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**Fiscal policy in an endogenous growth model  
with public capital and pollution**

by

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# Fiscal policy in an endogenous growth model with public capital and pollution

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## **Abstract**

In this paper we study growth and welfare effects of fiscal policy in an endogenous growth model with public capital and environmental pollution. As to pollution we assume that it is due to aggregate production. Pollution does not have direct effects as concerns production possibilities but it only reduces utility of the household. The paper then studies growth effects of fiscal policy for the model on the balanced growth path and taking into account transition dynamics. Further, welfare effects of fiscal policy are analyzed and it is demonstrated that the growth maximizing values of tax rates may be different from those values which maximize the long run balanced growth rate.

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# 1 Introduction

One strand in endogenous growth theory assumes that the government can invest in productive public capital which stimulates aggregate productivity. This approach goes back to Arrow and Kurz (1970) who presented exogenous growth models containing that assumption in their book. The first model in which productive public spending leads to sustained per capita growth in the long run was presented by Barro (1990). In his model, productive public spending positively affects the marginal product of private capital and makes the long run growth rate an endogenous variable. However, the assumption that public spending as a flow variable affects aggregate production possibilities is less plausible from an empirical point of view as pointed out in a study by Aschauer (1989).

Futagami et al. (1993) have extended the Barro model by assuming that public capital as a stock variable shows positive productivity effects and then investigated whether the results derived by Barro are still valid given their modification of the Barro model. As to the question of whether public spending can affect aggregate production possibilities at all the empirical studies do not reach unambiguous results. However, this is not too surprising since these studies often consider different countries over different time periods and the effect of public investment in infrastructure for example is likely to differ over countries and over time. A good survey of the empirical studies dealing with that subject can be found in Sturm et al. (1998).

Another line of research in economics are studies which try to understand the links between economic growth and pollution. Basically, there are two different ways of integrating the environment in economic models. On the one hand, models have been constructed in which economic activities generate environmental degradation. That line of research goes back to Forster (1973) and was extended by Gruver (1976).

On the other hand, there are so-called resource models in which a stock of natural resources is exploited in order to take up production. From the technical point of view, an equivalent formulation is the assumption that economic activities lead to pollution which

negatively affects the environmental quality. Models of this type within an endogenous growth framework are for example in the papers by Bovenberg and Smulders (1995) or Gradus and Smulders (1993)<sup>1</sup>.

Most of the models dealing with environmental quality or pollution assume that pollution or the use of resources influences production possibilities either through affecting the accumulation of human capital or by directly entering the production function. In this paper we pursue a different approach. We intend to analyze a growth model where pollution only affects utility of a representative household but does not affect production possibilities directly by entering the aggregate production function. However, there is an indirect effect of pollution on output because we suppose that resources are used for abatement activities. As concerns pollution we assume that it is an inevitable by-product of production and that it can be reduced to a certain degree by investing in abatement activities. As to the growth rate we suppose that it is determined endogenously and that public investment in a productive public capital stock brings about sustained long run per capita growth. Thus, we adopt that type of endogenous growth models which was initiated by Barro (1990) and Futagami et al. (1993) we mentioned above.

Within this framework we intend to analyze how the long run balanced growth rate reacts to fiscal policy and to the introduction of a less polluting technology. Further, we also study the effects of fiscal policy taking into account transition dynamics and we analyze welfare effects of fiscal policy for the model on the balanced growth path.

The rest of the paper is organized as follows. In section 2 we present the structure of our model and in section 3 we solve the model and show that there exists a unique balanced growth path. Section 4 analyzes growth effects of fiscal policy and growth effects of introducing a cleaner production technology for our model on the balanced growth path. Further, we also study the effects of fiscal policy as to the growth rates on the transition path. In section 5 we analyze welfare effects and section 6 concludes the paper.

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<sup>1</sup>For a good survey of how to model pollution in growth models see Smulders (1995) or Hettich (2000).

## 2 The economic model

We consider an economy with a household sector, a productive sector, and the government. First, we describe the household sector.

### 2.1 The household

Our economy is represented by one household. The goal of this household is to maximize a discounted stream of utility arising from consumption  $C(t)$  over an infinite time horizon subject to its budget constraint:

$$\max_{C(t)} \int_0^{\infty} e^{-\rho t} V(t) dt, \quad (1)$$

with  $V(t)$  the instantaneous subutility function which depends positively on the level of consumption and negatively on effective pollution.  $V(t)$  takes the logarithmic form  $V(t) = \ln C(t) - \ln P_E(t)$ , with  $\ln$  giving the natural logarithm.<sup>2</sup>  $\rho$  in (1) gives the subjective discount rate.

The budget constraint is given by<sup>3</sup>

$$\dot{K} = (w + rK)(1 - \tau) - C. \quad (2)$$

The budget constraint (2) states that the individual has to decide how much to consume and how much to save, thus increasing consumption possibilities in the future. The depreciation of physical capital is assumed to equal zero.

$w$  in the budget constraint is the wage rate. The labour supply  $L$  is constant, supplied inelastically, and we normalize  $L \equiv 1$ .  $r$  is the return to per capita capital  $K$  and  $\tau \in (0, 1)$  gives the income tax rate.

To derive necessary conditions we formulate the Hamiltonian function as  $\mathcal{H}(\cdot) = \ln C - \ln P_E + \lambda(-C + (w + rK)(1 - \tau))$ , with  $\lambda$  the costate variable. The necessary optimality

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<sup>2</sup>For a survey of how to incorporate pollution in the utility function see Smulders (1995), p. 328-29.

<sup>3</sup>In what follows we will suppress the time argument if no ambiguity arises.

conditions are given by

$$\lambda = C^{-1}, \quad (3)$$

$$\dot{\lambda}/\lambda = \rho - r(1 - \tau), \quad (4)$$

$$\dot{K} = -C + (w + rK)(1 - \tau). \quad (5)$$

Since the Hamiltonian is concave in  $C$  and  $K$  jointly, the necessary conditions are also sufficient if in addition the transversality condition at infinity  $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda(t) K(t) \geq 0$  is fulfilled. Moreover, strict concavity in  $C$  also guarantees that the solution is unique (cf. Seierstad and Sydsaeter (1987), pp. 234-235).

## 2.2 The productive sector

The productive sector in our economy is represented by one firm which chooses inputs in order to maximize profits and which behaves competitively. As to pollution  $P(t)$ , we suppose that it is a by-product of aggregate production  $Y$ . In particular, we assume that  $P(t) = \varphi Y(t)$ , with  $\varphi = \text{const.} > 0$ . Thus, we follow the line invited by Forster (1973) and worked out in more details by Luptacik and Schubert (1982).

