u	σ_v/σ_u	σ_p/σ_u
$\beta_{nn} = 3.00$	sine wave calibration:				6.67	2.14	0.57	0.29	0.50
	0.293	0.080	0.015	0.050	6.50	2.09	0.57	0.29	0.59
	0.293	0.070	0.000	0.045	6.75	2.12	0.57	0.29	0.62
	0.294	0.070	0.000	0.045	5.92	1.98	0.55	0.29	0.53
	0.294	0.070	0.002	0.045	6.75	2.18	0.57	0.28	0.61
$\beta_{nn} = 2.50$	0.296	0.105	0.015	0.050	6.58	2.08	0.57	0.29	0.60
$\Delta\beta_{we} = -0.05$	—	—	0.025	0.100	6.00	2.36	0.53	0.27	0.49
$\Delta\beta_{wv} = +0.05$	—	—	0.020	0.050	6.00	2.09	0.55	0.28	0.51
$\Delta\beta_{pu} = +0.03$	—	—	0.020	0.050	6.08	2.05	0.54	0.28	0.53
$\Delta\beta_{pv} = -0.20$	—	—	0.025	0.100	6.58	2.10	0.57	0.30	0.51

Table 5: Cyclical statistics of the full model under selected parameter changes.

Note: T is the cycle period, a hyphen indicates that the previous value is taken over. The upper half in the table has $\beta_{nn} = 3.00$, the lower half has $\beta_{nn} = 2.50$ underlying; the other parameters as reported in Section 4.5. $\Delta\beta_{we} = -0.05$ means the coefficient has *c.p.* been changed to $\beta_{we} = 0.75 - 0.05$, the other coefficients correspondingly.

It turns out from these simulation experiments that the variations in the coefficients of the investment function affect the variability of the real wage and the wage share only mildly, if at all, whereas they have a much stronger impact on the standard deviation of the price level. The price for the required decrease in σ_p, however, is an undue shortening of the cycle period. The examples may be sufficiently illustrative to demonstrate that a mere tuning of the investment reaction coefficients will not give us the desired features. It seems that other parameters have to be involved, too.

Since we are here concerned with the cyclical features of the wage-price dynamics, an obvious idea is to try suitable modifications of one of the parameters in the price or wage Phillips curve. But notice the problem that then each such parameter change shifts the narrow hatched region in Figure 3 where $6.5 < T < 7.0$, so that the original values for β_{Iu} and β_{Iq} may easily yield too short or too long a period T. Hence in each case β_{Iu} and β_{Iq} would have to be set anew.

To defuse this problem of a lack in robustness, we first look for another parameter in the model that enlarges the hatched area near the stability frontier. In this way we can expect that after a moderate change in the Phillips curve parameters, the initially chosen pair (β_{Iu}, β_{Iq}) is still contained in this critical region. Because of the limited effects of β_{nn} on the cyclical features in Section 4.4, we examine how the regions in Figure 3 are affected by changes in this stock adjustment speed. Figure 4 evidences that our intermediate goal can indeed be achieved by reducing β_{nn} from 3.00 to 2.50. It may also be remarked in passing that the effect of the change in β_{nn} on local stability is ambiguous: in the lower left of the parameter plane the stability region has slightly moved back, while it has gained territory in the upper right.

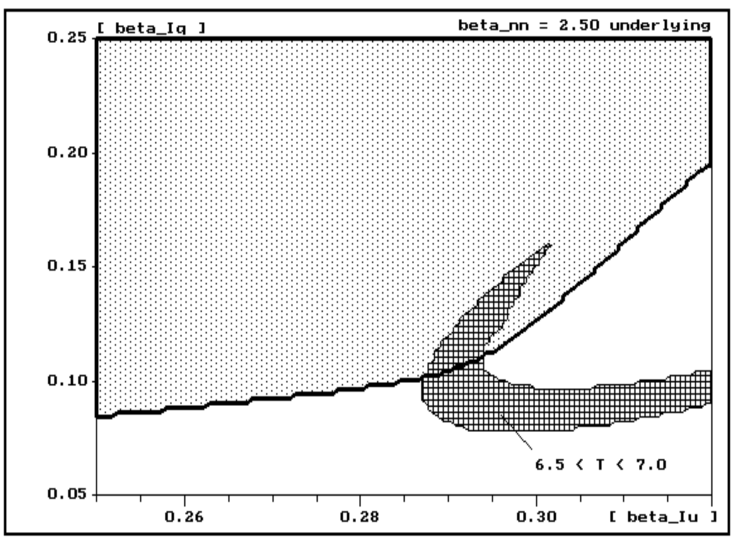

Figure 4: Parameter diagram of (β_{Iu}, β_{Iq}) with $\beta_{nn} = 2.5$ underlying.

Thus, fix β_{nn} at the new value 2.50 and let us proceed on this basis, the other parameters listed in Section 4.5 remaining unaffected. Very similar results as in the first scenario of Table 5 (second row in the upper half) are obtained if we put β_{Iu}, etc., as reported in the first row of the table's lower half. Let us then consider in turn *ceteris paribus* changes of the main four Phillips curve coefficients β_{we}, β_{wv}, β_{pu}, β_{pv}. The numerical values for β_{Iu} and β_{Iq} are maintained, while d_1 and d_2 are adjusted so that the standard deviation of utilization σ_u happens to be somewhat above 2%.

