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Externalities of Investment, Education and Economic Growth

by

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Abstract

We present a growth model in which investment in physical capital shows positive externalities which build up knowledge capital. A prerequisite for these spillovers to take place is that a country devotes time to education. Externalities associated with investment need education to raise the stock of knowledge capital. Analyzing the competitive economy we demonstrate that the model may explain why some low income countries show convergence whereas others do not. Further, we demonstrate that in the social optimum the level of investment is always higher than in the competitive economy whereas the time spent for education may be lower or higher. We also show how the competitive economy may replicate the social optimum for an appropriate choice of a lump-sum tax and an investment subsidy. Empirical evidence is provided in order to demonstrate the plausibility of our model.

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1 Introduction

With the emergence of the 'new' growth theory, economic growth has again become a major issue in macroeconomics. That theory attempts to explain sustained per-capita growth as an endogenous phenomenon. There are several ways how endogenous growth can arise[1].

One view is to assume that investment has positive external effects which generate constant or increasing returns of capital on the aggregate level of an economy. That approach goes back to Romer (1986) who demonstrated that neoclassical growth models may lead to sustained per-capita growth with an endogenously determined growth rate if externalities of investment are taken into account[2]. The significance of investment as concerns economic growth is empirically confirmed, too, because that variable is robust in explaining the growth rate of market economies (see Levine and Renelt (1992) or Sala-i-Martin (1997)).

Another approach which generates endogenous growth is the model by Lucas (1988) and Uzawa (1965). In that sort of models individuals permanently devote a certain part of their time to education which raises the stock of human capital and generates sustained per-capita growth. From the empirical point of view, that approach also seems to be confirmed (see Barro and Sala-i-Martin (1995), chap. 12.3, and Levine and Renelt (1992)), although it must be conceded that education is not a robust variable in explaining economic growth in the study by Sala-i-Martin (1997).

However, the Lucas-Uzawa model, as well as other AK-style endogenous growth models, have been criticised because they are not compatible with time series evidence. This has been pointed out in a series of papers by Jones (cf. Jones (1995, 1995a, 1997)). From the time series perspective one is confronted with the prediction of endogenous growth

[1]For a survey see the book by Barro and Sala-i-Martin (1995).

[2]The presence of positive external effects of investment in physical capital seems to be confirmed by empirical studies, cf. DeLong and Summers (1991) and Hamilton and Monteagudo (1998).

models that a rise in the level of an economic variable, like an increase in human capital or knowledge capital, implies strong and lasting effects on the growth rate of the economy. In fact, in Lucas (1988) and in Romer (1990) the growth rate is predicted to monotonically increase with level variables. But, as stylized facts show, level variables such as education, human capital or research intensity in most advanced countries have dramatically increased, while the growth rate of GDP did not increase. This gives rise to the question to what extent level variables of modern growth models have effects on growth rates. This indeed is a serious question since one would like to know if a country can expect a higher growth rate if it spends more time on education or if it builds up its stock of knowledge as a result of R&D spending. Further, the Lucas-Uzawa model implies that countries with more education have a higher balanced growth rate, which does not seem to hold universly.

In this paper we will build a model of economic growth which combines the two approaches mentioned above and try to avoid the shortcoming of the Lucas-Uzawa model. To do so, we assume that investment in physical capital is associated with positive externalities which build up a stock of knowledge capital, but those external effects only occur if individuals devote time to education. Thus, we acknowledge that workers must undergo education in order to be able to operate machines efficiently.

With that assumption we intend to formalize in a way what Abramovitz (1986, 1994) has summarized under the rubric social capability, which is a necessary condition to achieve economic growth and prosperity. According to that concept countries must be able to adopt existing technologies and to produce with them in order to achieve economic growth. That is more important than to develop new products or methods of production. That approach seems of particular relevance for less developed countries which intend to catch up with highly developed economies. A prerequisite for the ability to adopt modern technologies and to achieve economic growth is that economies dispose of a sufficiently high social capability[3]. With social capability Abramovitz refers to technical competence, which determines the ability to adopt modern methods of production, but also to other factors such as the stability of governments and of the monetary sector and the attitude towards wealth and capitalism for example.