Effective pollution  $P_E$  which affects utility of the household is that part of pollution which remains after investing in abatement activities. This means that abatement activities reduce pollution but cannot eliminate it completely. As to the modeling of effective pollution we follow Gradus and Smulders (1993) and Lighthart and van der Ploeg (1994) and take the following specification

$$P_E = \frac{P}{A^\beta}, \quad 0 < \beta \leq 1. \quad (6)$$

The limitation  $\beta \leq 1$  assures that a positive growth rate of aggregate production goes along with an increase in effective pollution,  $\beta < 1$ , or leaves effective pollution unchanged,  $\beta = 1$ . We make that assumption because we think that it is realistic to assume that higher production also leads to an increase in pollution, although at a lower rate because of abatement. Looking at the world economy that assumption is certainly justified.

Pollution is taxed at the rate  $\tau_p > 0$  and the firm takes into account that one unit of output causes  $\varphi$  units of pollution for which it has to pay  $\tau_p\varphi$  per unit of output. The per capita production function is given by,

$$Y = K^\alpha H^{1-\alpha}, \quad (7)$$

with  $H$  denoting the stock of productive public capital and  $\alpha \in (0, 1)$  gives the per capita capital share. Recall that  $K$  denotes per capita capital and that  $L$  is normalized to one.

Assuming competitive markets and taking public capital as given the first-order conditions for a profit maximum are obtained as

$$w = (1 - \tau_p\varphi)(1 - \alpha)K^\alpha H^{1-\alpha}, \quad (8)$$

$$r = (1 - \tau_p\varphi)\alpha K^{\alpha-1} H^{1-\alpha}. \quad (9)$$

### 2.3 The government

The government in our economy uses resources for abatement activities  $A(t)$  which reduce total pollution. Abatement activities  $A \geq 0$  are financed by the tax revenue coming from the tax on pollution, i.e.  $A(t) = \eta\tau_p P(t)$ , with  $\eta > 0$ . If  $\eta < 1$  not all of the pollution tax revenue is used for abatement activities and the remaining part is spent for public investment in the public capital stock  $I_p$ ,  $I_p \geq 0$ , in addition to the tax revenue resulting from income taxation. For  $\eta > 1$  a certain part of the tax revenue resulting from the taxation of income is used for abatement activities in addition to the tax revenue which is gained by taxing pollution. As to the interpretation of public capital one can think of pure public like infrastructure capital. However, one could also interpret public capital in a broader sense so that it also includes human capital which is built up as a result of public education.

The government in our economy runs a balanced budget at any moment in time. Thus, the budget constraint of the government is written as

$$I_p = \tau_p P(1 - \eta) + w\tau_w + rK\tau_K. \quad (10)$$

The evolution of public capital is described by

$$\dot{H} = I_p, \quad (11)$$

where for simplicity we again assume that there is no depreciation of public capital. As to the governmental decision rules, we do not try to find out the second best optimal level for the tax rates or the amount of abatement activities nor the socially optimal decisions for consumption and the fiscal parameters. Instead, we only consider how the growth rate reacts to changes in fiscal policy. This seems to be of higher relevance for real world economies with a democratic government because government behaviour may be hampered by bureaucracy and by political or institutional constraints (as to this argumentation see also van Ewijk and van de Klundert (1993)).

### 3 Equilibrium conditions and the balanced growth path

Combining the budget constraint of the government and the equation describing the evolution of public capital over time, the accumulation of public capital can be written as  $\dot{H} = -\eta\varphi\tau_p K^\alpha H^{1-\alpha} + \tau_p\varphi K^\alpha H^{1-\alpha} + \tau(w + rK) = K^\alpha H^{1-\alpha}(\varphi\tau_p(1 - \eta) + (1 - \varphi\tau_p)\tau)$ , where we have used (8) and (9). To obtain the other differential equations describing our economy we note that the growth rate of private consumption is obtained from (3) and (4), with  $r$  taken from (9) and where we have used  $\dot{P}_E/P_E = (1 - \beta)\dot{Y}/Y$ . Using (8) and (9)  $\dot{K}/K$  is obtained from (5). It should be noted that the accumulation of public capital which is positive for  $I_p > 0$  is the source of sustained economic growth in our model and makes the growth rate an endogenous variable.

Thus, the dynamics of our model are completely described by the following differential equation system:

$$\frac{\dot{C}}{C} = -\rho + (1 - \tau)(1 - \varphi\tau_p)\alpha \left(\frac{H}{K}\right)^{1-\alpha}, \quad (12)$$



$$\frac{\dot{K}}{K} = -\frac{C}{K} + \left(\frac{H}{K}\right)^{1-\alpha} (1 - \varphi\tau_p)(1 - \tau), \quad (13)$$

$$\frac{\dot{H}}{H} = \left(\frac{H}{K}\right)^{-\alpha} (\varphi\tau_p(1 - \eta) + (1 - \varphi\tau_p)\tau). \quad (14)$$

The initial conditions  $K(0)$  and  $H(0)$  are given and fixed and  $C(0)$  can be chosen freely by the economy. Further, the transversality condition  $\lim_{t \rightarrow \infty} e^{-\rho t} K(t)/C(t) \geq 0$  must be fulfilled.

In the following we will first examine our model as to the existence and stability of a balanced growth path (BGP). To do so, we define a BGP.

**Definition** A balanced growth path (BGP) is a path such that  $\dot{C}/C = \dot{K}/K = \dot{H}/H \equiv g > 0$  holds, with  $g$  constant and  $C$ ,  $K$  and  $H$  strictly positive.

This definition shows that on a BGP the growth rates of economic variables are positive and constant over time. It should be noticed that aggregate output and pollution grow at the same rate on the BGP. This implies that effective pollution is not constant in the long run (unless  $\beta = 1$  holds). Nevertheless, one may say that the BGP is sustainable if one adopts the definition given in Byrne (1997). There, sustainable growth is given if  $\dot{V}$  is positive. For our model this holds on the BGP since  $\dot{V} = \dot{C}/C - \dot{P}_E/P_E = \dot{C}/C - (1 - \beta)\dot{Y}/Y = \beta g > 0$ .