The second row in the lower half of Table 5 shows the outcome if β_{we} is reduced from 0.75 to 0.70, which brings down the variability of the price level to $\sigma_p/\sigma_u = 0.49$. Once again, however, this has to be bought at the price of an unacceptably short cycle period. Interestingly, almost the same effects are produced by small increases of β_{wv} (from 1.00 to 1.05) or β_{pu} (from 0.24 to 0.27).

Our problem is fortunately solved by the price Phillips curve coefficient β_{pv}, if it is reduced from 1.00 to 0.80. The last row in table 5 shows a desired price level variability of $\sigma_p/\sigma_u = 0.51$, where this time the cycle period remains at 6.58 years, and the standard deviations of the real wage and the wage share hardly change, either. So $\beta_{wv} = 0.80$ together with the other numerical parameters constitutes a scenario we have been looking for so far.

5.3 The final parameter scenario

We are nevertheless not through, yet. Since Table 5 focuses on a few statistics only, we still have to check if also the other cyclical features are within an acceptable range. To this end Table 6 is put forward, which presents all the statistics we are interested in. In particular, it distinguishes between the system's behaviour near the peaks and near the troughs of utilization, thus revealing a weak asymmetry in most of the variables. In this respect it may be added that the peak-to-trough period of utilization is with 3.33 years just one month longer than the trough-to-peak period with its 3.25 years.

A first observation for the results to be meaningful is the amplitude of the endogenous capital growth rate, recalling that the deviations of the exogenous sine waves of g_k from equilibrium were a fraction 0.30 of the deviations of utilization u. Table 6 reports a ratio of the standard deviations of the two series of 0.27, so that this important hurdle can be considered to be cleared. On the other hand, g_k takes a slight lead before u. A lag would have been preferable, but the discussion of the investment function and its stabilizing nonlinearity in the outer regions of the state space should have made clear that exactly such a lead had to be expected.

Table 6 may be compared to the summary of the results of the sine wave experiments in Table 4, or directly to the desirable features advanced in Table 1. It is thus seen that the employment rate, the real wage, the wage share, as well as the price level match the cyclical behaviour we have aimed at quite closely. Besides, the table makes the cyclical characteristics of the rate of inflation $\hat{p}$ explicit, which, given the countercyclical motions of the price level, must of necessity lag utilization by about a quarter of the cycle. Certainly, as already discussed in connection with eq. (20), the inflation climate π must lag $\hat{p}$, which explains the long lag of π behind u.

We had already come to terms with the limited amplitudes of excess demand and the

variable x	x^o	in peak	in trough	σ_x/σ_u	lag (peak)	lag (trough)
u	100.00	+ 2.80	− 3.11	––	––	––
g_k	3.00	+ 0.77	− 0.84	0.27	−0.17	−0.08
e	100.00	+ 1.79	− 1.94	0.64	0.58	0.42
ω	70.00	+ 1.14	− 1.22	0.57	0.92	0.83
v	70.00	+ 0.87	− 0.90	0.30	1.92	1.75
$-p/p^o$	0.00	+ 1.48	− 1.54	0.51	−0.17	−0.17
$\hat{p}$	3.00	+ 1.43	− 1.48	0.49	1.67	1.50
π	3.00	+ 0.54	− 0.54	0.18	2.33	2.25
$-\xi$	+0.69	+ 0.55	− 0.62	0.20	−0.75	−0.42
$-C/Y$	−64.59	+ 0.72	− 0.80	0.26	−0.75	−0.50
n	16.13	+ 0.30	− 0.33	0.11	0.92	0.83
m	14.00	+ 0.24	− 0.25	0.60	−0.58	−0.58
i	7.00	+ 1.20	− 1.31	0.43	0.83	0.50
q	9.00	+ 1.11	− 1.09	0.37	−2.42	−2.17

Table 6: Cyclical statistics of the calibrated endogenous model.

Note: All variables multiplied by 100. x^o (p^o) denotes the steady state (or trend) value of variable x (or p); the standard deviations of ω and m are divided by ω^o and m^o, respectively. Detrending of p is by HP 1600.

consumption-output ratio. They are, of course, equally limited in the full model; there is only a slight increase of the standard deviation of C/Y from 0.25 in Table 4 to 0.26, which should not be exaggerated. Regarding the comovements of ξ and C/Y, however, some improvement toward countercyclicality is observed; the lead of $-(\xi - \xi^o)$ is even strictly within the desirable range of Table 1. The statistics of the other four variables in the table are given for completeness (the inventory ratio $n = N/K$ resulting from excess demand, the real balances ratio $m = M/pK$ together with the bond rate i to which it gives rise, and the return differential $q = r - (i - \pi)$, the difference between the operative rate of profit and the real interest rate).