The rest of the paper is organized as follows. In the next section, we introduce our model and derive optimality conditions for both the competitive economy and the social optimum. In section 3 we discuss our model and section 4 presents some empirical evidence. Section 5, finally, concludes the paper.

2 The Growth Model with Externalities of Investment and Education

We consider an economy which consists of a representative household and a representative firm. Further, there is a positive externality associated with investment provided that the working household devotes time to education.

2.1 The Competitive Economy

We start with the description of the competitive economy. With competitive economy we refer to a situation where neither the household nor the firm take into account that investment is associated with positive externalities.

The Productive Sector

The productive sector is represented by a firm which produces a homogeneous good Y with a Cobb-Douglas production function[4]:

$$Y = (uAL)^{\alpha} K^{1-\alpha} \equiv (uA)^{\alpha} K^{1-\alpha}.$$

[3]This term was first employed by Ohkawa and Rosovsky (1973).

[4]We suppress the time argument if no ambiguity arises.

α denotes the labour share in the production function and labour L is constant over time and normalized to one. K and A denote the stock of physical and knowledge capital respectively where A raises the labour productivity and is taken as given by the firm in solving its optimization problem. $u \in (0, 1]$ is the time devoted to production and is determined by the household. The total amount of time available to the household is normalized to one and $1 - u$ is the fraction of time devoted to education.

The firm behaves competitively yielding

$$r = (1-\alpha)K^{-\alpha}(uA)^{\alpha} \tag{1}$$

$$w = \alpha\, u^{\alpha-1}A^{\alpha}K^{1-\alpha} \tag{2}$$

The External Effect

The stock of knowledge capital $A(t)$ is assumed to be a by-product of cumulated past gross investment (cf. Arrow (1962), Levhari (1966), Sheshinski (1966) or Romer (1986)), but in our case it is also affected by the educational effort. In contrast to the usual assumption we suppose that investment at certain dates shows different weights concerning its contribution to the current stock of knowledge capital (as to the use of weighting functions in growth models see e.g. Ryder and Heal (1973) or Wan (1970)). Formally, this stock can be expressed as

$$A(t) = \varphi(u) \int_{-\infty}^{t} e^{\eta(s-t)} I(s) ds, \tag{3}$$

with I gross investment, $\eta \geq 0$ depreciation rate of knowledge capital and $\varphi(\cdot) \geq 0$ the contribution of one unit of investment to the formation of knowledge capital. This becomes clearer by differentiating $A(t)$ with respect to time leading to

$$\dot{A} = \varphi(u)I - \eta A\,, \tag{4}$$

for a time invariable u. Thus, this formulation implies that the stock of knowledge may be subject to depreciation which can be justified by adopting a more Schumpeterian perspective in which new investment and education raises the stock of knowledge but, at the same time, makes a part of the existing knowledge obsolete.

$\varphi(u)$ represents the contribution of one unit of investment to the formation of the stock of knowledge capital and is assumed to be a positive function of the time devoted to education, $1-u$ or, equivalently, a negative function of the time used for production, i.e. $\varphi'(u) < 0$. The larger the fraction of time devoted to education the stronger the external effect of investment on the formation of knowledge. Further, we suppose that $\varphi(u) \rightarrow 0$ for $u \rightarrow 1$. This assumption states that without education no learning effect takes place and individuals are not capable to build up knowledge as a by-product of investment in new machines. In that case, investment does not show any externalities. This assumption can be justified by requiring that employees and workers must undergo a minimum level of education, for example be able to read and write, in order to be able to increase their skills as a by-product of investment. However, in the industrialized or newly industrializing countries the case $u = 1$ will not be observed for the average individual because governmental regulations prescribe that any citizen has to undergo a minimum of education. Thus, education without investment will not generate much growth. In our view, investment and education are complementary in the sense that neither of these activities is capable of increasing the stock of knowledge capital by itself.