To analyze our model further, we first have to perform a change of variables. Defining  $c = C/K$  and  $h = H/K$  and differentiating these variables with respect to time we get  $\dot{c}/c = \dot{C}/C - \dot{K}/K$  and  $\dot{h}/h = \dot{H}/H - \dot{K}/K$ . A rest point of this new system then corresponds to a BGP of our original economy where all variables grow at the same constant rate. The system describing the dynamics around a BGP is given by

$$\dot{c} = c \left( c - \rho - (1 - \alpha)(1 - \tau)(1 - \varphi\tau_p)h^{1-\alpha} \right), \quad (15)$$

$$\dot{h} = h \left( c - h^{1-\alpha}(1 - \varphi\tau_p)(1 - \tau) + h^{-\alpha}(\varphi\tau_p(1 - \eta) + (1 - \varphi\tau_p)\tau) \right). \quad (16)$$

Concerning a rest point of system (15) and (16) it should be noted that we only consider interior solution. That means that we exclude the economically meaningless stationary

point  $c = h = 0$ . As to the uniqueness and stability of a BGP we can state proposition 1.

**Proposition 1** *Assume that  $\tau_p \varphi < 1$  and  $(1 - \tau_p \varphi)\tau + (1 - \eta)\tau_p \varphi > 0$ . Then there exists a unique BGP which is saddle point stable.*

*Proof:* To prove that proposition we first calculate  $c^\infty$  on a BGP which is obtained from  $\dot{h}/h = 0$  as

$$c^\infty = h^{1-\alpha}(1 - \varphi\tau_p)(1 - \tau) - h^{-\alpha}(\varphi\tau_p(1 - \eta) + (1 - \varphi\tau_p)\tau).$$

Inserting  $c^\infty$  in (15) gives after some modifications

$$f(\cdot) \equiv \dot{c}/c = -\rho + (1 - \tau)(1 - \varphi\tau_p)\alpha h^{1-\alpha} - h^{-\alpha}(\varphi\tau_p(1 - \eta) + (1 - \varphi\tau_p)\tau),$$

with  $\lim_{h \rightarrow 0} f(\cdot) = -\infty$  (for  $I_p > 0$ ) and  $\lim_{h \rightarrow \infty} f(\cdot) = \infty$ . A rest point for  $f(\cdot)$ , i.e. a value for  $h$  such that  $f(\cdot) = 0$  holds, then gives a BGP for our economy. Further, we have

$$\partial f(\cdot)/\partial h = (1 - \tau)(1 - \varphi\tau_p)(1 - \alpha)\alpha h^{-\alpha} + \alpha h^{-\alpha-1}(\varphi\tau_p(1 - \eta) + (1 - \varphi\tau_p)\tau) > 0,$$

for  $I_p > 0$ . Note that on a BGP  $\dot{H}/H > 0$  must hold implying  $I_p > 0$  and, thus,  $\varphi\tau_p(1 - \eta) + (1 - \varphi\tau_p)\tau > 0$ .  $\partial f/\partial h > 0$  for  $h$  such that  $f(\cdot) = 0$  means that  $f(\cdot)$  cannot intersect the horizontal axis from above. Consequently, there exists a unique  $h^\infty$  such that  $f(\cdot) = 0$  and, therefore, a unique BGP.

The saddle point property is shown as follows. Denoting with  $J$  the Jacobi matrix of (15) and (16) evaluated at the rest point we first note that  $\det J < 0$  is a necessary and sufficient condition for saddle point stability, i.e. for one negative and one positive eigenvalue. The Jacobian in our model can be written as

$$J = \begin{bmatrix} c & -c h^{-\alpha}(1 - \alpha)^2(1 - \tau)(1 - \tau_p \varphi) \\ h & -v \end{bmatrix},$$

with

$$v = (1 - \alpha)h^{1-\alpha}(1 - \tau)(1 - \tau_p \varphi) + \alpha h^{-\alpha}(\varphi\tau_p(1 - \eta) + (1 - \varphi\tau_p)\tau).$$

The determinant can be calculated as

$$\det J = -ch\alpha \left( h^{-\alpha-1}(\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)\tau) + (1-\alpha)h^{-\alpha}(1-\tau)(1-\tau_p\varphi) \right) < 0.$$

Thus, proposition 1 is proved.  $\square$

That proposition states that our model is both locally and globally determinate, i.e. there exists a unique value for  $c(0)$  such that the economy converges to the BGP in the long run.<sup>4</sup> The assumption  $(1-\varphi\tau_p) > 0$  is necessary for a positive growth rate of consumption and is sufficient for a positive value of  $c^\infty$ .<sup>5</sup> The second assumption  $(\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)\tau) > 0$  must hold for a positive growth rate of public capital. It should be noted that  $(\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)\tau) = I_p/Y$  stating that the second assumption in proposition 1 means that on a BGP the ratio of public investment to GDP must be positive.

## 4 Growth effects of fiscal policy

In the last section we demonstrated in proposition 1 that there is a unique BGP under slight additional assumptions. Thus our model including the transition dynamics is completely characterized. In this section we will analyze how the growth rate in our economy reacts to fiscal policy. We do this for the model on the BGP and taking into account transition dynamics.

### 4.1 The BGP

Before we analyze growth effects of fiscal policy we study effects of introducing a less polluting production technology, i.e. the impact of a decline in  $\varphi$ .

The balanced growth rate which we denote with  $g$  is given by (14) as

$$g = \dot{H}/H = (h)^{-\alpha} (\varphi\tau_p(1-\eta) + (1-\varphi\tau_p)\tau).$$

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<sup>4</sup>For a definition of local and global determinacy see e.g. Benhabib and Perli (1994) or Benhabib, Perli, and Xie (1994).

<sup>5</sup>This is realized if  $c^\infty$  is calculated from  $\dot{c}/c = 0$  as  $c^\infty = \rho + (1-\alpha)(1-\tau)(1-\tau_p\varphi)h^{1-\alpha}$ .

