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On the Efficiency-Effects of Private (Dis-)Trust in the Government

by

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Abstract

We consider a continuous-time version of Ireland's Neo-Keynesian reinterpretation of the seminal Kydland-Prescott model, assuming now an heterogenous private sector. In each period, a fraction of the private agents naively believes the policy announcements made by the government. The other agents, who know the current number of non-believers in the economy, are utility-maximizers. The fraction of agents who believe the government changes over time according to a Word of Mouth learning process, that depends upon the difference between the payoffs they obtain and the payoffs realized by the non-believers. The government minimizes its cumulated loss through its choice of policy announcement and realized policy.

We show that the economy can have two stable equilibria. At one of these, all agents act rationally. At the other equilibrium, which is associated with a higher average utility of the private sector, a positive percentage of the agents trusts the government. The two equilibria are separated by a Skiba point associated with an unstable spiral of the canonical system. Thus, the initial fraction of believers in the economy can have drastic consequences for the economic policy followed and the losses experienced by the different agents. Moreover, the flexibility of the private sector in reacting to the losses' difference proves to be crucial. Independently of the number of believers in the economy, the government losses monotonically increase with the flexibility. The private sector, on the other hand, is best off for an intermediate level of flexibility.

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1 Introduction

One of the most ubiquitous and stable characteristics of real-life policy-making appears to be that decision-makers repeatedly make announcements and promises that later they do not respect. An important strand of economic literature argues that this type of behavior leads to a loss of trust in and of reputation for the decision-makers, and is necessarily detrimental to all economic agents. Thus, it may be indispensable to impose strict rules to insure that announcements are respected.

One may wonder why announcements are made, and why they influence individual behaviors, if they are detrimental. In this paper, we show that promises that are not respected can reduce the losses not only of the government, but also of all private agents, if the private sector includes any number of naive *believers* who take the announcements at face value. Moreover, if agents tend to adopt the behavior of other, more successful ones, a stable equilibrium may exist where a positive fraction of the population consists of believers. This equilibrium Pareto-dominates the one where all agents act rationally, that is, optimize their objective function under perfect information. To attain this superior equilibrium, the government builds reputation and leadership by insuring good results for the believers, rather than by pre-committing to its announcements.

This Pareto-superior equilibrium is not the only possible one. Depending upon the model parameters – most crucially: upon the initial fraction of believers and upon the intensity of information transmission among private agents, i.e., upon the *flexibility* of the private sector – it may be rational for the government to steer the Pareto-inferior rational equilibrium. This paper, thus, stresses the importance of the initial confidence level in the population and of the private flexibility in explaining the policies followed by a government, the welfare level realized, and the persistence or decay of private confidence in the governmental announcements.

There are two main prerequisites for the existence of a Pareto-superior equilibrium where part of the population believes the governmental announcements. The first one is the absence of major conflict of interest among private agents and between the private agents and the government. The second one is a poor, inefficient outcome for all agents and for the government under the standard rationality hypothesis. In the model, this poor outcome is due in particular to the atomistic nature of the private sector and to the game-theoretic character of the equilibrium. These properties are shared by numerous models in economics, suggesting that the approach delineated here can readily be applied to other contexts.

The potential usefulness of deliberately employing misleading announcements to Pareto-improve upon standard game-theoretic equilibrium solutions was suggested in Vallée and Deissenberg (1998) and in subsequent papers by the same authors for the case of general linear-quadratic dynamic games. An application to the credibility problem in monetary economics was developed in Deissenberg and Alvarez (2002), who use a model similar to the one presented here but with different learning processes and dynamics. Among other papers of related interest, Cho and Matsui (1995) and Ireland (2000) develop models of monetary policy making with boundedly rational agents where the government can build credibility by adopting a policy unilaterally.

The paper is organized as follows. In Section 2, we first present the static economic model underlying in this paper and its main properties. We then introduce the word of mouth learning process that generates a constant flow of private agents who adopt the strategy (to believe or not to believe) that performed best in the near past. This allows us to define the intertemporal optimization problem of the government. In Section 3, we present the main insights obtained from the model. The final Section 4 summarizes the mechanisms at work in the model and the main results, and hints at possible further applications or extensions.

2 The Model

2.1 The economy

We consider an economy consisting of a continuum of private agents i, and of a government or central bank G. Time t is continuous. To keep notation simple, we do not index the variables with t, unless it is useful for a better understanding.

At any time t:

1. The government makes an announcement y^a about the anticipated inflation rate at time t. This announcement has no real effect on the economy but may influence the beliefs of the private sector.

2. The private sector builds expectations x about the inflation rate at time t. These expectations are public knowledge.

3. The government determines the actual inflation rate y at time t.

More specifically, and in the tradition of recent reformulations of the Kydland-Prescott (1977) seminal model – see e.g. Sargent (1999) – we assume that each private agent i makes independently a prediction x^i about the current rate of inflation in the economy, y. The amount of work currently supplied by the agent depends upon x^i and y. The agent's unemployment rate u_i is given at any moment by an expectations augmented Philips curve:

$$u^i = \bar{u} - \theta(y - x^i), \tag{1}$$

with θ a strictly positive constant and where $\bar{u}$, another strictly positive constant, is the natural rate of unemployment. Note that this is the rate of unemployment that would prevail at the symmetric equilibrium when all agents perfectly predict y, that is, when $x^i = y \ \ \forall i$.

The welfare loss function of any private agent i is given by:

$$J^i(x^i, y) = \frac{1}{2}\left[(y - x^i)^2 + y^2\right] = \frac{1}{2}\left[(\frac{\bar{u} - u^i}{\theta})^2 + y^2\right]. \tag{2}$$

All agents are identical, save for the individual predictions x^i and their consequences in terms of unemployment u_i and welfare J^i. The functions (1) and (2) are common knowledge.

The formulation (1)-(2) would be a standard variant of the Kydland-Prescott model if all private agents chosed the same x^i, that is, were symmetric – as e.g. in Sargent (1999). In this article, however, we allow for heterogenous predictions. Thus, the level of unemployment u^i will vary from agent to agent, and the validity of the individual Philips curves (1) and loss functions (2) outside of the symmetric equilibrium requires to be economically justified. It turns out that (1) and (2) can easily be derived from a simple version of Ireland's (1997) New Keynesian model, see also Stokey (2002). Suppose that each private agent supplies a differentiated commodity it produces using its labor as only input. Interpret the agent's predicted inflation, x^i, as the wage it sets before the current inflation rate y is known, wages being sticky for one period. Setting a wage above or below the realized economy-wide inflation rate implies that the private agent will work less or more than individually optimal in the current period. Thus, the first term of the loss function J^i reflects the loss in utility associated with wrong wage setting. The second one, that depends only on the actual rate of inflation, is the "shoe leather" cost of inflation. The government controls the inflation rate y through money creation.

There are two types of private agents in the economy: (1) Believers B who trust the announcement of the government, and use this announcement as the sole information[1] while forming their expectations; and (b) Non-Believers NB who discard the announcement as cheap talk and optimally use all information available to them. All agents within a group are symmetric. We

[1] In addition to their primitives (1) and (2).

denote the individual (and average) expectation about the inflation rate of the believers by x^B, that of the non-believers by x^{NB}. The current fraction π of believers in the private sector is common knowledge for the government and all private agents. For every y, there is a unique symmetric equilibrium where each agent predicts exactly the current inflation y, has the unemployment rate $\bar{u}$, and has no additional welfare loss besides the cost of inflation *per se*, that is $J^i = \frac{1}{2}y^2$.

In the Kydland-Prescott tradition, it is natural to assume that the government G attempts to minimize the sum of the squared average unemployment and squared inflation in the economy, that is the instantaneous loss function:

$$J^G = \frac{1}{2}\left[\left\{\pi u^B + (1-\pi)u^{NB}\right\}^2 + y^2\right]. \tag{3}$$

For any y, the loss function J^G is minimized when the unemployment rate is null, $u = 0$ (Remember that the private agents prefer the positive unemployment rate $\bar{u} > 0$). A zero unemployment rate, however, is not to be interpreted in the sense of the physically maximal possible employment. Rather, the variable u is scaled in such a way as to take the value zero at the government's optimum. Sufficiently small negative values of u are possible, which can be interpreted as a situation where (sectors of) the economy (are) is overheating without hitting physical capacity.