But it should be underlined that our assumption concerning $\varphi(\cdot)$ does not necessarily mean that in economies with a higher level of education any unit of investment undertaken raises the stock of knowledge capital to a higher degree. Instead $\varphi(u)$ may differ between countries even if they spend the same amount of time for education. This holds because other factors, such like institutional, cultural or environmental factors, will also influence the social capability of a country and, thus, its ability to build up knowledge as a by-product of investment.

With the function $\varphi(\cdot)$ we intend to formalize in a way what Abramovitz (1986, 1994) has summarized as social capability. If a large fraction of time is devoted to education the ability to handle new machines without difficulties is increased. Then, any new investment increases the stock of knowledge capital to a great degree. If little time is devoted to education the external effect of investment is likely to be very small, or in the extreme case, cannot be observed at all. In that case, the effect of investment consists merely in raising the stock of physical capital.

However, it should be mentioned that our model is only a very modest attempt to put Abramovitz's ideas in a formal framework. In particular, it should be emphasized that we only take into account one aspect determining the social capability of a country which in reality comprises more factors than the one we have modelled.

The Household Sector

The household maximizes the discounted stream of utility resulting from consumption $C(t)$ over an infinite time horizon:

$$\max_{C(t),u(t)} \int_0^{\infty} e^{-\rho t} U(C(t)) dt. \tag{5}$$

$\rho > 0$ gives the rate of time preference and $U(\cdot)$ is the utility function, with $U'(\cdot) > 0$ and $U''(\cdot) < 0$. We assume a CRRA utility function implying that the intertemporal elasticity of substitution of consumption, $1/\xi \equiv -U'/U''C$, is constant. The household's budget constraint is written as

$$C + \dot{K} + \delta K = \tilde{w} u + rK. \tag{6}$$

$\delta \geq 0$ is the depreciation rate of physical capital and r is the return to physical capital. Recall that the labour supply is assumed to be constant and normalized to one so that all variables give per-capita quantities.

$\tilde{w}$ is the wage rate the household expects to receive for its labour input. The household assumes the wage rate to be a function which positively depends on the time spent for education. This holds because education has positive impact on the marginal product of labour and, thus, on the household's income. The economic justification for this assumption is that in modern economies higher wages positively co-vary with education. It is implicitly acknowledged that better trained workers are more productive over time because they are able to build up knowledge capital according to (4). Therefore, we assume that the wage rate, that the households expects to receive, is a positive function of the time spent for education and of exogenous variables, or equivalently, a negative function of the time used for production, i.e. $\tilde{w} = f(u, \cdot)$, with $f_u(u, \cdot) < 0$, where $\cdot$ stands for exogenous variables which we do not consider in detail. Further, we also assume that $f(u, \cdot)$ is such that there exists an interior maximum for $\tilde{w}u$, the wage income of the household.

In equilibrium, however, the household is paid the marginal product of labour, i.e. $\tilde{w} = w$ holds, with w given by (2). Now, what would happen if the household expected to get the equilibrium wage rate, that is if it sets w from (2) in its budget constraint? Then, it is immediately seen that the household would not undergo any education at all, that is it would set $u = 1$ so that its wage income $w(\cdot)u$ is maximized because then it could not expect to get any remuneration for education. That holds since in the competitive economy the positive external effect of investment, i.e. the formation of knowledge capital according to (4), is not taken into account. But then, we would have the conventional Ramsey type growth model with zero per-capita growth in the long-run. The same outcome would be obtained if the household took w as given in solving its optimization problem. Then, the household would set $u = 1$, that is it would spend no time for education.

However, the household's expectation that the wage rate positively varies with the time spent for education indeed makes it better off compared to the situation where the household takes the wage rate as given. This holds because then the household spends time for education which brings about positive external effects of investment which may lead to sustained per-capita growth as will be seen below. So, education is a merit good for society because it is the prerequisite for positive externalities of investment in the competitive economy.

An alternative specification, which could be imagined, is that u is given exogenously.

Then, the wage rate w and u are parameters for the household which it takes as given in solving the optimization problem. The economic reason behind that assumption might be that the government fixes how much basic education the population has to undergo. It is that sort of education which contributes to the productivity of an economy. For example, it is often argued that the increase in basic education in the fast growing economies of South East Asia has been a major reason for their high per-capita growth rates.