Differentiating  $g$  with respect to  $\varphi$  gives

$$\frac{\partial g}{\partial \varphi} = h^{-\alpha} \tau_p (1 - \eta - \tau) - (\varphi \tau_p (1 - \eta) + (1 - \varphi \tau_p) \tau) \alpha h^{-\alpha-1} \frac{\partial h}{\partial \varphi}.$$

$\partial h / \partial \varphi$  is obtained by implicit differentiation from  $f(\cdot) = 0$  (from the proof of proposition 1) as

$$\frac{\partial h}{\partial \varphi} = \frac{\tau_p (1 - \eta - \tau) + \tau_p (1 - \tau) \alpha h}{\alpha (1 - \tau) (1 - \varphi \tau_p) (1 - \alpha) + \alpha h^{-1} (\varphi \tau_p (1 - \eta) + (1 - \varphi \tau_p) \tau)}.$$

For  $(1 - \eta - \tau) = 0$  we get  $\partial g / \partial \varphi < 0$ . To get results for  $(1 - \eta - \tau) \neq 0$  we insert  $\partial h / \partial \varphi$  in  $\partial g / \partial \varphi$ . That gives

$$\begin{aligned} \frac{\partial g}{\partial \varphi} &= h^{-\alpha} \tau_p (1 - \eta - \tau) \cdot \\ &\left( 1 - \frac{(\varphi \tau_p (1 - \eta) + (1 - \varphi \tau_p) \tau) [(1 - \eta - \tau) + h \alpha (1 - \tau)]}{(1 - \eta - \tau) [(\varphi \tau_p (1 - \eta) + (1 - \varphi \tau_p) \tau) + h (1 - \tau) (1 - \tau_p \varphi) (1 - \alpha)]} \right). \end{aligned}$$

From that expression it can be seen that the expression in brackets is always positive for  $(1 - \eta - \tau) < 0$  such that  $\partial g / \partial \varphi < 0$ . For  $(1 - \eta - \tau) > 0$  it is immediately seen that

$$\frac{\partial g}{\partial \varphi} > = < 0 \Leftrightarrow (1 - \varphi \tau_p) (1 - \alpha) (1 - \eta - \tau) > = < \alpha (\varphi \tau_p (1 - \eta) + (1 - \varphi \tau_p) \tau),$$

which simplifies to

$$\frac{\partial g}{\partial \varphi} > = < 0 \Leftrightarrow (1 - \eta) (1 - \alpha) > = < \varphi \tau_p (1 - \eta) + (1 - \varphi \tau_p) \tau.$$

The r.h.s. of that expression is equivalent to  $I_P / Y$ . Thus we have proved the following proposition.

**Proposition 2** *If  $(1 - \eta - \tau) \leq 0$  the use of a less polluting technology raises the balanced growth rate. For  $(1 - \eta - \tau) > 0$  the use of a less polluting technology raises (leaves unchanged, lowers) the balanced growth rate if*

$$\frac{I_P}{Y} > (=, <) (1 - \alpha) (1 - \eta).$$

To interpret that result we first note that a cleaner production technology (i.e. a lower  $\varphi$ ) shows two different effects: on the one hand it implies that less resources are needed for abatement activities leaving more resources for public investment. That effect leads to a higher ratio  $H/K$ , thus raising the marginal product of private capital  $r$  in (9). That is the return on investment rises. Further, a less polluting technology implies that the firm has to pay less pollution taxes (the term  $(1 - \tau_p\varphi)$  rises) which has also a stimulating effect on  $r$ , which can be seen from (9) and which also raises the incentive to invest. On the other hand, however, less pollution implies that the tax revenue resulting from the taxation of pollution declines and, thus, productive public spending. That effect tends to lower the ratio  $H/K$  and, therefore, the marginal product of private capital. This tends to lower the balanced growth rate.

If  $\eta \geq 1 - \tau$ , i.e. if much of the pollution tax is used for abatement activities a cleaner technology always raises the balanced growth rate. In that case, the negative growth effect of a decline in the pollution tax revenue is not too strong since most of that revenue is used for abatement activities which are non-productive anyway. If, however,  $\eta < 1 - \tau$ , i.e. a good deal of the pollution tax is used for productive government spending, a cleaner technology may either raise or lower economic growth. It increases the balanced growth rate if the share of public investment per GDP is larger than a constant which positively depends on the elasticity of aggregate output with respect to public capital and negatively on  $\eta$ , and vice versa.

Let us next study growth effects of varying the income tax rate. Proposition 3 demonstrates that a rise in that tax may have positive or negative growth effects and that there exists a growth maximizing income tax rate.

**Proposition 3** *Assume that there exists an interior growth maximizing income tax rate. Then this tax rate is given by*

$$\tau = (1 - \alpha) - \alpha\varphi\tau_p(1 - \eta)/(1 - \varphi\tau_p).$$

*Proof:* To calculate growth effects of varying  $\tau$  we take the balanced growth rate  $g$  from

(14) and differentiate it with respect to that parameter. Doing so gives

$$\frac{\partial g}{\partial \tau} = h^{-\alpha}(1 - \tau_p \varphi) \left( 1 - \frac{\alpha((1 - \tau_p \varphi)\tau + (1 - \eta)\tau_p \varphi)}{1 - \tau_p \varphi} \frac{\partial h}{\partial \tau} \frac{1}{h} \right),$$

where  $\partial h/\partial \tau$  is obtained by implicit differentiation from  $f(\cdot) = 0$  leading to

$$\frac{\partial h}{\partial \tau} = \frac{(1 - \varphi \tau_p)(1 + \alpha h)h}{h(1 - \tau)(1 - \varphi \tau_p)(1 - \alpha) + \alpha((1 - \tau_p \varphi)\tau + (1 - \eta)\tau_p \varphi)}.$$

Inserting  $\partial h/\partial \tau$  in  $\partial g/\partial \tau$  we get

$$\frac{\partial g}{\partial \tau} = h^{-\alpha}(1 - \tau_p \varphi) \left( 1 - \frac{((1 - \tau_p \varphi)\tau + (1 - \eta)\tau_p \varphi)(1 + \alpha h)}{h(1 - \tau)(1 - \varphi \tau_p)(1 - \alpha) + ((1 - \tau_p \varphi)\tau + (1 - \eta)\tau_p \varphi)} \right),$$

showing that

$$\frac{\partial g}{\partial \tau} > = < 0 \Leftrightarrow (1 - \tau)(1 - \varphi \tau_p)(1 - \alpha) > = < \alpha((1 - \tau_p \varphi)\tau + (1 - \eta)\tau_p \varphi).$$

Solving for  $\tau$  gives

$$\frac{\partial g}{\partial \tau} > = < 0 \Leftrightarrow \tau < = > (1 - \alpha) - \alpha \varphi \tau_p (1 - \eta) / (1 - \varphi \tau_p)$$

That shows that the balanced growth rate rises with increases in  $\tau$  as long as  $\tau$  is smaller than the expression on the r.h.s. which is constant.  $\square$

That proposition shows that the growth maximizing income tax rate does not necessarily equal zero in our model which was to be expected since the government finances productive public spending with the tax revenue. There are two effects of going along with variations of the income tax rate: on the one hand, a higher income tax lowers the marginal product of private capital and, therefore, is a disincentive for investment. On the other hand, the government finances productive public spending with its tax revenue leading to a rise in the ratio  $H/K$ , which raises the marginal product of private capital  $r$  and which has, as a consequence, a positive effect on economic growth. However, boundary solutions, i.e.  $\tau_K = 0$  or  $\tau_K = 1$ , cannot be excluded. Whether there exists an interior or a boundary solution for the growth maximizing capital income tax rate depends on the numerical specification of the parameters  $\varphi$ ,  $\tau_p$ , and  $\eta$ . Only for  $\varphi \tau_p = 0$  or  $\eta = 1$  the

growth maximizing tax rate is always in the interior of  $(0, 1)$  and equal to the elasticity of aggregate output with respect to public capital.