The function J^G is poorly specified in the sense that for any $\pi > 0$ it allows the government to minimize its loss by choosing a very large and possibly infinite negative value for y^a – this in turn implying, as we shall see, a very large negative value for u^B. To insure that u^B remains in an economically acceptable range, we introduce soft constraints on its value by using the following governmental loss function:

$$J^G = \frac{1}{2}\left[\pi\left(u^B\right)^2 + (1-\pi)\left(u^{NB}\right)^2 + y^2\right] \tag{4}$$

instead of (3).[2]

2.2 The static problem

In the model presented above, there are no dynamics: The current values of the variables do not depend on their past values. The preferences of the actors are defined in terms of instantaneous loss functions. Accordingly, the actors solve in every t a static optimization problem, independent from the

[2] Using (3) together with the hard constraint $u^B \geq u^B_{\max}$ for some $u^B_{\max} < 0$ would be very cumbersome from an analytical point of view. The formulation (4) simplifies the mathematics without affecting the qualitative flavor of our results.

past and the future of the economy. Before introducing dynamics in the model, it is useful to investigate the solution of this static problem.

To solve the static problem note first that, once the expectations of believers and non-believers, x^B and x^{NB}, are formed, the government chooses the inflation rate:

$$y = R^G(x^B, x^{NB}; \pi) = \frac{\theta}{1+\theta^2}\left[\bar{u} + \theta\pi x^B + \theta(1-\pi)x^{NB}\right] \tag{5}$$

in order to minimize its loss J^G.

Assume now that the government announces that it will realize an inflation rate y^a. The optimal reaction function of the representative believer, that follows from minimizing (2) w.r.t. x^i subject to the constraint $y = y^a$, is trivially:

$$x^B = R^B(y^a) = y^a. \tag{6}$$

On the other hand, the representative non-believer uses all available information in forming his expectation. He knows that the π believers form their expectations according to (6), and that the government chooses the inflation rate according to (5). Substituting these two reactions functions in (2), minimizing the resulting expression w.r.t. x^i treating $(1-\pi)\,x^{NB}$ as a constant[3], and solving for the equilibrium value $x^i = x^{NB}$, one obtains for the optimal decision of the representative non-believer:

$$x^{NB} = R^{NB}(y^a; \pi) = \frac{\theta^2\pi y^a + \theta\bar{u}}{1+\theta^2\pi}. \tag{7}$$

Given the reaction functions (6) and (7) of believers and non-believers, the problem of the government reduces to a simple optimization exercise. Straightforward calculations yield for the optimal announced and realized inflation:

$$y^{a*} = -\frac{\bar{u}}{\theta}, \tag{8}$$

$$y^* = \frac{\theta(1-\pi)}{1+\theta^2\pi}\bar{u}. \tag{9}$$

While the announced inflation does not depend upon π, the realized inflation y^* decreases with π. Note that $y^* \geq y^{a*}$, and that the discrepancy between announced and realized inflation $y^* - y^{a*} > 0$ decreases with π.

[3] This, since the representative non-believer knows that he is too small to influence the average expectation $(1-\pi)\,x^{NB}$.

The individually optimal choices of believers and non-believers are $x^{B*} = y^{a*}$ and $x^{NB*} = y^{*}$. Thus, the non-believers correctly anticipate the realized inflation while the believers' expectations coincide with the announcement. Since $y^{*} - y^{a*}$ is decreasing in π, so is $x^{NB*} - x^{N*}$. As the proportion of believers in the economy grows, the non-believers (rationally) tend to increasingly mimic the behavior of the non-believers.

The loss of the government at equilibrium is:

$$J^{G*} = \frac{1+\theta^2}{1+\theta^2\pi}(1-\pi)\bar{u}. \tag{10}$$

It is 0 for $\pi = 1$.

The loss of the representative believer is:

$$J^{B*} = \frac{1}{2}\frac{\left(1+2\theta^2+2\theta^4-2\pi\theta^4+\pi^2\theta^4\right)}{\theta^2\left(1+\pi\theta^2\right)^2}\bar{u}, \tag{11}$$

and the loss of the representative non-believer, that reduces to the shoe leather cost of inflation, is:

$$J^{NB*} = \frac{1}{2}y^{*2} = \frac{1}{2}\left[\frac{\theta(1-\pi)}{1+\theta^2\pi}\bar{u}\right]^2. \tag{12}$$

The average loss in the private sector, finally, is given by:

$$J^{P*} = \pi J^{B*} + (1-\pi)J^{NB*}. \tag{13}$$

Several properties of this solution are worth mentioning. On the one hand, for any value of $\pi \in (0,1)$, and as one would intuitively expect, the believers' losses are always higher that the non-believers' – that is, that $J^{B*} > J^{NB*} \quad \forall\pi$, with the difference $J^{B*} - J^{NB*}$ decreasing in π. On the other hand, the losses J^{G*}, J^{B*}, J^{NB*} are strictly decreasing in π.[4] Thus, *all* private agents *and* the government would be better off in a society where every private agent believes the government. To recognize why this is so, let's examine what happens in the extreme cases $\pi = 0$ and $\pi = 1$, that is, in the cases where there is an homogenous population of non-believers or of believers only. For $\pi = 0$, the solution reduces exactly to the standard Stackelberg equilibrium with the representative private agent as an atomistic leader that one would obtain in the standard Kydland-Prescott framework[5]. This

[4] The average private loss, J^{P*}, is strictly decreasing in π for $\theta > 1$. It is constant in π for $\theta = 1$.

[5] For a more detailled discussion of the possible solutions in the Kydland-Prescott model see Deissenberg and Alvarez (2002).

solution is inefficient for two reasons: (a) The game-theoretic Stackelberg solution is not efficient in itself; and: (b) Being atomistic, the non-believers do not internalize the impact of their aggregate decisions on the outcome. At the other extreme, for $\pi = 1$, the government efficiently solves a standard optimization problem where the impact of all private decisions is properly internalized. The objective function of this optimization problem is almost identical to the one of the representative private agent – the main difference being a preference for a lower rate of unemployment. The efficiency gain obtained by switching from the game-theoretic to the optimization solution overweighs for all private agents the loss due to the fact that the government uses its objective function J^G rather than the private objective function J^P or J^i at the latter solution. Note that at $\pi = 1$ the government has a loss of 0, the private agents a strictly positive loss, reflecting the difference between private and governmental objective functions. Note also that in this economy announcements that are not respected do reduce all losses if there are at least *some* gullible agents. This suggests that attempting to insure central bank efficiency by subjecting it to strict transparency rules may be more questionable than usually assumed.

For $\pi \in (0, 1)$, the solution is (somewhat simplifying) a mixed average between the strongly inefficient Stackelberg and the better optimization solutions. As π increases, non-believers enjoy the benefits of a decreasing inflation, while their unemployment rate remains fixed at $\bar{u}$. Likewise, believers benefit from the decrease in inflation, and from an increase of their employment from some negative value[6] to the government's optimum 0. The government gains from the decrease in inflation and from the convergence of the unemployment rate of the believers towards its preferred value 0 – this last effect being reinforced by the increase of the number of believers in the population. The decrease of the inflation rate is made rational by the existence of the believers, who are led by the government's announcements to work less than they would otherwise.

The above results would not be qualitatively affected if we assumed that the government and the non-believers played a Nash game instead of a Stackelberg one. In fact, it is easy to show that the losses of all actors are greater under the Nash assumption than under the Stackelberg one for all $\pi < 1$. Thus, if anything, using the Nash solution would reinforce the flavor of our results. On the other hand, giving up the hypothesis that the private agents behave atomistically leads to qualitatively different results. We shall not elaborate here on this point, save for stressing its potential importance: It

[6] Remember that the variable u is scaled in a way as to meaningfully take negative values.

hints that much of the discussion based on the assumption of a unique representative agent, as in the original Kydland-Prescott model, may be strongly biased. However, the consequences of assuming atomistic agents do not appear to have been much considered in the monetary policy/credibility literature yet.

2.3 Introducing dynamics

In the model previously presented, the government has a natural role as a leader, in the sense that for any $\pi > 0$ its announcement reduces the losses of all agents compared to the Stackelberg solution (and to the Nash) solution when all agents act as non-believers. In other words, a core of believers, however small, gives the government the needed leverage to Pareto-improve the economic situation through its announcements. In this section, we reinforce this role by assuming that the government cares not only for the current period, but also for the future.