To derive necessary optimality conditions for a maximum of (5) subject to (6) we formulate the current-value Hamiltonian

$$H(\cdot) = U(C) + \gamma_1(-C - \delta K + f(u, \cdot)u + rK),$$

with γ_1 the current value co-state variable. The maximum principle gives

$$\gamma_1 = U'(C), \tag{7}$$

$$0 = f_u(u, \cdot)u + f(u, \cdot), \tag{8}$$

implicitly defining C and $u^\star$, the optimal value of u. The second order condition guaranteeing that $u^\star$ yields a maximum is fulfilled if $f_{uu}(u^\star, \cdot) < 2(f_u(u^\star, \cdot))^2/f(u^\star, \cdot)$ holds. That inequality holds if $f(u, \cdot)$ is linear in u, exponential or isoelastic with an absolute elasticity larger one[5].

The evolution of γ_1 is described by

$$\dot{\gamma}_1 = (\rho + \delta)\gamma_1 - r\gamma_1.$$

Furthermore, we need the limiting transversality condition $\lim_{t\to\infty} e^{-\rho t}\gamma_1(t)K(t) = 0$ to hold which is automatically fulfilled if the growth rate of $K(t)$ is smaller than ρ. Combining the condition (7) with the equation for $\dot{\gamma}_1$ gives the growth rate of private consumption as

$$\frac{\dot{C}}{C} = -\frac{\rho + \delta}{\xi} + \frac{r}{\xi}. \tag{9}$$

[5]For example, the function $f(u, \cdot) = (1 - u)^{0.5} + a$ yields $u^\star = 0.93$ $(0.91, 0.86, 0.81, 0.74, 0.67)$ for $a = 1.5$ $(1.2, 0.8, 0.5, 0.2, 0)$.

Equilibrium Conditions

The use of equilibrium conditions can be justified by supposing a theoretical dichotomy between growth and business cycles and by arguing that growth theory is primarily concerned with the long-run behaviour of economies. Since components, which are fixed in the short run, become flexible in the long-run adjustment mechanisms may take effect such that the economy attains an equilibrium.

The houshold's budget constraint, (6), together with (1) and (2), which give the equilibrium wage rate and the interest rate, describes the evolution of the physical capital stock. The growth rate of consumption is given by (9) and the growth rate of knowledge capital, finally, is described by (4) with $I = Y - C$. This leads to the following differential equation system, which completely describes our competitive economy.

$$\frac{\dot{C}}{C} = -\frac{\rho+\delta}{\xi} + \frac{(1-\alpha)(u^\star)^\alpha K^{-\alpha} A^\alpha}{\xi} \tag{10}$$

$$\frac{\dot{K}}{K} = -\delta - \frac{C}{K} + (u^\star)^\alpha \left(\frac{A}{K}\right)^\alpha \tag{11}$$

$$\frac{\dot{A}}{A} = -\eta + \varphi(u^\star)\left((u^\star)^\alpha \left(\frac{A}{K}\right)^{\alpha-1} - \frac{C}{A}\right). \tag{12}$$

The initial conditions are $K(0) = K_0 > 0$, $A(0) = A_0 > 0$ and $C(0) > 0$ can be chosen freely. Further, the transversality condition $\lim_{t\to\infty} e^{-\rho t}\gamma_1(t)K(t) = 0$ must be fulfilled, with $\gamma_1 = \gamma_1(C)$ determined by the maximum principle. Next, we study the social optimum.

2.2 The Social Optimum

In solving the optimization problem, a social planner takes into account that investment is associated with positive externalities. The optimization problem, therefore, is:

$$\max_{C,u} \int_0^\infty e^{-\rho t} U(C) dt, \tag{13}$$

subject to

$$\dot{K} = (uA)^\alpha K^{1-\alpha} - C - \delta K, \quad K(0) = K_0, \tag{14}$$

$$\dot{A} = \varphi(u)((uA)^{\alpha}K^{1-\alpha} - C) - \eta A, \quad A(0) = A_0. \tag{15}$$