As to the tax on pollution the growth maximizing income tax rate negatively varies with that tax if  $\eta < 1$ . For  $\eta > 1$  the growth maximizing income tax rate is the higher the higher the tax on pollution  $\tau_p$ . The interpretation of that result is as follows: if  $\eta < 1$  the government uses a part of the pollution tax revenue for the creation of public capital which has positive growth effects. Increasing the tax on pollution implies that a part of the additional tax revenue is used for productive investment in public capital. Consequently, the income tax rate can be reduced without having negative growth effects. It should be noticed that a decrease in the income tax rate shows an indirect positive growth effect because it implies a reallocation of private resources from consumption to investment. In contrast to that, if  $\eta > 1$  the whole pollution tax revenue is used for abatement activities. Raising the pollution tax rate in that situation implies that the additional tax revenue is used only for abatement activities but not for productive public spending. Consequently, the negative indirect growth effect of a higher pollution tax (through decreasing the return on capital  $r$ ) must be compensated by an increase in the income tax rate. It should be noted that the latter also has a negative indirect growth effect but that one is dominated in this case by the positive direct growth effect of higher productive public spending.

Let us next analyze long run growth effects of a rise in the pollution tax rate. Proposition 4 gives the result.

**Proposition 4** *For  $(1 - \eta - \tau) \leq 0$  a rise in the pollution tax rate always lowers the balanced growth rate. If  $(1 - \eta - \tau) > 0$  the pollution tax rate maximizing the balanced growth rate is determined by*

$$\tau_p = \left( \frac{1}{\varphi} \right) \left( \frac{1 - \eta - \tau - \alpha(1 - \eta)}{1 - \eta - \tau} \right)$$

*which is equivalent to*

$$\frac{I_p}{Y} = (1 - \alpha)(1 - \eta).$$

*Proof:* To calculate growth effects of varying  $\tau_p$  we take the balanced growth rate  $g$  again from (14) and differentiate it with respect to that parameter. Doing so gives

$$\frac{\partial g}{\partial \tau_p} = h^{-\alpha} \varphi (1 - \eta - \tau) \cdot \left( 1 - \frac{(\varphi \tau_p (1 - \eta) + (1 - \varphi \tau_p) \tau) [(1 - \eta - \tau) + h \alpha (1 - \tau)]}{(1 - \eta - \tau) [(\varphi \tau_p (1 - \eta) + (1 - \varphi \tau_p) \tau) + h (1 - \tau) (1 - \tau_p \varphi) (1 - \alpha)]} \right).$$

From that expression it can be seen that the expression in brackets is always positive for  $(1 - \eta - \tau) < 0$  such that  $\partial g / \partial \tau_p < 0$ . For  $(1 - \eta - \tau) = 0$  the result can directly be seen by multiplying out the expression above. For  $(1 - \eta - \tau) > 0$  it is seen that

$$\frac{\partial g}{\partial \tau_p} > = < 0 \Leftrightarrow (1 - \varphi \tau_p) (1 - \alpha) (1 - \eta - \tau) > = < \alpha (\varphi \tau_p (1 - \eta) + (1 - \varphi \tau_p) \tau),$$

which simplifies to

$$\frac{\partial g}{\partial \tau_p} > = < 0 \Leftrightarrow \tau_p < = > \left( \frac{1}{\varphi} \right) \left( \frac{1 - \eta - \tau - \alpha (1 - \eta)}{1 - \eta - \tau} \right)$$

and is equivalent to

$$\frac{\partial g}{\partial \tau_p} > = < 0 \Leftrightarrow \frac{I_p}{Y} < = > (1 - \eta) (1 - \alpha).$$

Thus, the proposition is proved.  $\square$

The interpretation of that result is straightforward. An increase in the pollution tax rate always lowers the balanced growth rate if  $(1 - \eta - \tau) \leq 0$ . In that case, too much of the additional tax revenue (gained through the increase in  $\tau_p$ ) goes in abatement activities so that the positive growth effect of a higher pollution tax revenue (i.e. the increase in the creation of the stock of public capital) is dominated by the negative indirect one of a reduction of the rate of return to physical capital  $r$ . The latter effect namely implies a reallocation of private resources from investment to consumption which reduces economic growth. For  $(1 - \eta - \tau) > 0$ , however, there exists a growth maximizing pollution tax rate.<sup>6</sup> In that case, the pollution tax has to be set such that public investment per GDP

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<sup>6</sup>But it must kept in mind that  $1 - \tau_p \varphi > 0$  must hold so that a BGP exists. Therefore, the boundary condition  $\tau_p = \varphi^{-1} - \epsilon$ ,  $\epsilon > 0$ , cannot be excluded.



equals the elasticity of aggregate output with respect to public capital multiplied with that share of the pollution tax revenue which is not used for abatement activities but for productive public spending.

Further, it should be noticed that the growth maximizing value of  $\tau_p$ <sup>7</sup> is the higher the less of the pollution tax revenue is used for abatement activities. In the limit ( $\eta = 0$ ) we get the same result as in Barro (1990) and Futagami et al. (1993) that the growth maximizing share of public investment per GDP equals the elasticity of aggregate output with respect to public capital.

It should also be noted that the conditions for a positive growth effect of an increase in the pollution tax rate are just reverse to the conditions which must be fulfilled such that the introduction of a less polluting technology raises economic growth.