Specifically, we assume that the government minimizes over an infinite horizon its cumulated discounted loss V^G with respect to its current and future controls $\{y^a(t), y(t)\}_{t\in[0,\infty)}$:

$$V^G = \int_{t=0}^{\infty} e^{-\rho t}\frac{1}{2}\left[\pi\left(u^B\right)^2 + (1-\pi)\left(u^{NB}\right)^2 + y^2\right] dt. \tag{14}$$

In order to introduce learning in the private sector, we suppose that at each point of time each private agent observes the strategy (to believe or not to believe) and the payoff of another agent. If this payoff is higher than his own, the agent adopts the other's strategy with a probability proportional to the payoffs difference. The resulting dynamics of π is given by[7]:

$$\dot{\pi} = \beta\pi(1-\pi)\left[J^{NB} - J^B\right]. \tag{15}$$

Notice that $\dot{\pi}$ reaches its maximum for $\pi = \frac{1}{2}$ (the value of π for which the probability of encounter between agents with different payoffs is maximized), and tends towards 0 for $\pi \to 0$ and $\pi \to 1$ (for extreme values of π, almost all agents have the same payoffs). The parameter $\beta \geq 0$, that depends on the adoption probability of the other's strategy, measures the speed of the information flow between believers and non-believers. It may be interpreted as a measure of willingness to change strategies, that is, of the flexibility of the private agents.

[7]See e.g. Hofbauer and Sigmund (1998) or Dawid (1999) for a derivation of these dynamics in a discrete-time setup.

Equation (15) implies that by choosing the value of (y^a, y) at time t, the government now not only influences its instantaneous payoffs but also the future proportion of believers in the economy. This, in turn, has an impact on the government's future payoffs. Hence, although there are no explicit dynamics for the economic variables, the government faces a non-trivial intertemporal optimization problem.

The private agents are assumed to minimize in each t the instantaneous loss function J^i.[8]

Summarizing, the government's optimization problem is given by:

$$\max_{\{y^a(t),y(t)\}} (14) \quad \text{s.t.} \quad (15) \quad \text{and} \quad \pi(0) = \pi_0, \tag{16}$$

with $x^B = x^{B*}$, $x^{NB} = x^{NB*}$, $u^B = u^{B*}$ and $u^{NB} = u^{NB*}$ formed (as a function of the current value of π) according to (6), (7) in conjunction with (1). To distinguish between the static and the dynamic solutions, we shall use the superscript $^{\$}$ to designate the optimal values for above dynamic problem (16).

We analyze this optimization problem using Pontryagin's maximum principle – see e.g. Leonard and Van Long (1996). The Hamiltonian for the problem is:

$$\begin{aligned} & H(\pi, y^a, y, \lambda) \\ = \; & \frac{1}{2}\left[\pi\left(u^B\right)^2 + (1-\pi)\left(u^{NB}\right)^2 + y^2\right] + \lambda\beta\pi(1-\pi)\left[(y-x^B)^2 - \left(y-x^{NB}\right)^2\right], \end{aligned}$$

where the co-state λ is the shadow price of the stock of believers for the government. To find the optimal controls $y^a(t), y(t)$ as a function of state π and co-state λ we have to solve the system of first order conditions $H_y = 0, H_{y^a} = 0$ for y and y^a. Since this system is linear, the solution can be given in closed form. Doing so requires tedious but straightforward calculations that are omitted here. The result is summarized in the following lemma:

Lemma 1 *Let:*

$$\begin{aligned} a_{11} &= -(1+\theta^2)(1+\theta^2\pi), & a_{21} &= (1+\theta^2\pi)(\theta^2(1+\theta^2) - \lambda\beta\theta^2(1-\pi)), \\ a_{12} &= \pi(\theta^2(1+\theta^2) + \lambda\beta(1-\pi)), & a_{22} &= -\theta^2 - \pi(2\theta^4 + \theta^6) - \lambda\beta(1-\pi)(1+2\pi\theta^2), \\ a_{13} &= \theta\left[1+\theta^2 - \lambda\beta\pi(1-\pi)\right], & a_{23} &= \theta\left[-(1+\theta^2+\theta^4) - \pi\theta^2 + \lambda\beta\theta^2\pi(1-\pi)\right]. \end{aligned}$$

[8] Alternatively, we could assume that they minimize their discounted cumulated losses over an infinite horizon, but believe at each instant that $\dot{\pi} = 0$. Note in that context that the decision of any *single* agent to switch strategy from x^B to x^{NB} or vice-versa has no impact on $\dot{\pi}$, that is, $\partial\dot{\pi}/\partial\Delta x^i = 0$.

Given that the condition:

$$a_{11}a_{22} - a_{12}a_{21} > 0 \tag{17}$$

holds, the optimal controls of the government are given by:

$$y^{a\$} = \frac{a_{13}a_{21} - a_{11}a_{23}}{a_{11}a_{22} - a_{12}a_{21}}\bar{u}, \tag{18}$$

$$y^{\$} = \frac{a_{12}a_{23} - a_{13}a_{22}}{a_{11}a_{22} - a_{12}a_{21}}\bar{u}, \tag{19}$$

whenever both values are in an arbitrary finite interval $[-\bar{Y}, \bar{Y}]$*. Otherwise at least one control is on the boundary of* $[-\bar{Y}, \bar{Y}]$*.*

In the rest of the paper, and without loss of generality, we shall assume that the interval $[-\bar{Y}, \bar{Y}]$ is sufficiently large to be never binding at the optimal solution. Notice that $y^{a\$}$ and $y^{\$}$ both depend upon π.

In (18) and (19), the numerator is a polynomial of degree 4 in π and of degree 2 in λ. The denominator is a polynomial of degree 3 in π and of degree 2 in λ. Furthermore, the canonical system of differential equations for the state/co-state dynamics is given by:

$$\dot{\pi} = \frac{\beta\pi(1-\pi)}{2(1+\theta^2\pi)^2}(y^{a\$} - \theta\bar{u})\left[2(1+\theta^2\pi)y^{\$} - (1+2\theta^2\pi)y^{a\$} - \theta\bar{u}\right], \tag{20}$$

$$\dot{\lambda} = \frac{1}{2(1+\theta^2\pi)^3}\left[2(1+\theta^2\pi)^3\rho\lambda - h(\pi,\lambda)\right], \tag{21}$$

where:

$$\begin{aligned} h(\pi,\lambda) = \ & -2(1+\theta^2\pi)^3\left[\bar{u} + \theta(y^{a\$} - y^{\$})\right] \\ & +(1+\theta^2\pi)\left[(1+(1+\pi)\theta^2\right]\bar{u} + \theta^3\pi y^{a\$} - \theta(1+\theta^2\pi)y^{\$2} \\ & -2\theta^3(1-\pi)(y^{a\$} - \theta\bar{u})\left[(1+(1+\pi)\theta^2\right]\bar{u} + \theta^3\pi y^{a\$} - \theta(1+\theta^2\pi)y^{\$} \\ & +\lambda\beta(1+\theta^2\pi)(1-2\pi)(y^{a\$} - \theta\bar{u})\left[2(1+\theta^2\pi)y^{\$} - (1+2\theta^2\pi)y^{a\$} - \theta\bar{u}\right] \\ & -2\lambda\beta\pi(1-\pi)\theta^2(y^{a\$} - \theta\bar{u})\left[(1+\theta^2\pi)y^{\$} - \theta^2\pi y^{a\$} - \theta\bar{u}\right]. \end{aligned}$$

According to the maximum principle only state-costate trajectories that satisfy the canonical system (20), (21) and the transversality condition:[9]

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t) = 0 \tag{22}$$

[9] For infinite horizon optimal control problems such as the one considered here, it is not always the case that the transversality condition (22) is a necessary optimality condition. In our case, however, we have a finite control space thanks to the restriction $Y \in [-\bar{Y}, \bar{Y}]$. Moreover, for all $\pi \in (0,1)$, the origin is a feasible interior point of the right hand side of (15). A result by Michel (1982) then establishes that any optimal solution has to satisfy the transversality condition (22).

are *candidates* for optimal solutions. However, since the Hamiltonian of this problem is not jointly concave in state π and control (y^a, y), these optimality conditions are not sufficient. Thus, several candidates optimal solutions may exist for a given π_0.

