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Business Cycles, Wage Stickiness and Labor Market Disequilibrium

by

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Abstract

Real Business Cycles are often studied in the context of the general equlibrium framework with competitive markets, flexible wages and prices. This restricts the effective application of intertemporal models to the real world business cycles. Stickiness of wages and prices have been considered in recent monopolistic competition models. In this paper, we go a step further and separate labor supply and labor demand and allow for different variants of nonclearing labor markets. Calibration for U.S. economy shows that such model variants will produce a higher volatility in employment and a more realistic correlation between consumption and employment and thus fit the data significantly better than the standard model.

Keywords: Wage stickiness, disequilibrium, RBC models, stochastic dynamic models.

JEL classification: E32, C61.

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1 Introduction

The real business cycle (RBC) model has become one of the major approaches in macroeconomics to explain observed economic fluctuations. Despite its rather simple structure, it can explain, at least partially, the volatility of some major macroeconomic variables such as output, consumption and capital stock. However, to explain the actual variation in employment the model generally predicts an excessive smoothness of labor effort in contrast to empirical data. Another problem (which is also related to the labor market) is that the model implies an excessively high correlation between consumption and employment while empirical data only indicate a weak correlation.

The excessive smoothness of labor effort, and thus the low variation in the employment series is a well-known puzzle in the RBC literature. A recent discussion on this failure of the RBC model is given in Schmidt-Grohe (2001). The excessive correlation between consumption and labor has, to our knowledge, not been sufficiently studied in the literature. We will also explore this puzzle in section 3 when we calibrate the model.

The above mentioned labor market puzzles in the RBC literature are related to the specification of the labor market structure, which is assumed to be competitive, with flexible wages that clear the labor market in all periods. For the model to effectively replicate employment variation, it seems necessary to make improvement upon labor market specifications. One perspective for such an improvement is to allow for nonclearing of the labor market.

Attempts have been made that introduce variants of Keynesian features into the RBC model. There are models of wage contracts and efficiency wages where nonclearing of the labor market can arise, see, for instance, Benassy (1995, 2002), Danthine and Donaldson (1990, 1995) and Uhlig and Xu (1996). [1] In all these papers with nonclearing labor market, an explicit labor demand function is introduced, which is derived from the marginal product of labor. However, the decision rule with regard to labor supply in those models is often dropped because the labor effort no longer appears in the household's utility function. Consequently, the moments of labor effort become purely demand-determined. [2]

New Keynesian literature presents models with imperfect competition and sluggish price and wage adjustments where labor effort is endogenized. Important work of this type can be found in Rotemberg and Woodford (1995, 1999), King and Wollman (1999), Gali (1999), Erceg, Henderson and Levin (2000) and Woodford (2003). However, the market in those models is still cleared since the producer is assumed to supply the output according to what the market demands for the existing price. A similar consideration is also assumed to hold for the labor market. Here the wage rate is set optimally by a representative of the household according to the expected market a demand curve for labor. Once the wage has been set, it is assumed to be sticky for some time period and only a fraction of wages are set optimally in each period. In those models there will be a gap again between the optimal wage and existing wage,

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[1] Another line of recent research on modelling unemployment in dynamic optimization framework can be found in the work by Merz (1999) who employs search and matching theory to model the labor market.

[2] The labor supply in these models is implicitly assumed to be given exogenously, such as set to 1. Hence disequilibrium occurs if the demand is not equal to 1.

yet the labor market is still cleared since the household is assumed to supply labor whatever the market demand is at the given wage rate.[3]

In this paper, we shall present a dynamic model that allows for a noncleared labor market. Wages are presumed to be sticky, which could be seen to be caused by staggered wage as described by Taylor (1980), Calvo (1983) or other theories of sluggish wage adjustment. The objective to construct a model such as ours is to approach the two aforementioned labor market puzzles coherently within a single model of dynamic optimization. Unlike the model with efficiency wage and nonclearing labor market, we however do not drop the labor effort from the decisions of the household. We view the decision concerning the labor effort derived from dynamic optimization as a natural reflection of the agent's willingness to supply labor. With the introduction of labor demand, the two basic forces in the labor market can thus be formalized. One of the advantages from this formulation, as will become clear, is that a variety of disequilibrium rules could be adopted to specify the realization of actual employment when labor market are not cleared. In order to rationalize the existence of nonclearing markets we introduce a multiple stage decision process which is shown to generate such an outcome. We also indicate how such a model can be used to explain the secular change in unemployment.

The remainder of this paper is organized as follows. Section 2 is a theoretical preparation by presenting a model with noncleared labor market. Section 3 estimates and calibrates our different model variants for the U.S. economy. Section 4 discusses some international differences in labor market institutions and what it would predict for our model. Section 5 concludes the paper. The appendix presents the derivation of some decision rules.