## 4.2 The model with transition dynamics

In this subsection we study how the growth rates of consumption and of public and private capital react to a change in the income tax and pollution tax rate taking into account transition dynamics. To do this we proceed as follows. We assume that initially the economy is on the BGP when the government changes the tax rates at time  $t = 0$  and then we characterize the transition path to the new BGP which is attained in the long run.

First, we consider the effects of an increase in the income tax rate  $\tau$ . To do this we state that the  $\dot{c} = 0$  and  $\dot{h} = 0$  isoclines are given by

$$c|_{\dot{c}=0} = \rho + (1 - \alpha)(1 - \tau)(1 - \tau_p\varphi)h^{1-\alpha}, \quad (17)$$

$$c|_{\dot{h}=0} = h^{1-\alpha}(1 - \tau)(1 - \tau_p\varphi) - h^{-\alpha}((1 - \tau_p\varphi)\tau + (1 - \eta)\tau_p\varphi). \quad (18)$$

Calculating the derivative  $dc/dh$  it can easily be seen that the  $\dot{h} = 0$  isocline is steeper than the  $\dot{c} = 0$  isocline. Further, for the  $\dot{c} = 0$  isocline we have  $c = \rho$  for  $h = 0$  and

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<sup>7</sup>Note that  $I_p/Y$  positively varies with  $\tau_p$  for  $(1 - \eta - \tau) > 0$ .

$c \rightarrow \infty$  for  $h \rightarrow \infty$ . For the  $\dot{h} = 0$  isocline we have  $c \rightarrow -\infty$  for  $h \rightarrow 0$ ,  $c = 0$  for  $h = ((1 - \tau_p\varphi)\tau + (1 - \eta)\tau_p\varphi)/((1 - \tau)(1 - \tau_p\varphi))$  and  $c \rightarrow \infty$  for  $h \rightarrow \infty$ . This shows that there exists a unique  $(c^\infty, h^\infty)$  where the two isoclines intersect.

If the income tax rate is increased it can immediately be seen that the  $\dot{h} = 0$  isocline shifts to the right and the  $\dot{c} = 0$  isocline turns right with  $c = \rho$  for  $h = 0$  remaining unchanged. This means that both on the new  $\dot{c} = 0$  and on the new  $\dot{h} = 0$  isocline any given  $h$  goes along with a lower value of  $c$  compared to the isoclines before the tax rate increase. This implies that the increase in the income tax rate raises the long run value  $h^\infty$  and may reduce or raise the long run value of  $c^\infty$ . Further, the capital stocks  $K$  and  $H$  are predetermined variables which are not affected by the tax rate increase at time  $t = 0$ . These variables react only gradually. This implies that  $\partial h(t = 0, \tau)/\partial \tau = 0$ . To reach the new steady state<sup>8</sup>  $(c^\infty, h^\infty)$  the level of consumption adjusts and jumps to the stable manifold implying  $\partial c(t = 0, \tau)/\partial \tau < 0$  in figure 1.

Figure 1 about here

Over time both  $c$  and  $h$  rise until the new BGP is reached at  $(c^\infty, h^\infty)$ . That is we get  $\dot{c}/c = \dot{C}/C - \dot{K}/K > 0$  and  $\dot{h}/h = \dot{H}/H - \dot{K}/K > 0$  implying that on the transition path the growth rates of consumption and of public capital are larger than that of private capital for all  $t \in [0, \infty)$ . The impact of a rise in  $\tau$  on the growth rate of private consumption is obtained from (12) as

$$\frac{\partial}{\partial \tau} \left( \frac{\dot{C}(t = 0, \tau)}{C(t = 0, \tau)} \right) = -h^{1-\alpha} \alpha (1 - \tau_p \varphi) < 0,$$

where we again use that  $K$  and  $H$  are predetermined variables implying  $\partial h(t = 0, \tau)/\partial \tau = 0$ . This shows that at  $t = 0$  the growth rate of private consumption reduces as a result of the increase in the income tax rate  $\tau$  and then rises gradually as the new BGP is approached. The same must hold for the private capital stock since we know from above

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<sup>8</sup>The economy in steady state means the same as the economy on the BGP.

that the growth rate of the private capital stock is smaller than that of consumption on the transition path. The impact of a rise in  $\tau$  on the growth rate of public capital is obtained from (14) as

$$\frac{\partial}{\partial \tau} \left( \frac{\dot{H}(t=0, \tau)}{H(t=0, \tau)} \right) = h^{-\alpha}(1 - \tau_p \varphi) > 0,$$

where we again use that  $h$  does not change at  $t = 0$ . This result states that the growth rate of public capital rises and then declines over time as the new BGP is approached. This result was to be expected since an increase in the income tax rate at a certain point in time means that the instantaneous tax revenue rises. Since a certain part of the additional tax revenue is spent for public investment the growth rate of public capital rises.

We summarize the results of our considerations in the following proposition.

**Proposition 5** *Assume that the economy is on the BGP. Then, a rise in the income tax rate leads to a temporary decrease in the growth rates of consumption and of private capital but to a temporary increase in the growth rate of public capital. Further, on the transition path the growth rates of public capital and of consumption exceed the growth rate of private capital.*

Next, we analyze the effects of a rise in the pollution tax rate  $\tau_p$ . To do so we proceed analogously to the case of the income tax rate. Doing the analysis it turns out that we have to distinguish between two cases. If  $1 - \eta - \tau > 0$  the results are equivalent to those we derived for an increase in the income tax rate.<sup>9</sup> If  $1 - \eta - \tau < 0$  two different scenarios are possible.<sup>10,11</sup> First, the long run values  $h^\infty$  and  $c^\infty$  decline. Analogously to a rise in the income tax rate the  $\dot{c} = 0$  isocline turns right with  $c = \rho$  for  $h = 0$  remaining unchanged. Further, the new  $\dot{h} = 0$  isocline lies above (below) the old  $\dot{h} = 0$  isocline, i.e.

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<sup>9</sup>A detailed derivation is available on request.

<sup>10</sup>For  $1 - \eta - \tau = 0$  the analysis is equivalent to that of a rise in the income tax rate with the only difference that  $\partial(\dot{H}(t=0, \tau)/H(t=0, \tau))/\partial \tau = 0$  holds.

<sup>11</sup>Note that in this case the balanced growth rate declines.

the isocline before the increase in  $\tau_p$ , for  $h < (>) - (1 - \eta - \tau)/(1 - \tau)$ . Since  $K$  and  $H$  are predetermined values, the level of consumption must decrease and jump to the stable manifold implying  $\partial c(t = 0, \tau)/\partial \tau < 0$  to reach the new steady state  $(c^\infty, h^\infty)$ . Figure 2 shows the phase diagram.