It is easy to see that the isocline $\dot{\pi} = 0$ will in general have 3 branches – one for $\pi = 0$, one for $\pi = 1$, and one where both believers and non-believers make the same prediction error,i.e. where $y^{\$} - y^{a\$} = R^{NB}(y^{a\$}) - y^{\$}$. In principle, a branch where $y^{a\$} = \theta\bar{u}$ is also possible. It can be neglected, since one can show that choosing this (very high) value for the announcement is never optimal.

Obtaining further analytical results appear illusory given the complexity of the expressions for the optimal controls and co-state dynamics. In particular, this complexity practically excludes the analytical determination of the steady-states of the canonical system and investigation of their stability properties. We therefore study numerically the qualitative properties of the optimal solution. The computations were done using our own Mathematica routines and (in what concerns the computation of the value functions and Skiba points) a dedicated program kindly provided by Lars Grüne, whose support is most gratefully acknowledged. This last program uses a proprietary dynamic programming algorithm with flexible grid size. See Grüne and Semmler (2002) for details.

3 Main results

Depending upon the parameter values, the model can have either two stable equilibria separated by a so-called Skiba point, or a unique stable equilibrium. In the latter case, the unique equilibrium coincides with the static solution * of Section 2.2 for $\pi = 0$. We discuss first the more interesting case of multiple equilibria, before addressing the question of the bifurcations leading to a situation with a unique equilibrium. Unless otherwise specified, the results presented pertain to the reference parameter configuration $\bar{u} = 5.5, \theta = 1, \beta = 1, \rho = 3$. They are robust with respect to another choice of parameters: The same qualitative insights would be obtained for arbitrary alternative parameter configurations.

From the onset, let us stress the fundamental mechanism that underlies most of the results. *Ceteris paribus*, the government would like to face as many believers as possible, since $\partial J^{G*}/\partial\pi < 0$. However, it can not implement the actions y^{a*}, y^* that maximizes its instantaneous utility since in that case $J^{NB*} > J^{B*}$, see (11) and (12), which implies by (15) a decreasing number of believers, $\dot{\pi} < 0$. Instead, it will use the actions $y^{a\$}$, $y^{\$}$ that insure an optimal compromise between maintaining the value of $\dot{\pi}$ as high as possible, and maximizing its instantaneous utility. Depending on the parameter values and on the current value of π, the optimal value $\dot{\pi}^{\$}$ of $\dot{\pi}$ may be negative. In that case, the economy will tend towards an equilibrium where nobody believes the government, $\pi = 0$.

Alternatively, $\dot{\pi}^{\$}$ may be positive. Then, there will be a positive number of believers at the equilibrium. Since for any given actions y^a, y the speed of learning $\dot{\pi}$ is first increasing and then decreasing in π, see (15), it will not come as a surprise if for certain parameter constellations there exists a value of π that defines a threshold between two domains of convergence, towards $\pi = 0$ respectively towards some $\pi_R > 0$.

3.1 Multiple equilibria

3.1.1 Phase diagram

The phase diagram for the reference parameter values $\bar{u} = 5.5, \theta = 1, \beta = 1, \rho = 3$, is given in Figure 1. In this Figure, the area that does not satisfy the necessary condition (17) is shaded.

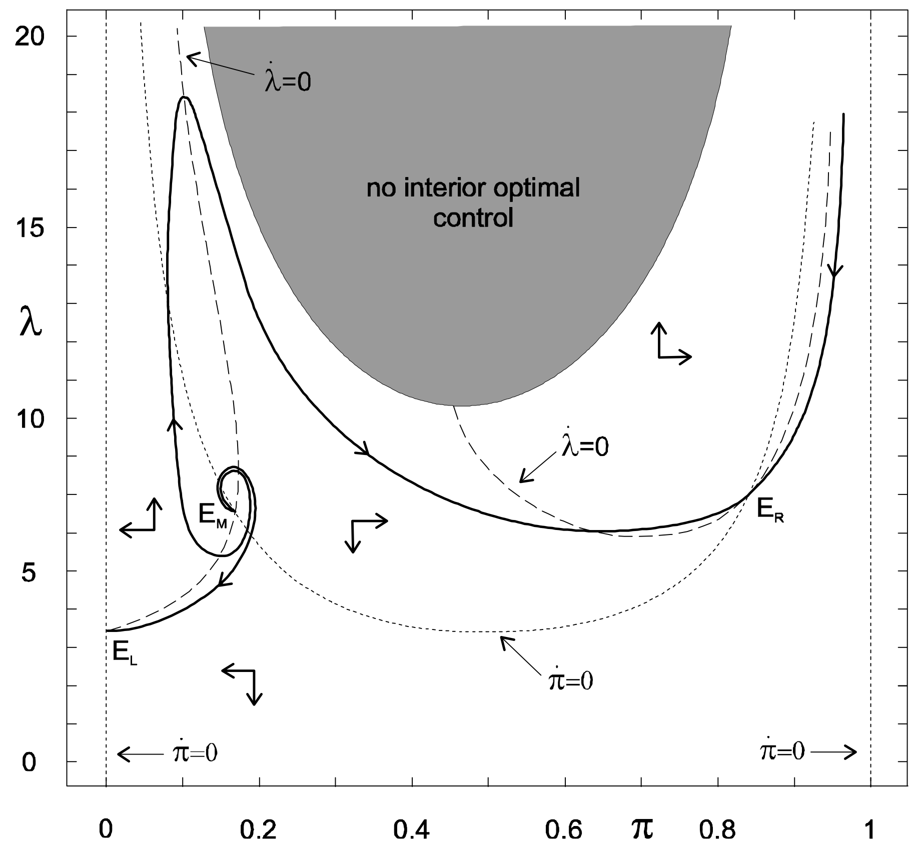

Figure 1: Phase portrait for $\bar{u} = 5.5, \theta = 1, \beta = 1, \rho = 3$

The Figure shows in dotted lines the three branches of the $\dot{\pi} = 0$ isocline mentioned above and defined by $\pi = 0$, $\pi = 1$, and $y^{\$} - y^{a\$} = R^{NB}(y^{a\$}) - y^{\$}$ respectively. Furthermore, Figure 1 includes (also as dotted lines) the $\dot{\lambda} = 0$ isocline. This isocline has two branches. For $\lambda \to \infty$, the left branch approaches asymptotically the line $\pi = 0$, and the right one the line $\pi = 1$. The thick continuous line represents the dynamics of the system (20)-(22), that is, the candidate optimal solution. There are two stable equilibria, $E_L = (0, \lambda_L)$ and $E_R = (\pi_R, \lambda_R)$ surrounding an unstable one, $E_M = (\pi_M, \lambda_M)$, with $0 < \pi_M < \pi_R < 1$.

The above configuration – an unstable equilibrium surrounded by two stable ones – implies the existence of a threshold π_S such that it is optimal for the government to follow a policy leading in the long run to the stable equilibrium $E_L = (0, \lambda_L)$ whenever the initial value $\pi(0) = \pi_0$ of π is smaller than the threshold value, $\pi_0 < \pi_S$.

In other words, when the initial fraction of believers is less than π_S, it is rational for the government to let the number of believers go to zero, although the resulting stationary equilibrium is associated with high losses for all agents. Whenever $\pi_0 > \pi_S$, the government's optimal policy leads to the stable equilibrium $E_R = (\pi_R, \lambda_R)$, that is to a situation where there is in permanence a strictly positive number of believers and where all agents experience relatively low losses.

As previously mentioned, the existence of the threshold is closely associated with the properties of the learning function (15). For any actions of the government y^a, y (that is equivalently: for any values of J^{B*} and J^{NB*}) the learning speed $\dot{\pi}$ will be slower for small or large values of π than for intermediate ones. Note that, for any $\pi_0 < 1$, it is never optimal for the government to follow a policy that would ultimately insure that all agents are believers, $\pi = 1$. There are two concurrent reasons for that. On the one hand, as π increases, the government has to deviate more and more from y^* in order to make believing more profitable than not believing. On the other hand, the learning speed slows down. Thus, the discounted benefits from increasing π decrease. The rationale for steering the inferior equilibrium E_L starting from any $\pi_0 < \pi_S$ lies in the fact that, for the reasons mentioned above, the governmental losses along the optimal path towards E_R are sufficiently higher than the ones needed to steer the economy towards E_L to make the second preferable.

Above results suggest that, for example, the policy of a newly elected government may be very different, depending on the fraction of the population that is ready to give it blind trust just after the election. If the initial confidence in the government is high, the costs of building a large fraction of believers are compensated, in the long run, by accrued benefits. The private sector as a whole profits. However, if the government's trustworthiness is low from the onset, its interest is to exploit its initial credibility for short term gains, although this leads to an inferior situation where ultimately nobody trusts the government.