2   An Economy with Nonclearing Labor Market

We shall still follow the usual assumptions of identical households and identical firms. Therefore we are considering an economy that has two agents: the representative household and the representative firm. There are three markets in which the agents exchange their products, labor and capital. The household owns all the factors of production and therefore sells factor services to the firm. The revenue from selling factor services can only be used to buy the goods produced by the firm either for consuming or for accumulating capital. The representative firm owns nothing. It simply hires capital and labor to produce output, sells the output and transfers the profit back to the household.

Unlike the typical RBC model, in which one could assume a once-for-all market, we, however, in this model shall assume that the market to be re-opened at the beginning of each period t. This is necessary for a model with nonclearing markets in which

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[3]See, for example, Woodford (2003, ch. 3). There are also traditional Keynesian models that allow for disequilibria, see Benassy (1984) among others. Yet, the well-known problem of these earlier disequilibrium models was that they disregard intertemporal optimizing behavior and never specify who sets the price. This has now been resolved by the modern literature of monopolistic competition as can be found in Woodford (2003). However, while resolving the price setting problem, the decision with regard to quantities seems to be ignored. The supplier may no longer behave optimally concerning their supply decision, but simply supplies whatever the quantity the market demands for at the current price.

adjustments should take place which leads us to a multiple stage adaptive optimization behavior. Yet, let us first describe how prices and wages are set.

2.1 Price and Wage Setting

As usual we presume that both the household and the firm express their desired demand and supply on the basis of given prices, including the output price p_t, the wage rate w_t and the rental rate of the capital stock r_t, we shall first discuss how the period t prices are determined at the beginning of period t. Note that there are three commodities in our model. One of them should serve as a numeraire, which we assume to be the output. Therefore, the output price p_t always equals 1. This indicates that the wage w_t and the rental rate of capital stock r_t are all measured in terms of the physical units of output.[4] As to the rental rate of capital r_t, it is assumed to be adjustable so as to clear the capital market. We can then ignore its setting. Indeed, as will become clear, one can imagine any initial value of the rental rate of capital when the firm and the household make the quantity decisions and express their desired demand and supply. This leaves us to focus the discussion only on the wage setting. Let us first discuss how the wage rate might be set.

Most recent literature, in discussing wage setting,[5] assumes that it is the supplier of labor, the household, or its representative, that sets the wage rate whereas the firm is simply a wage taker. On the other hand, there are also models that discuss how firms set the wage rate.[6] In actual bargaining it is likely, as Taylor (1999) has pointed out, that wage setting is an interacting process between firms and households. Despite this variety of wage setting models, we, however, follow the recent approach. We assume that the wage rate is set by a representative of the household which acts as a monopolistic agent for the supply of labor effort as Woodford (2003, ch. 3) has suggested. Woodford (2003:221) introduces different wage setting agents and monopolistic competition since he assumes heterogenous households as different suppliers of differentiated types of labor. In appendix 1, in close relationship to Woodford (2003, ch.3) and Erceg et al (2000) we present a wage setting model, where wages are set optimally, but a fraction of wages may be sticky. We neglect, however, differentiated types of labor and refer only to aggregate wages.

We want to note, however, that recently many theories have been developed to explain wage and price stickiness. There is the so-called menu cost for changing prices (though this seems more appropriate for the output price). There is also a reputation cost for changing prices and wages.[7] In addition, changing the price, or wage, needs information, computation and communication, which may be costly.[8] All these efforts cause costs which may be summarized as adjustment costs of changing the price or wage. The adjustment cost for changing the wage may provide some reason for the

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[4]For our simple representative agent model without money, this simplification does not effect our major result derived from our model. Meanwhile, it will allow us to save some effort to explain the nominal price determination, a focus in the recent New Keyensian literature.

[5]See, for instance, Erceg, Henderson and Levin (2000), Christiano, Eichenbaum and Evans (2001) and Woodford (2003) among others.

[6]These are basically the efficiency wage models that are mentioned in the introduction.

[7]This is emphasized by Rotemberg (1982)

[8]See the discussion in Christiano, Eichenbaum and Evans (2001) and Zbaracki, Ritson, Levy, Dutta and Bergen (2000).

representative of the household to stick to the wage rate even if it is known that current wage may not be optimal. One may also derive this stickiness of wages from wage contracts as in Taylor (1980) with the contract period to be longer than one period.

Since workers, or their respective representative, enter usually into long term employment contracts involving labor supply for several periods with a variety of job security arrangements and termination options, a wage contract may also be understood from an asset price perspective, namely as derivative security based on a fundamental underlying asset such as the asset price of the firm. In principle a wage contract could be treated as a debt contract with similar long term commitment as exists for other liabilities of the firm.[9] As in the case of the pricing of corporate liabilities the wage contract, the value of the derivative security, would depend on some specifications in contractual agreements. Yet, in general it can be assumed to be arranged for several periods.