Figure 2 about here

Over time both  $c$  and  $h$  decline until the new BGP is reached at  $(c^\infty, h^\infty)$ . That is we get  $\dot{c}/c = \dot{C}/C - \dot{K}/K < 0$  and  $\dot{h}/h = \dot{H}/H - \dot{K}/K < 0$  implying that on the transition path the growth rates of consumption and of public capital are smaller than that of private capital for all  $t \in [0, \infty)$ . The impact of a rise in  $\tau_p$  on the growth rate of private consumption is obtained from (12) as

$$\frac{\partial}{\partial \tau} \left( \frac{\dot{C}(t = 0, \tau_p)}{C(t = 0, \tau_p)} \right) = -h^{1-\alpha} \alpha (1 - \tau) \varphi < 0,$$

where we again use that  $K$  and  $H$  are predetermined variables implying  $\partial h(t = 0, \tau)/\partial \tau = 0$ . This shows that at  $t = 0$  the growth rate of private consumption falls as a result of the increase in the tax rate  $\tau_p$  and then continues to decline gradually as the new BGP is approached. The impact of a rise in  $\tau_p$  on the growth rate of public capital is obtained from (14) as

$$\frac{\partial}{\partial \tau} \left( \frac{\dot{H}(t = 0, \tau)}{H(t = 0, \tau)} \right) = h^{-\alpha} (1 - \eta - \tau) \varphi < 0, \quad \text{for } 1 - \eta - \tau < 0$$

where we again use that  $h$  does not change at  $t = 0$ . This result states that the growth rate of public capital declines and then continues to decline over time as the new BGP is approached. As with a rise of the income tax rate a higher pollution tax rate implies an instantaneous increase of the tax revenue. However, if  $\eta$  is relatively large, so that  $1 - \eta - \tau < 0$ , a large part of the additional tax revenue is used for abatement activities so that the growth rate of public capital declines although the tax revenue rises. The growth rate of the private capital stock may rise or decline. What we can say as to the

the growth rate of the private capital stock on the transition path is that it is always larger than those of consumption and of public capital.

Second, the long run value  $h^\infty$  rises while  $c^\infty$  may rise or fall. In this case, the phase diagram is the same as the one in figure 1 with the exception that the  $\dot{h} = 0$  isocline before and after the rise in the tax rate intersect at  $h = -(1 - \eta - \tau)/(1 - \tau)$ . Another difference to the effects of a rise in the income tax rate is that the growth rate of public capital at  $t = 0$  declines. The rest of the analysis is analogous to that of a rise in the income tax rate. In particular, we have again  $\dot{c}/c = \dot{C}/C - \dot{K}/K > 0$  and  $\dot{h}/h = \dot{H}/H - \dot{K}/K > 0$ .

We can summarize our results in the following proposition.

**Proposition 6** *Assume that the economy is on the BGP. Then, a rise in the pollution tax rate shows the same temporary effects as concerns the growth rates of consumption, of private capital and of public capital as a rise in the income tax rate if  $1 - \eta - \tau > 0$ . If  $1 - \eta - \tau < 0$  two situations are feasible: First,  $h^\infty$  declines and the temporary growth rates of consumption and of public capital decline while the growth rate of private capital may rise or fall. Further, the temporary growth rates of consumption and of public capital are smaller than that of private capital. Second,  $h^\infty$  rises and the temporary growth rates of consumption, of public capital and of private capital fall. Further, on the transition path the growth rates of public capital and of consumption exceed the growth rate of private capital.*

In the next section we analyze welfare effects of fiscal policy assuming that the economy is on the BGP.

## 5 Welfare effects of fiscal policy

In analyzing welfare effects we confine our considerations to the model on the BGP. That is we assume that the economy immediately jumps to the new BGP after a change in

fiscal parameters. In particular, we are interested in the question of whether growth and welfare maximization are identical goals.

To derive the effects of fiscal policy on the BGP arising from increases in tax rates at  $t = 0$  we first compute (1) on the BGP as

$$J(\cdot) \equiv \arg \max_{C(t)} \int_0^\infty e^{-\rho t} (\ln C(t) - \ln P_E(t)) dt. \quad (19)$$

Denoting the balanced growth rate by  $g$ , (19) can be rewritten as

$$J(\cdot) = \rho^{-1} \left( \ln C_0 + \beta g/\rho + \beta \ln \eta + \ln \tau_p - (1 - \beta) \ln \varphi - (1 - \beta) \ln K_0^\alpha H_0^{1-\alpha} \right), \quad (20)$$

with  $C_0 = C(0)$ ,  $K_0 = K(0)$  and  $H_0 = H(0)$ . From (12) and (13) we get

$$g = \alpha(1 - \tau)(1 - \tau_p \varphi) h^{1-\alpha} - \rho \quad \text{and} \quad C_0 = K_0 \left( (1 - \tau)(1 - \tau_p \varphi) h_0^{1-\alpha} - g \right).$$

Combining these two expressions leads to  $C_0 = K_0(\rho + g(1 - \alpha))/\alpha$ . Inserting  $C_0$  in (20)  $J$  can be written as

$$J(\cdot) = \rho^{-1} \left( \ln(\rho/\alpha + g(1 - \alpha)/\alpha) + \beta g/\rho + \ln \tau_p + C_1 \right), \quad (21)$$

with  $C_1$  a constant given by  $C_1 = \ln K_0 + \beta \ln \eta - (1 - \beta) \ln \varphi - (1 - \beta) \ln(K_0^\alpha H_0^{1-\alpha})$ . (21) shows that welfare in our economy positively varies with the growth rate on the BGP, i.e. the higher the growth rate the higher welfare. Differentiating (21) with respect to  $\tau$  and  $\tau_p$  yields

$$\frac{\partial J}{\partial \tau} = \frac{\partial g}{\partial \tau} \left( c_0 \frac{1 - \alpha}{\alpha \cdot \rho} + \frac{\beta}{\rho^2} \right), \quad \frac{\partial J}{\partial \tau_p} = \frac{1}{\tau_p \rho} + \frac{\partial g}{\partial \tau_p} \left( c_0 \frac{1 - \alpha}{\alpha \cdot \rho} + \frac{\beta}{\rho^2} \right). \quad (22)$$

With the expressions in (22) we can summarize our results in the following proposition.