The dynamics of the canonical system at the unstable middle equilibrium E_M form a spiral. Therefore, the threshold value π_S is typically distinct from π_M, i.e. $\pi_S \neq \pi_M$, and the threshold takes the particularly challenging form of a Skiba point. In particular, no local analytical expression exists that characterize a Skiba point. Thus, Skiba points must be computed numerically. It is important to recognize that the candidate solution represented by the thick line in Figure 1 is not optimal in some unknown neighborhood of π_s, since it does not satisfy the sufficient conditions for an optimum in this neighborhood. For a reference article on thresholds and Skiba points in economic models, see Deissenberg et al. (2003).

3.1.2 Sensitivity analysis

Numerical analyses show that an increase in the private sector's flexibility, i.e. an increase in β, shifts the stable equilibrium E_R to the right and the unstable equilibrium E_M as well as of the Skiba point π_S to the left. In other words, if the population reacts quickly to payoff differences, the government will follows a policy that converges towards the upper equilibrium even if the initial core of believers is relatively small. Indeed, the fast speed of reaction of the private sector means that the cumulated costs incurred by the government *en route* to E_R will be small and easily compensated by its gains around and at E_R. Reinforcing this, the government does not have to make believers much better off than non-believers in order to insure a fast reaction. As a result, for β large, the equilibrium E_R is characterized by a large proportion of believers, and thus insures a small stationary loss to all agents.

Not surprisingly, an increase of the discount factor ρ has the opposite effect. Impatient governments will want to build up confidence only if the initial proportion of believers is high. The resulting equilibrium value of π will be relatively low since the time and efforts needed now for an additional increase of the stock of believers weighs heavily compared to the expected future benefits.

Like an increase of β, an increase in the natural rate of unemployment $\bar{u}$ implies a higher value of π_R and a lower one of π_M. This principally reflects the fact that the static governmental loss J^{G*} is linearly increasing in $\bar{u}$, and decreasing with diminishing slope in π, see (10). A higher $\bar{u}$ thus creates the incitation for following a policy that insures higher π-values.

The impact of an augmentation of the parameter θ, that measures the sensitivity of the individual unemployment rate with respect to prediction errors, is more ambiguous. For small values of θ, π_R increases while π_M may increase or decrease. Otherwise π_R and π_M both decrease.

3.1.3 Optimal value functions

In Figure 2, we plotted the government's optimal value function $V^{G\$}$:

$$V^{G\$} = V^{G\$}(\pi_0) := \min_{\{y^a, y\}_{t\in[0,\infty)}} V^G(\pi_0)$$

for the reference values of the parameters[10] (continuous line). This value function is non-differentiable at the Skiba point π_S. One recognizes that

[10]The optimal value function $V^{G\$}$ gives the government's minimum cumulated discounted loss over $t \in [0, \infty)$ when the initial proportion of believers is π_0.

$\pi_S < \pi_M$, that is, that the Skiba point is on the left of the unstable middle equilibrium. This is generically true in the model, independently of the underlying parameter values.

Figure 2 also shows (dotted line) the optimal value function of the government when $\beta = 0$, that is when the agents do not learn and thus when the proportion of believers and non-believers in the economy does not change over time. Since there is no learning, there are no longer any dynamics or threshold. Rather, for any value of π the government is from the onset and forever at its stationary optimum. The value function for $\beta = 0$ is everywhere below the value function for $\beta = 1$. Thus, the government would always prefer facing a population that that does not learn than one that does, independently of the initial proportion of believers and non-believers. Numerical investigations not presented here[11] show that the important result that $V^{G\$}{}_{|\beta=0} < V^{G\$}{}_{|\beta>0}$ is generic for the model and does not depend upon the values of other the parameters.

If the initial stock of believers is so small that it is optimal for the government to approach the equilibrium E_L for a given positive β, a simple consideration establishes that the government would indeed always prefer complete public inflexibility. If $\beta = 0$, the government should choose for any given π the constant controls $y^{a*}(\pi)$ and $y^*(\pi)$ that maximize its instantaneous utility at π. If $\beta \neq 0$ it must choose for the same value of π the controls $y^{a\$}(\pi)$ and $y^{\$}(\pi)$ that solve (16). These latter controls do not maximize the government's instantaneous utility at π, since they must insure an optimal balance between high instantaneous utility and favorable dynamics of π. Additionally, the stock of believers decreases over time for positive β. This also induces higher losses compared to $\beta = 0$ where the stock of believers stays at π_0. Thus, a positive β increases the instantaneous and future losses of the government through its impact on J^G and on the dynamics of π. This argument, repeated for any $\pi \leq \pi_S$, leads after integration to the conclusion that the optimal value function for the dynamic problem must lie above the static return function everywhere on the intervall $(0, \pi_S)$.

The above statement, however, cannot be extended to the interval $(\pi_S, 1)$. For the reasons already mentioned, $(y^{a\$}, y^{\$})$ generates higher instantaneous losses than (y^{a*}, y^*) on this interval. However, contrary to what happened in the case $\pi_0 < \pi_S$, the stock of believers increases along the dynamically optimal path $\{y^{a\$}, y^{\$}\}$ for $\pi_0 > \pi_S$ and $\beta > 0$. This limplies a reduction of the future governmental losses. Thus, a positive β increases the instantaneous but tends to reduce the future losses of the government.The two effects formerly identified work now in opposite directions. The overall effect

[11] See, however, Section 3.3 and most particularly Figure 8.

of a positive β on the losses of the government is therefore unclear. Hence, for $\pi_0 \in [\pi_S, 1]$ the numerically observed property that $V^{G\$}{}_{|\beta=0} < V^{G\$}{}_{|\beta>0}$ is most likely partly related to the shape of the underlying functions – a point that we didn't investigate in more detail.

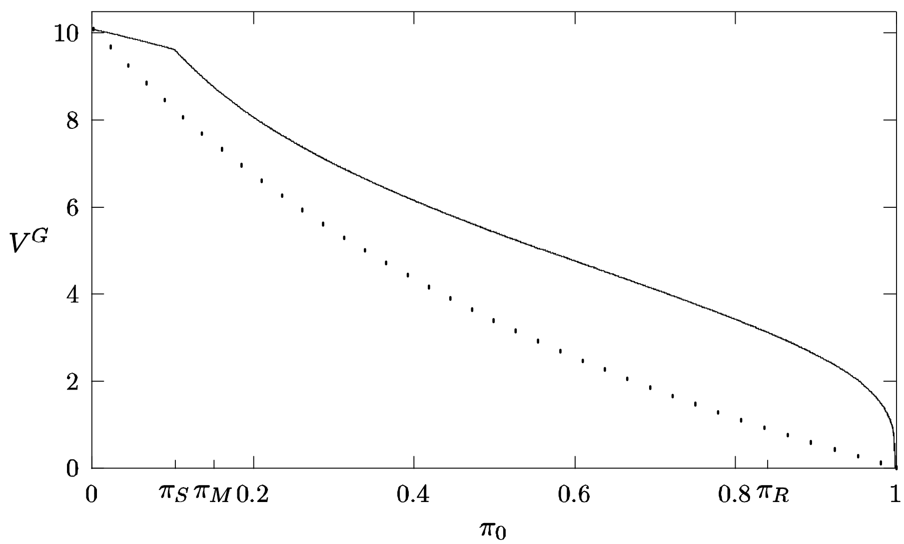

Figure 2: Impact of π_0 on the government's value function, $\beta = 0$ and $\beta = 1$.

On the other hand, Figure 3 shows that the average cumulated discounted loss of the private sector[12]:

$$V^P = \int_0^\infty \frac{1}{2} e^{-\rho t} \left\{ \pi \left[\left(y^\$ - y^{a\$} \right)^2 + (y^\$)^2 \right] + (1-\pi) \left[(y^\$ - R^{NB}(y^{a\$}))^2 + (y^\$)^2 \right] dt \right\}$$

is larger for $\beta > 0$ (thick line) than for $\beta = 0$ (dotted line). *Ceteris paribus,* the government is better off when there is a larger population of believers. Thus, whenever it can influence π, i.e. whenever $\beta > 0$, it rationally chooses an announced and a realized inflation rate that benefits the believers. This choice either slows down the convergence towards the equilibrium E_L, or speeds up the convergence towards the equilibrium E_R and shifts the latter to the right, see sub-section 3.1.2. Thus, word of mouth learning benefits the private sector as a whole compared to the situation where agents do not learn, as we shall now see in more detail. Notice that the cumulated loss

[12]That is with some abuse of language, the private sector's loss function.