As noted above we do not have to posit that the wage rate, w_t, to be completely fixed in contracts and never responds to the disequilibrium in the labor market. One may imagine that the dynamics of the wage rate, for example, follows the updating scheme as suggested in Calvo's staggered price model (1983) or in Taylor's wage contract model (1980). In Calvo's model, for example, there is always a fraction of individual prices to be adjusted in each period t.[10] This can be expressed in our model as the expiration of some wage contracts, to be reviewed in each time period and therefore new wage contracts will be signed in each t. The new signed wage contracts should respond to the expected market conditions not only in period t but also through t to $t + j$, where j can be regarded as the contract period.[11] Through such a pattern of wage dynamics, wages are only partially adjusted.[12]

Explicit formulation of wage dynamics of a Calvo type of updating scheme, particularly with differentiated types of labor, is studied in Erceg et al (2000) and Woodford (2003, ch. 3) and briefly sketched, as underlying our model, for an aggregate wage in appendix 1 of our paper. A more explicit treatment is not the task of this paper. Indeed, as will become clear in section 3, the empirical study of our model does not rely on how we formulate the wage dynamics. All we need to presume is that, wage contracts are only partially adjusted, giving rise to a sticky aggregate wage.

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[9]For such a treatment of the wages as derivative security, see Uhlig (2003). For further details of the pricing of such liabilities, see Grüne and Semmler (2004).

[10]These are basically those prices that have not been adjusted for some periods and there the adjustment costs (such as the reputation cost) may not be high.

[11]This type of wage setting is used in Woodford (2003, ch. 4) and Erceg et al. (2000).

[12]Strictly speaking, the so-called labor market clearing should be defined as the condition that the firm's willingness to demand factors is equal to the household's willingness to supply factors. Such concept has somehow disappeared in the new Keynesian literature in which the household supplies the labor effort according to the market demand and therefore it does not seem to face excess demand or supply. Yet, even in this case, the household's willingness to supply labor effort is not necessarily equal to its actual supply or the market demand. This further indicates that even if there are no adjustment costs so that the household can adjust the wage rate at every time period t, the disequilibrium in the labor market may still exist.

2.2 The Household's Desired Transactions

The next step in our multiple stage decision process is to model the quantity decisions of the households. When the price, including the wage, has been set, the household is then going to express its desire of demand for goods and supply of factors. We define the household's desired demand and supply as those that can allow the household to obtain the maximum utility on the condition that these demand and supply can be realized at the given set of prices. We can express the household's desired demand and supply as a sequence of output demand and factor supply $\{c_{t+i}^d, i_{t+i}^d, n_{t+i}^s, k_{t+i+1}^s\}_{i=0}^{\infty}$, where i_{t+i} is referred to investment. Note that here we have used the superscripts d and s to refer to the agent's desired demand and supply. The decision problem for the household to derive its demand and supply can be formulated as

$$\max_{\{c_{t+i}, n_{t+i}\}_{i=0}^{\infty}} E_t \left[\sum_{i=0}^{\infty} \beta^i U(c_{t+i}^d, n_{t+i}^s) \right] \tag{1}$$

subject to

$$c_{t+i}^d + i_{t+i}^d = r_{t+i}k_{t+i}^s + w_{t+i}n_{t+i}^s + \pi_{t+i} \tag{2}$$

$$k_{t+i+1}^s = (1-\delta)k_{t+i}^s + i_{t+i}^d \tag{3}$$

Above π_{t+i} is the expected dividend. Note that (2) can be regarded as a budget constraint. The equality holds due to the assumption $U_c > 0$. Next, we shall consider how the representative household calculates π_{t+i}.

Assuming that the household know the production function $f(\cdot)$ while it expects that all its optimal plans can be fulfilled at the given price sequence $\{p_{t+i}, w_{t+i}, r_{t+i}\}_{i=0}^{\infty}$, we thus obtain

$$\pi_{t+i} = f(k_{t+i}^s, n_{t+i}^s, A_{t+i}) - w_{t+i}n_{t+i}^s - r_{t+i}k_{t+i}^s \tag{4}$$

Explaining π_{t+i} in (2) in terms of (4) and then substituting from (3) to eliminate i_t^d, we obtain

$$k_{t+i+1}^s = (1-\delta)k_{t+i}^s + f(k_{t+i}^s, n_{t+i}^s, A_{t+i}) - c_{t+i}^d \tag{5}$$

For the given technology sequence $\{A_{t+i}\}_{i=0}^{\infty}$, equations (1) and (5) form a standard intertemporal decision problem. The solution to this problem can be written as:

$$c_{t+i}^d = G_c(k_{t+i}^s, A_{t+i}) \tag{6}$$

$$n_{t+i}^s = G_n(k_{t+i}^s, A_{t+i}) \tag{7}$$

We shall remark that although the solution appears to be a sequence $\{c_{t+i}^d, n_{t+i}^s\}_{i=0}^{\infty}$ only (c_t^d, n_t^s) along with (i_t^d, k_t^s), where $i_t^d = f(k_t^s, n_t^s, A_t) - c_t^d$ and $k_t^s = k_t$, are actually carried into the market by the household for exchange due to our assumption of re-opening market.