Introducing the current-value Hamiltonian

$$H = U(C) + \gamma_2((uA)^{\alpha}K^{1-\alpha} - C - \delta K) + \gamma_3(\varphi(u)((uA)^{\alpha}K^{1-\alpha} - C) - \eta A),$$

the necessary conditions are obtained as

$$\begin{aligned} U'(C) &= \gamma_2 + \gamma_3\varphi(u), && (16)\\ \varphi'(u)I &= -\alpha u^{\alpha-1}K^{1-\alpha}A^{\alpha}(\varphi(u) + \gamma_2/\gamma_3), && (17)\\ \dot{\gamma}_2 &= (\rho+\delta)\gamma_2 - \gamma_2(1-\alpha)K^{-\alpha}(uA)^{\alpha} - \gamma_3\varphi(u)(1-\alpha)K^{-\alpha}(uA)^{\alpha}, && (18)\\ \dot{\gamma}_3 &= (\rho+\eta)\gamma_3 - \gamma_2\alpha A^{\alpha-1}u^{\alpha}K^{1-\alpha} - \gamma_3\varphi(u)\alpha A^{\alpha-1}u^{\alpha}K^{1-\alpha}, && (19) \end{aligned}$$

with $I = (uA)^{\alpha}K^{1-\alpha} - C$ investment. If the matrix $\partial^2 H/\partial i \partial j$, $i,j = C,u$, is negative semidefinite and if the limiting transversality condition $\lim_{t\to\infty} e^{-\rho t}(\gamma_2(t)K(t) + \gamma_3(t)A(t)) \geq 0$ is fulfilled the necessary conditions are again sufficient. That holds because the maximized Hamiltonian is concave in A and K together (see Seierstad and Sydsæter, 1987, p. 107/108).

3 Discussion of the Model

Looking at the competitive economy, which is described by equations (10)-(12), we realize that for a constant level of knowledge capital $A(t)$ the growth rates $\dot{C}/C$ and $\dot{K}/K$ become negative for $K \to \infty$ implying that in this case sustained per-capita growth is not feasible and our model is equal to the conventional neoclassical Ramsey type growth model and does not reveal sustained per-capita growth. Only if the external effect of investment concerning the formation of knowledge capital is strong enough so that the marginal product of physical capital does not converge to $\rho + \delta$ in the long-run endogenous growth is feasible. This, for its part, is only possible if the coefficient φ exceeds a certain threshold level, which is given by $\varphi(u^{\star}) = \eta A I^{-1}$ and which is sufficient for a positive growth rate of knowledge capital. In economic terms this means that there must be a sufficiently high social capability so that through any unit of investment the knowledge capital can be increased. For $\eta = 0$ and $\varphi > 0$ that condition is trivially satisfied and we can observe endogenous growth as long as investment is positive.

A point we should also like to emphasize is that in case of endogenous growth the balanced growth rate, which is given by (10), crucially depends on the marginal product of physical capital which positively varies with the stock of knowledge capital. Thus, countries with a lower stock of pure physical capital tend to have higher growth rates than countries with a higher stock. But this is neither a sufficient nor necessary condition for high growth rates. Instead, the level of knowledge capital plays an important role, too. So, economies with a high stock of knowledge capital may compensate a high stock of physical capital and countries with a very small stock of physical capital may nevertheless have only a relatively small marginal product of physical capital and, thus, little growth if they are endowed with a very small knowledge capital stock. Consequently, the growth rate will be the highest in those countries in which the stock of physical capital is small but the stock of knowledge capital relatively large. This combination gives a very high marginal product of physical capital and, as a consequence, high growth rates. Therefore, those countries will show convergence in the long-run. Thus, that framework can be used to explain the high growth rates of Germany and Japan after the second World War. Next, we will analyze how the competitive economy differs from the social optimum.

Comparing equations (7) and (16), which determine the optimal level of consumption and, thus implicitly, of investment in the competitive economy and in the social optimum, it is immediately seen that in the social optimum the level of investment is higher and, consequently, the balanced growth rate, too. That holds because in the social optimum investment is not only paid its shadow price γ_2 but also an additional weighted shadow price $\varphi(\cdot)\gamma_3$. Therefore, as usual, the government has to give incentives for investment by, for example, raising a lump-sum tax which is used to subsidize investment. It should be noted that the higher φ, i.e. the more any unit of investment contributes to the growth of the stock of knowledge, the higher the subsidy has to be.