$V^{P\$}$ of the private sector jumps at the Skiba point for $\beta = 1$. This follows from the fact that the optimal policy of the government is discontinuous at the Skiba point, a phenomenon that is typical for Skiba points and never happens at non-Skiba thresholds.

Moreover, $V^{P\$}$ is (typically significantly) lower for $\pi \in (\pi_S, 1)$ than for $\pi \in (0, \pi_S)$. This hints that the welfare of a population may depend in a non-smooth way upon the initial trust accorded initially to a government. If this trust is by mischance low, even a government that is receptive to the private welfare will choose to let the economy converge towards the inferior equilibrium E_L. Thus, the initial level of confidence enjoyed by a government may have profound implication for its later behavior and for the well-being of society as a whole.

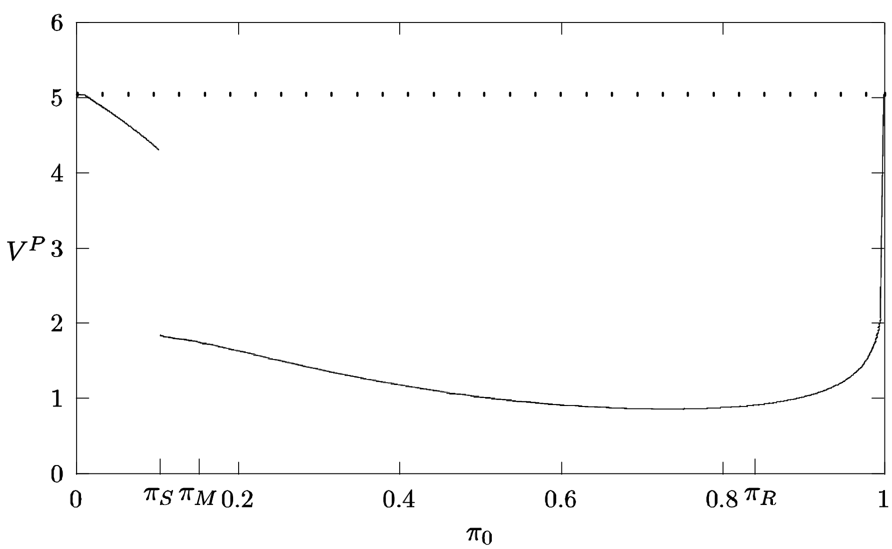

Figure 3: Impact of π_0 on the private sector's value function, $\beta = 0$ and $\beta = 1$.

3.2 Bifurcation towards a unique equilibrium

The scenario with two stable equilibria separated by a Skiba point can change qualitatively as the parameter values are altered. For small values of $\bar{u}, \theta, \beta$, respectively for large values of ρ the equilibrium E_R collides with S_R and disappears. The bifurcation values (all other parameters being held at the reference values) are $\tilde{u} \approx 3.51, \tilde{\beta} \approx 0.41, \tilde{\theta} \approx 0.355, \tilde{\rho} \approx 7.4$. For these values, the only stable equilibrium is E_L. Thus, in the long-run, the proportion of believers goes to 0. The resulting stationary solution is the one for the standard Kydland-Prescott model without believers, and will not be discussed further.

In the phase space these collisions are (inverse tangent) bifurcations. Before they occur the two branches of the λ-isocline collide and re-connect, as shown by the phase diagrams of Figures 4 to 6. Notice that the phase diagram of Figure 4 is modulo some scaling the same as in Figure 1.

For very small values of β, $\beta < 0.15$, the λ-isocline has an intersection with the line $\pi = 1$. Thus, there exists an additional equilibrium. However, this equilibrium is a repelling node and its existence does not alter the qualitative properties of the model.

The occurrence of bifurcation towards a unique equilibrium can be easily interpreted for the different parameters considered. For instance, let us focus on ρ. A government that is discounting heavily (ρ large) will act almost myopically and choose a policy close to the one that maximizes its instantaneous utility, y^{a*}, y^*. Then $J^B > J^{NB}$, and π goes to 0 for all initial π_0.

Our main focus in the remainder of the paper, however, lies on the impact of changes in the public flexibility parameter β on the qualitative properties of the equilibrium outcome and on the value functions of the government and the private sector.