**Proposition 7** *Assume that the economy is in steady state and that there exist interior growth maximizing values for the income and pollution tax rates. Then, the welfare maximizing pollution tax rate is larger than the growth maximizing rate and the welfare maximizing income tax rate is equal to the growth maximizing income tax rate.*

*Proof:* The fact that the growth maximizing income tax rate also maximizes welfare follows immediately from (22). Since the pollution tax rate  $\tau_p$  maximizes the balanced growth we have  $\partial g/\partial\tau_p = 0$ . But (22) shows that for  $\partial g/\partial\tau_p = 0$ ,  $\partial J/\partial\tau_p > 0$  holds. Thus, the proposition is proved.  $\square$

This proposition states that welfare maximization may be different from growth maximization. If the government sets the income tax rate it can be assured that the income tax rate which maximizes economic growth also maximizes welfare if one neglects transition dynamics. However, if the government chooses the pollution tax rate it has to set the pollution tax rate higher than that value which maximizes the balanced growth rate in order to achieve maximum welfare. The reason for this outcome is that the pollution tax rate exerts a direct positive welfare effect by reducing effective pollution in contrast to the income tax rate. This is seen from (21) where the expression  $\ln \tau_p$  appears explicitly but  $\tau$  does not.

The same also holds for variation of the parameter  $\varphi$  which determines the degree of pollution as a by-product of aggregate production. If  $\varphi$  declines, meaning that production becomes cleaner, there is always a positive partial and direct welfare effect going along with that effect. Again, this can be seen from (21) where  $\ln \varphi$  appears. The overall effect of a decline in  $\varphi$ , i.e. of the introduction of a cleaner technology, consists of this partial welfare effect and of changes in the balanced growth rate.

## 6 Conclusion

In this paper we have presented an endogenous growth model with public capital and pollution. The main novelty of our approach compared to the literature on endogenous growth and environmental pollution is the assumption that pollution only affects the utility of the household but not production possibilities directly.

Analyzing our model we derived the effects of fiscal policy on the long run balanced growth rate. We demonstrated that variations in both the income tax rate and the

pollution tax rate may have positive or negative growth effects and we derived conditions which must be fulfilled so that an increase in these tax rates generates a higher balanced growth rate. In particular, we could derive growth maximizing values of tax rates without resorting to numerical simulations.

Further, we studied the effects of a rise in the income and pollution tax rate on the growth rates of consumption, private capital and of public capital on the transition path and we have seen that the transition effects of fiscal policy may differ from the long run effects. Finally, we also demonstrated that growth maximization and welfare maximization need not be equivalent goals. In particular, it turned out that the welfare maximizing values of parameters are different from the growth maximizing values if the parameters have a direct impact on effective pollution and, thus, on utility.



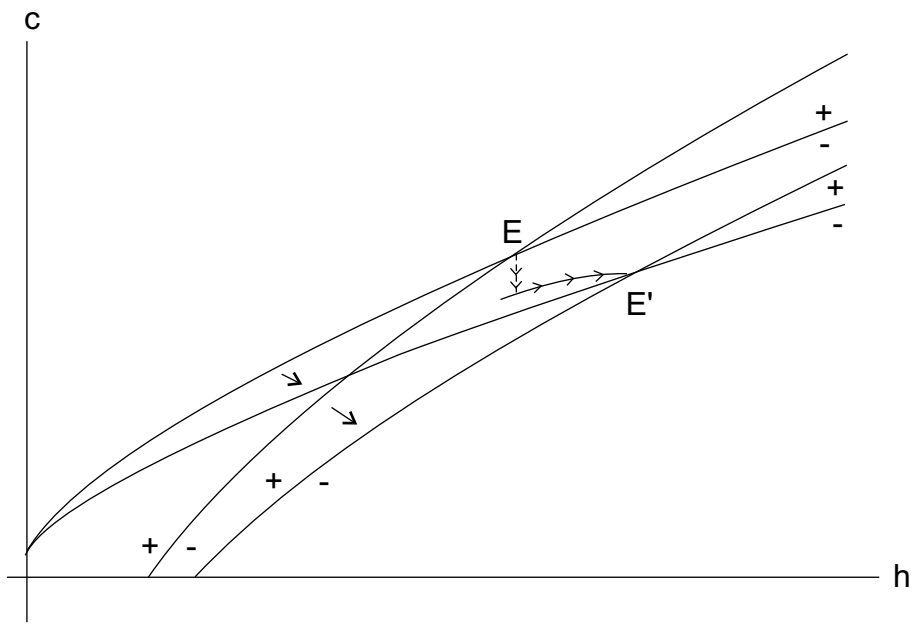


Figure 1

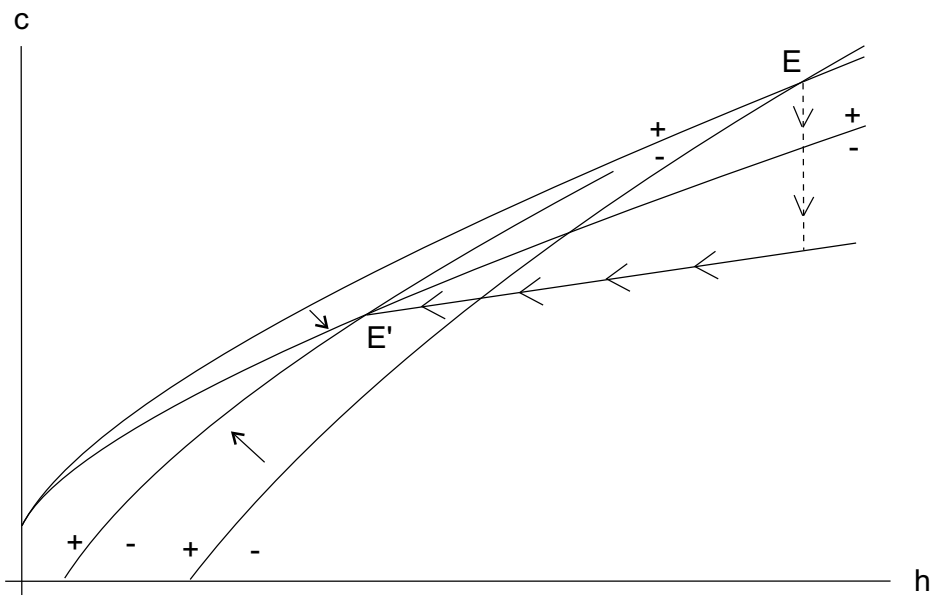


Figure 2

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