To be more concrete we assume that a government levies a lump-sum tax, denoted by Γ, which is then used to finance an investment subsidy, denoted by θ. Further, we suppose that the budget of the government is balanced at any moment in time. The budget constraint of the household, then, is written as

$$\dot{K} = I(1+\theta) - \delta K - \Gamma,$$

where I denotes investment which is given by $I = (\tilde{w}u + rK - C)$ which is simply the difference between the household's income and its consumption spending. The maximum principle then yields

$$\gamma_1(1+\theta) = U'(C) \tag{20}$$

and the differential equation describing the evolution of γ_1 is

$$\dot{\gamma}_1 = (\rho+\delta)\gamma_1 - r\gamma_1. \tag{21}$$

If θ is chosen so that $\theta = \gamma_3\varphi(\cdot)/\gamma_1$ for all $t \in [0,\infty)$ investment in the competitive economy equals investment in the social optimum. This is easily seen if we insert in (20) and in (21) and if we use that the marginal product of capital is given by $r = (1-\alpha)K^{-\alpha}(uA)^{\alpha}$. Doing so immediately shows that the evolution of the shadow price of physical capital in the competitive economy, $\dot{\gamma}_1$, is equal to that of the social optimum, $\dot{\gamma}_2$, with $\gamma_1(0) = \gamma_2(0)$. Further, the optimality conditions determining investment, that is equations (7) and (16), are also the same for the competitive economy and for the social optimum.

It should be recalled that the lump-sum tax Γ has to be chosen such that the budget of the government is balanced. Along a balanced growth path (BGP) Γ will remain constant since $\varphi(\cdot)$ is constant and γ_1 and γ_3 grow at the same rate, implying that the ratio is constant. In order to set θ such that the competitive economy replicates the social optimum, the social planner has also to take care that the level of education in the competitive economy equals the one in the social optimum. Therefore, we next look at the level of education in the competitive economy and in the social optimum.

Equation (17) states that in optimum the social planner sets u such that the negative effect of a marginal increase in the time spent for production, which consists in a lower external effect of investment, just equals the positive effect of more time used for production, which consists in a higher level of production and a higher marginal product of private capital. We should also like to point out that the time spent for education in the social optimum is not necessarily higher than in the competitive economy. That depends on the function $f(u, \cdot)$ and demonstrates that the competitive economy may lead to a level of education which is higher than the socially optimal value. This holds because the household only knows that spending time for education makes him better off compared to the situation in which it does not take education at all. However, the household does not know how much education is optimal since it does not take into account the positive externalities associated with investment in physical capital. Therefore, the social planner has to give information about the true return to education implying that the household will select the optimal value for u.

Further, this outcome demonstrates that economies spending more time for education are not necessarily characterized by a higher balanced growth rate. This holds because a situation is feasible in which the household in the competitive economy expects a higher return to education compared to the social optimum so that it spends more time for education than in the social optimum. But, nevertheless, the competitive economy yields a lower balanced growth rate since the level of investment is always smaller in the competitive economy.

As to the dynamics of this model we first state that a BGP is defined as a path on which C, K and A grow at the same and constant rate and u is constant. Analyzing the competitive economy we find that it is characterized by a situation with a unique BGP or by a situation with two BGPs (with different growth rates in the long run) in case of endogenous growth, depending on the parameters of the model[6]. From an economic point of view two BGPs imply that the economy only converges to the path with the high growth rate in the long-run if the starting values of K, A and C take certain predetermined values. So, the initial conditions of knowledge and physical capital can crucially determine the long-run growth rate. In that case we can speak of lock-in effects or path dependence in the sense of Arthur (1988) implying that an economy with an initially lower stock of knowledge per physical capital possibly always lags behind another one and can never catch up.