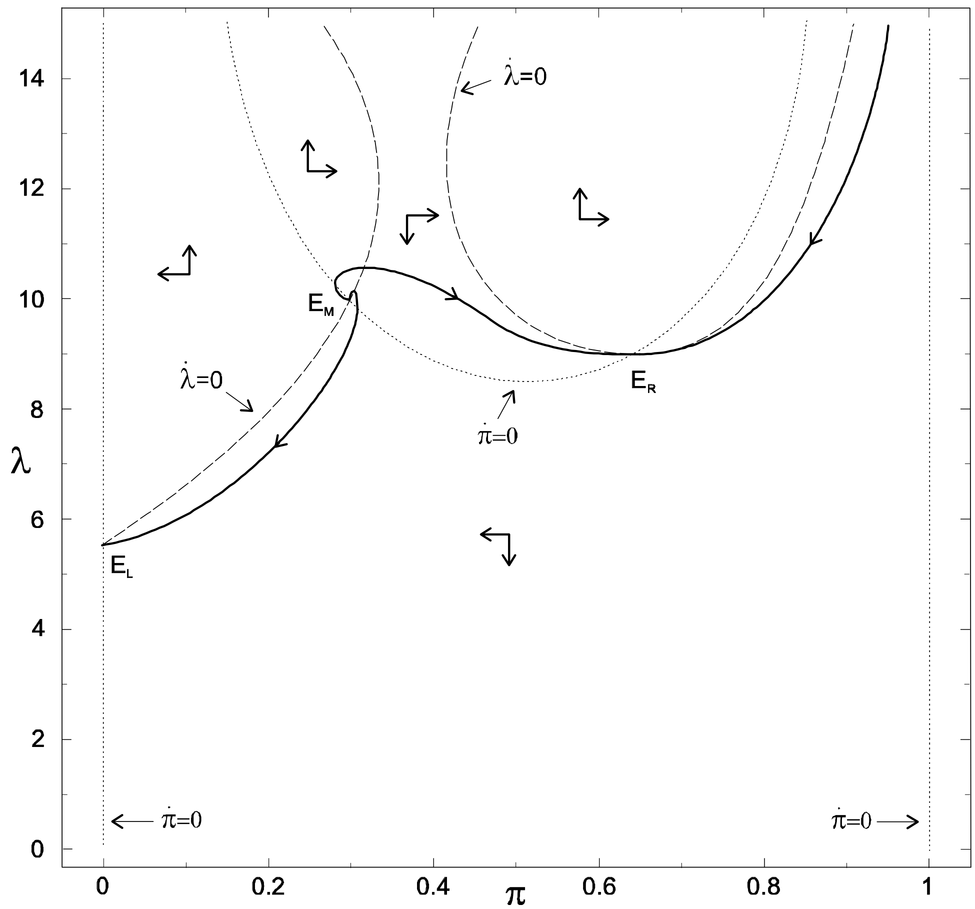

Figure 4: Phase diagram, $\beta = 0.5$

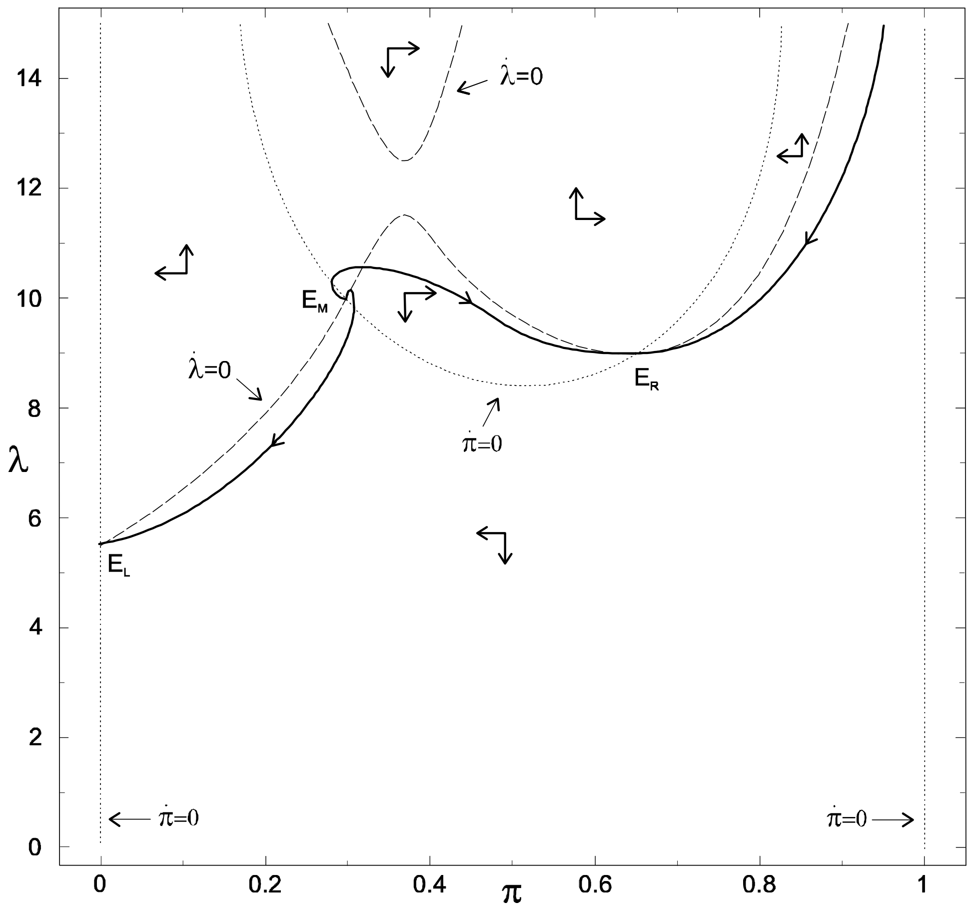

Figure 5: Phase diagram, $\beta = 0.485$

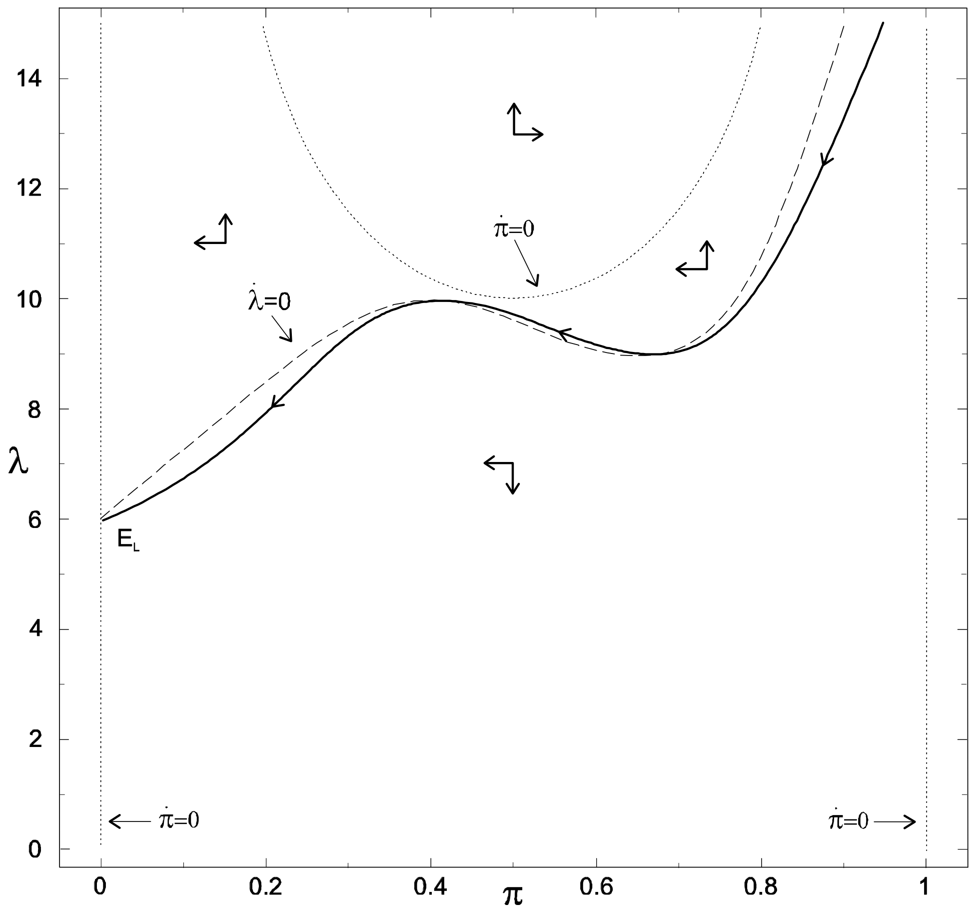

Figure 6: Phase diagram, $\beta = 0.48$

3.3 Impact of public flexibility

As discussed above, a sufficient decrease in the public flexibility parameter β leads to the disappearance of the equilibrium E_R. Figure 7 shows the corresponding bifurcation diagram. The fraction of believers in the equilibrium E_R goes down as public flexibility decreases whereas simultaneously the initial number of believers necessary to reach E_R under the optimal policy goes up. For $\beta < \tilde{\beta} \approx 0.41$, public flexibility is so low that the government prefers to exploit the current stock of believers, leading to a slow decrease of π towards zero regardless of the initial number of believers.

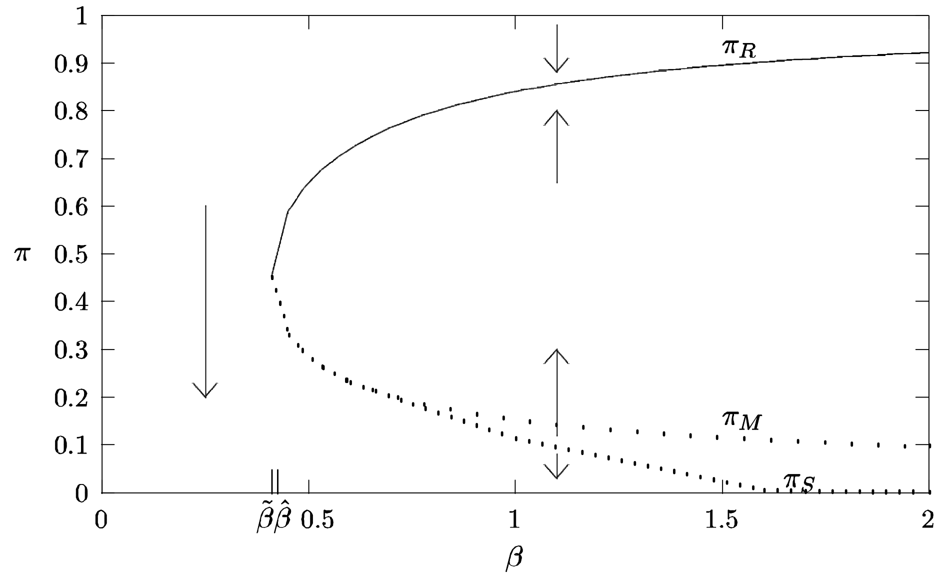

Figure 7: Bifurcation diagram, reference configuration.

Noteworthy on Figure 7 is the fact that the Skiba value π_S converges towards 0 and pratically reaches this value for β greater than 1.65 approximately. Thus, if β is larger than 1.65, the economy has only one non-trivial equilibrium, E_R, for all practical purpose The government will steer the economy towards this superior equilibrium for very small positive initial values of π. Thus, the inferior equilibrium E_L and only this equilibrium will be observed even if there are no almost no believers from the onset. Concurrently, the canonical system (20)-(22) has an unstable equilibrium E_M which, for the parameter constellation considered, does not approach 0 but stays above a level of $\pi_M = 0.1$. This equilibrium does not satisfy the sufficient conditions for an optimum and, thus, has no substantive value for the problem considered. A superficial analysis of the underlying phase diagram may nonetheless easily lead to the – erroneous – affirmation that E_M is a threshold separating two basins of attraction towards E_L and E_R respectively. One can find numerous examples of such mistakes in the economic litterature[13].