To make the statistical numbers of Table 6 more vivid, Figure 5 displays a diagram of three selected time series. It demonstrates the very regular motions of utilization and, almost

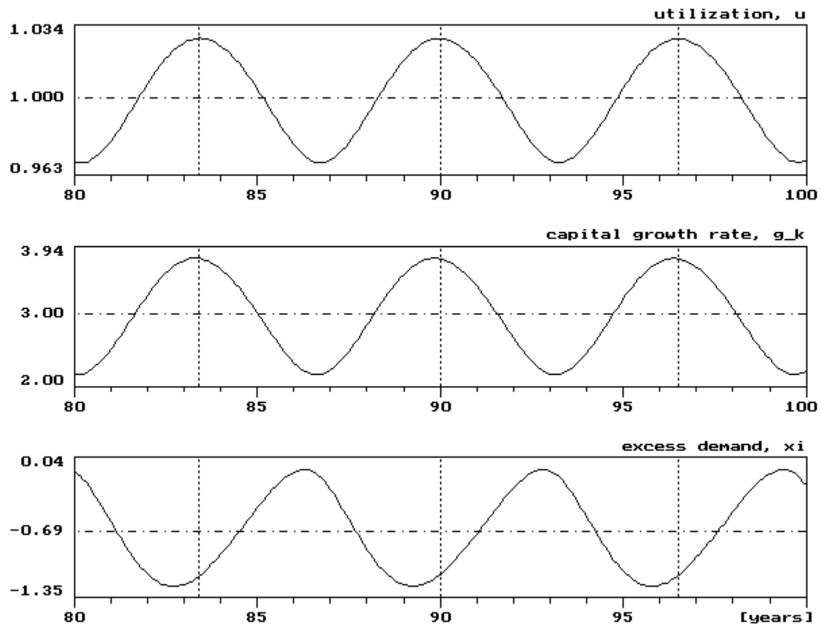

Figure 5: Selected time series of the calibrated endogenous model.

synchronously, the capital growth rate. As a matter of fact, it now turns out that the exogenous sine waves of the two variables in Sections 3 and 4 have had not only heuristic value, but they also come close to the model's reality in a deterministic setting. The bottom panel of Figure 5 illustrates the basically countercyclical pattern of excess demand ξ, as well as the small asymmetries it exhibits.

It will finally be convenient to collect the numerical parameter values on which we have settled down in an extra box.

The final parameter scenario

1:	$\beta_{zu} = 0.45$		
2:	$\beta_{pu} = 0.24$	$\beta_{pv} = 0.80$	$\kappa_{pw} = 0.00$
	$\beta_{we} = 0.75$	$\beta_{wv} = 1.00$	$\kappa_{wp} = 0.50$
3:	$\beta_{\pi} = 1.00$	$\kappa_{\pi p} = 0.50$	
4:	$\eta_{m,i} = 0.10$		
5:	$s_c = 0.75$	$\tau_w = 0.38$	$\beta_y = 12.0$
6:	$\beta_{nn} = 2.50$		
7:	$\beta_{Iu} = 0.296$	$\beta_{Iq} = 0.105$	
	$d_1 = 0.025$	$d_2 = 0.100$	

6 Conclusion

The paper has put forward a complete deterministic macro model of the business cycle that takes up theoretical elements which may be connected with, in particular, the names of Keynes, Metzler and Goodwin. The aim of the paper was a calibration of this disequilibrium model. Given stylized oscillations of two exogenous variables, capacity utilization and the capital growth rate, this procedure could be organized in a hierarchical structure. In this way the numerical coefficients need not all be determined simultaneously but could be chosen step by step. At each level of the hierarchy, the parameters here involved gave rise to motions of some few endogenous variables, and we tried to find parameters such that the thus generated cyclical patterns match the desired features that had been formulated in advance.

This type of analysis has ended up with numerical values of, on the whole, 14 parameters (apart from the steady state ratios and similar magnitudes). Subsequently, the lacking investment function was introduced to endogenize the hitherto exogenous variables, which added another four parameters. They were set such as to render the steady state position of the full model locally unstable, while a suitable nonlinearity in the investment function prevented the system from totally diverging. Hence, the model was able to produce persistent cyclical behaviour, which actually took the form of a limit cycle.

The main characteristics of the model's dynamics may be judged to be by and large satisfactory. On the one hand, this concerns the amplitudes of the oscillating variables, on the other hand the more or less procyclical motions of the capital growth rate, the employment rate and the (productivity-deflated) real wage rate, as well as the countercyclicality in the price level, relative excess demand and the consumption ratio (the latter two relative to total output). Of course, the cyclical statistics cannot always be perfect, but we have reached a stage where, within the given modelling framework, a single statistic could hardly be improved any further without seriously affecting another one.

In summing up the results, we may claim that the model-generated time series can well stand comparison with those obtained by the competitive equilibrium models of the real business cycle school. Therefore, the disequilibrium approach of the present Keynes-Metzler-Goodwin model does not only have its theoretical merits with its more transparent feedback mechanisms, but it is also a quantitatively operational alternative to the ruling RBC paradigm.

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