But even if two economies dispose of the same starting levels for physical and knowledge capital, they may converge to different BGPs in the long-run if they choose different starting levels for consumption. Thus, we see that our model may be globally indeterminate in the sense of Benhabib and Perli (1994). This can explain why two economies may exhibit different growth rates in the long-run although they have the same technology and preferences and also the same endowment of physical and knowledge capital. As to the social optimum, we can show that it is characterized by unique BGP which is a saddle point if education is constant.

4 Empirical Evidence

In order to find whether our model is compatible with real time series we consider the time paths of the variables of our model for Germany and Japan after the second World War. To do so we first state that the competitive economy is completely described by equations (10)-(12). In principle, this system could be estimated using time series methods. However, some difficulties arise in this estimation. First, there are no data for the stock of knowledge and these data must be constructed by using equation (12). Second, there are two many parameters to be estimated. For example, in equation (10) the constant to

[6]We do not go into the details. A detailed proof of this outcome is available from the authors on request.

be estimated comprises the parameters ρ, δ and ξ so that there are a lot of possible combinations which give the estimated constant. Therefore, we proceed as follows: we first fix some parameters at values which are generally considered as economically reasonable in calibration studies and, second, then estimate our model empirically.

To do so, we limit our considerations to the case of a logarithmic utility function, i.e. we set $\xi = 1$, which is considered as a plausible value (see e.g. Blanchard and Fischer (1989), p. 44). Then, the competitive economy can be described by the variable $c \equiv C/K$ with A given by (12). Differentiating c with respect to time gives $\dot{c}/c = \dot{C}/C - \dot{K}/K$ which is

$$\frac{\dot{c}}{c} = -\rho + (1-\alpha)\left(\frac{uA}{K}\right)^{\alpha} - \frac{I}{K}, \tag{22}$$

for $\xi = 1$ and with I investment. It should be noted that the left hand side of (22) is the growth rate of the ratio of private consumption to physical capital. The stock of knowledge in our economy is formed according to (12) and we will construct this variable, A, by using the perpetual inventory method (for a good description of the perpetual inventory method, see Park (1995)).

In constructing the variable A we set $\varphi(\cdot) = 0.4$. This implies that the external effect of investment is 40 percent. This is in line with the study by Benhabib and Farmer (1995) for example, who suggest that the externality associated with production is in the range of 40-60 percent. The depreciation rate of knowledge is set to 6 percent, i.e. $\eta = 0.06$. As to the time used for production we set $u = 0.85$ which is about the mean of the male participation rate in West Germany from 1970-1996.

To estimate equation (22) we replace the differential operator by first differences and consider one period to comprise one year. The equation, which is estimated using non-linear least squares, then is

$$D(\ln(C_t/K_t)) = c_1 + c_2\left(\frac{uA_t}{K_t}\right)^{1-c_2} - c_3\frac{I_t}{K_t} + \epsilon_t, \tag{23}$$

with $c_3 = 1$ and $u = 0.85$. ϵ_t is the residual we add to equation (22) which is normally distributed with zero mean and finite variance σ^2, i.e. $\epsilon \sim N(0, \sigma^2 I)$.

To estimate (23) for Germany we need data[7] for consumption and investment which are taken from Statistisches Bundesamt (1974) and Sachverstaendigenrat (1995). Further, we need data for private capital. These are obtained from Statistisches Bundesamt (1991, 1995). The data are annual and cover the period from 1950-1994. The result of our estimation[8] is given in table 1.

Table 1: Estimation of equation (23) for Germany

Parameter	Value	Standard Error	t-Statistics
c_1	-0.096	0.039	-2.468
c_2	0.37	0.06	6.2
R^2	0.38	DW	1.8

Table 1 shows that the capital share $c_2 = 1 - \alpha$ is 37 percent, which is a plausible value. Further, this value is statistically significant. The discount rate $\rho = -c_1$ is also statistically significant and takes a value of about 9.6 percent. In calibration studies annual discount rates of about 6.5 percent are generally considered as plausible (see e.g. Benhabib and Farmer, 1994, or Benhabib and Perli, 1994). So, our estimated value for ρ seems a bit high but we think that it can nevertheless be accepted.