The discussion of section 3.1.3 suggested that the government always prefers low public flexibility to high public flexibility. Is this indeed so and is the reverse true for the private sector? To help answer these questions, Figure 8 shows the optimal value functions of the government (continuous line) and of the private sector (dotted line) as a function of the flexibility β. As before,

[13]The authors are thankful to Franz Wirl for bringing this point to their attention.

it is assumed that initially half of the agents are believers, $\pi(0) = 0.5$. In the figure, $\tilde{\beta}$ is the bifurcation value introduced above. As can be seen, the qualitative properties of both value functions changes approximately at $\tilde{\beta}$. The crucial β value, however, is the value $\hat{\beta}$ for which the equilibrium value π_R equals the initial value $\pi(0) = 0.5$, that is, the solution of $\pi_R(\beta) = 0.5$. As can be seen from Figure 7 this value is slightly larger than $\tilde{\beta}$. We shall examine the two cases, $\beta < \hat{\beta}$ and $\beta > \hat{\beta}$, separately.

When $\beta < \hat{\beta}$, it is optimal for the government to act in such a way as to let the stock of believers decrease. For $\tilde{\beta} < \beta < \hat{\beta}$ the stock of believers will decrease to π_R, for $\beta < \tilde{\beta}$ it will ultimately go down to zero. However, since the government benefits from being trusted, the decrease in π should optimally be slower than the one under the static decisions y^{a*}, y^*. A small β implies a slow decrease of π even if the governmental actions make the non-believers significantly better off than the believers. The value of π will decrease, moreover, even if the governmental actions make non-believers better off than believers. Thus, the government does not have to take much into account the specific interests of the private sector and does not need to deviate much from y^{a*}, y^* in any t. Its loss is low, the private loss is high. As β increases, however, the government must pay more attention to keeping the gap in losses between believers and non-believers small, in order to insure a relatively slow decrease of π. As the governmental loss increases, the private loss decreases.

When $\beta > \hat{\beta}$, the government interest is to built up the stock of believers to the value $\pi_R > \pi(0)$. To do this, it must insure a positive value of $\dot{\pi}$,and thus make sure that believers have lower losses than non-believers. This implies choosing an inflation rate below the instantaneously optimal level y^*. As β increases, it becomes more and more easy to trigger a fast increase in the level of believers by making them only slightly better off than the non-believers. Thus, the government takes less and less into account the private interests. As the governmental loss decreases, the private loss increases. Both value functions are not very sensitive to the value of β. Notice that the minimum of $V^{P\$}$ is attained for β slightly larger than $\tilde{\beta}$.

It should be noted that whereas the long run attractor of π changes discontinuously as β crosses $\tilde{\beta}$ from above, both $V^{G\$}$ and $V^{P\$}$ are continuous in β. The function $V^{G\$}$ must be continuous since it is the value function of an optimization problem. The continuity of $V^{P\$}$, however, may appear counterintuitive. The reason for this continuity can be recognized from the Figures 5 and 6. For β slightly smaller than $\tilde{\beta}$, the optimal trajectory converges in the long run, starting from any $\pi_0 < 1$, to $\pi = 0$. However, this trajectory passes near the point where π_R used to be just before the bifurcation occurred. At this point, it is very close to the isoclines $\dot{\pi} = 0$ and $\dot{\lambda} = 0$ as well. Hence, the economy spends a very long time near the old equilibrium E_R, and the discounted losses of the private sector are continuous in β.

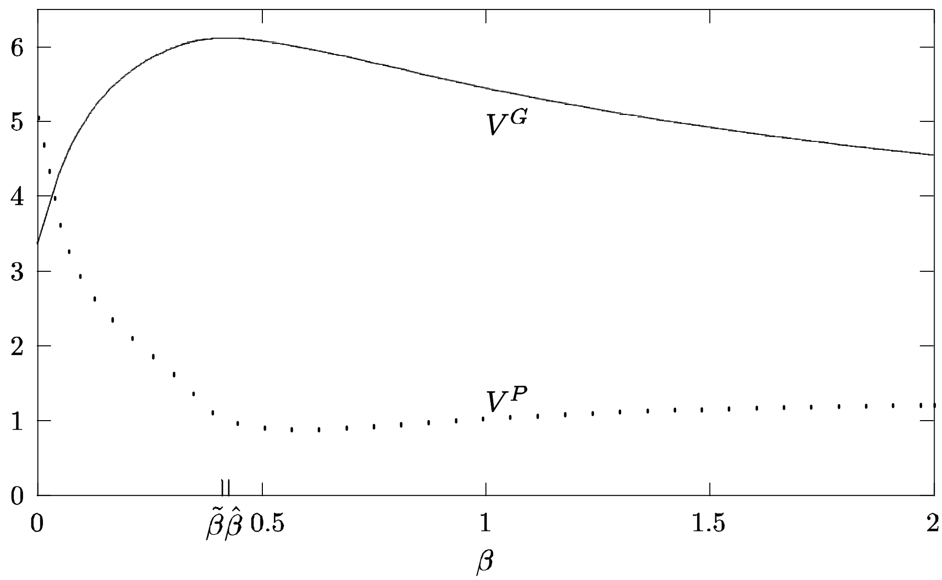

Figure 8: Impact of β on the value functions.

4 Conclusion

The starting point of this paper is a situation where standard optimizing behavior leads to a Pareto-inferior outcome, although there is no fundamental conflict of interest neither among the different agents nor between the private sector and the government. In addition to the monetary policy problem studied here, such a situation is common for instance in disaster relief, patent protection, capital levies, or default on debt – suggesting a wide applicability of the proposed approach.

This approach exploits a basic property of the 1997 static Kydland-Prescott and related models. The existence of a core of believers, however small, who take the government's announcements at face value is sufficient to Pareto-improve the outcome. This property crucially hinges on the fact that the private sector is atomistic, and thus, that single agents do not anticipate the collective impact of their individual decisions.

The static framework is extended by assuming that the proportion of believers and non-believers in the economy changes over time based on the losses' differences for the two kinds of agents. The change obeys a word of mouth process driven by the relative success of the one or the other private strategy – to believe or not to believe. It is assumed that the government recognizes its ability to influence the word to mouth dynamics by an appropriate choice of actions, and is interested not only in its instantaneous but also in its future losses. It is shown that worth to mouth learning is insufficient to insure a Pareto-optimal outcome. However, it may lead to a Pareto improvement if the government is sufficiently patient and if the *flexibility* of the private agents, i.e. the intensity with which they react to payoff differences, is sufficiently high.

For an intermediate range of flexibility, the initial fraction of believers in the population plays a distinguished role. If this fraction is low, the government optimal course of action ultimately leads to an equilibrium where nobody any longer believes the government, i.e., to the standard equilibrium of the static problem. The losses of the government and of the private agents are very high at this equilibrium. On the other hand, if the initial proportion of believers is sufficiently large, it is rational for the government to act in a way that ultimately insures that a positive (but strictly smaller than 1) fraction of the private agents are believers. The resulting equilibrium Pareto-dominates the one where nobody believes the government. The basins of attractions of both equilibria are separated by a Skiba point, i.e. by a value of the initial stock of believers at which the government is indifferent between steering the one or the other equilibrium. For a sufficiently large flexibility, the Skiba point almost coincides with the origin. That is, the government will steer to the better equilibrium for almost any strictly positive initial population of believers.

The cumulated losses of the private sector are discontinuous at the Skiba point. They are higher if the government chooses to converge towards the standard static equilibrium, lower otherwise. Moreover, they are typically higher when the initial proportion of believers is smaller than the Skiba value than in the opposite case.

An important insight from the model is that the level of private flexibility that minimizes the government's cumulated losses differs from the level that minimizes the private sector's losses. Somewhat surprisingly, the government is always better off when the population is totally inflexible and this, independently of the number of believers (When the private sector is completely inflexible, this number does not change over time). This reflects among others the fact that, when the population is inflexible, the government does not need to nurture the stock of believers by insuring good outcomes to its members. The private sector, by contrast, attains its lowest cumulated losses for a positive but not too high flexibility. As indicated above, a low flexibility reduces the incentive of the government to insure, at a cost, good outcomes for the believers. A high flexibility has a similar effect, since the proportion of believers then changes very quickly as a function of small payoff differences.

The solution is obtained under the assumption that the non-believers are boundedly rational, in the restrictive sense that while they know the instantaneous loss function of the government, they do not recognize that the government is solving an intertemporal optimization problem and that it is using the word of mouth process strategically. We leave to future research the non-trivial task of exploring the consequences of a relaxation of this hypothesis.

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