In a next step, we estimate equation (23) for Japan. To do so we proceed as for Germany and take the same prespecified values for u as well as for the parameters necessary to obtain A. The data for Japan are taken from the extended Summers and Heston (1991) database and comprise the period 1950-1992. Since this database does not contain data for the physical capital stock we had to construct this variable. We did this by applying the perpetual inventory method assuming an annual depreciation rate of 5 percent. In table 2 we show the results of our estimations.

[7] Data are for West Germany only.

[8] The estimations were done with Eviews, Version 2.0.

Table 2: Estimation of equation (23) for Japan

Parameter	Value	Standard Error	t-Statistics
c_1	-0.12	0.052	-2.402
c_2	0.34	0.082	4.097
c_3	0.47	0.078	6.068
R^2	0.55	DW	2.2

The capital share $c_2 = 1 - \alpha$ is statistically significant and takes a reasonable value of 34 percent. The discount rate is also statistically significant but the value of 12 percent seems a bit high. Nevertheless, a value of 12 percent need not be termed as unplausible and it may well be that the subjective discount factor in Japan is higher compared to other countries. It should also be mentioned that in our estimation with Japanese data we could not set $c_3 = 1$ but had to treat c_3 as a parameter to be estimated. Otherwise, there would have been serial correlation in the residuals which would have made the regression results unreliable.

R^2 in tables 1 and 2 seems low at first sight. However, one must be aware that our model does not intend to capture high-frequency oscillations caused by business cycles. So, the model and, thus, the fitted time series reflects the general evolution of the growth rates quite well, but it does not follow the peaks and troughs of the actual series.

This exercise demonstrates that our model is compatible with time series data in the German and Japanese economy after the second World War. The obtained structural parameters take reasonable values and are statistically significant. However, it must also be stated that the estimation of our model for the Japanese and German economies is not a proof of the model's relevance. In this respect, additional empirical research is necessary. But nevertheless, it demonstrates that with certain plausibly prespecified parameters the model does not have to be rejected because it is compatible with empirical data.

5 Conclusions

This paper has presented a simple combination of the Romer (1986) and Lucas (1988) and Uzawa (1965) models of endogenous growth. Investment in physical capital shows positive externalities which raise the stock of knowledge capital but only if the household spends time for education which positively influences the external effect of investment, i.e. the contribution of one unit of physical investment to the formation of knowledge capital. As a consequence, investment in physical capital does not raise knowledge and physical capital one for one so that these two stocks cannot be summarized within one state variable. It was demonstrated that both the competitive economy and the social optimum may generate endogenous growth. A prerequisite from the economic point of view consists in a sufficiently strong external effect of investment which positively affects the marginal product of private capital.

This approach combines the property of diminshing returns to physical capital of the Ramsey type growth model with endogenous growth and underlines the importance of the interrelation between knowledge and physical capital for economic growth. The model demonstrates that countries with a low stock of physical capital but a relatively large stock of knowledge capital have a very large marginal product of physical capital and, consequently, high growth rates. This could be observed for example for Japan and Germany after the second world war, where almost the whole stock of physical capital was destroyed, while the stock of knowledge capital, embodied in people, was still present (see Shell (1967), p. 78/79 for this example) and, in recent times, for countries such as Korea and Taiwan. But a low stock of physical capital is not a sufficient condition for high growth rates so that economic aid in form of capital goods to developing countries for example, which have not built up a sufficiently large stock of knowledge capital, is likely to fail if they are not accompanied by efforts to improve skills. Countries with a high capital stock can overcome diminishing returns to physical capital by large external effects of investment on the knowledge capital stock.

For the process of catching up, there are, of course, also other factors important. In particular, as Abramovitz (1986, 1994) states, the social capability of a country is of crucial importance and we intended to model this view by a function depending on the time spent for education. However, it must be conceded that this formulation is a very simple representation of what Abramovitz had in mind since only one aspect is modelled. In reality several factors will impact the process of catching up and some of them will also be exogenously given, determined by institutional, cultural and environmental conditions. Therefore, our approach must be considered as a very modest attempt to represent Abramovitz's idea in a formal model.

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