2.3 The Firm's Desired Transactions

As in the case of the household, the firm's desired demand for factors and supply of goods are those that maximize the firm's profit under the condition that all its intentions can be carried out at the given set of prices. The optimization problem

for the firm can thus be expressed as being to choose the input demands and output supply (n_t^d, k_t^d, y_t^s) that maximizes the current profit:

$$\max \ y_t^s - r_t k_t^d - w_t n_t^d$$

subject to

$$y_t^s = f(A_t, k_t^d, n_t^d) \tag{8}$$

For regular conditions on the production function $f(\cdot)$, the solution to the above optimization problem should satisfy

$$
\begin{aligned}
r_t &= f_k(k_t^d, n_t^d, A_t) \tag{9} \\
w_t &= f_n(k_t^d, n_t^d, A_t) \tag{10}
\end{aligned}
$$

where $f_k(\cdot)$ and $f_n(\cdot)$ are respectively the marginal products of capital and labor. Next we shall consider the transactions in our three markets. Let us first consider the two factor markets.

2.4 Transaction in the Factor Market and Actual Employment

We have assumed the rental rate of capital r_t to be adjustable in each period and thus the capital market is cleared. This indicates that

$$k_t = k_t^s = k_t^d$$

As we have discussed previously, due to the adjustment cost in setting the wage, it is feasible that the existing wage rate w_t cannot clear the labor market.[13] In this case, we shall have to specify what rule should apply regarding the realization of actual employment.

> Disequilibrium Rule: When disequilibrium occurs in the labor market either of the following two rules will be applied:

$$n_t = \min(n_t^d, n_t^s) \tag{11}$$

$$n_t = \omega n_t^d + (1 - \omega) n_t^s \tag{12}$$

> where $\omega \in (0, 1)$.

Above, the first is the famous short-side rule when disequilibrium occurs. It has been widely used in the literature on disequilibrium analysis (see, for instance, Benassy 1984 and 2002 among others). The second might be called the compromising rule. The rule indicates that when disequilibrium occurs in the labor market both firms and workers have to compromise. If there is excess supply, firms will employ more labor than what they wish to employ.[14] On the other hand, when there is excess demand, workers will have to offer more effort than they wish to offer.[15] Such mutual compromises may be due to institutional structures and moral standards of the society.[16] A detailed discussion of this rule is given in section 4.

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[13]See footnote 12 above.

[14]Labor market institutions that might support this proposition are discussed in section 4.

[15]This could be achieved by employing the same number of workers but each worker supplying more hours (varying shift length and overtime work); for a more formal treatment of this point, see Burnside et al. (1993).

[16]Note that if firms are off their supply schedule and workers off their demand schedule, a proper study would have to compute the firms' cost increase and profit loss and the workers' welfare loss. If,

2.5 Actual Employment and Transaction in the Product Market

After the transactions in these two factor markets have been carried out, the firm will engage in its production activity. The result is the output supply, which, instead of (8), is now given by

$$y_t^s = f(k_t, n_t, A_t) \tag{13}$$

Then the transaction needs to be carried out with respect to y_t^s. It is important to note that when a disequilibrium occurs in the labor market that means when the household faces a constraint in the labor market, the previous consumption plan as expressed by (6) becomes invalid due to the improper budget constraint (2), which further brings about an improper transition law of capital, (5), for deriving the plan. Therefore, the household will draw up a new consumption plan, which is derived from the following optimization program which represents a further step in our multiple stage decision process:[17]

$$\max_{(c_t^d)} U(c_t^d, n_t) + E_t \left[\sum_{i=1}^{\infty} \beta^i U(c_{t+i}^d, n_{t+i}^s) \right] \tag{14}$$

subject to

$$k_{t+1}^s = (1 - \delta)k_t + f(k_t, n_t, A_t) - c_t^d \tag{15}$$
$$k_{t+i+1}^s = (1 - \delta)k_{t+i}^s + f(k_{t+i}^s, n_{t+i}^s, A_{t+i}) - c_{t+i}^d \tag{16}$$
$$i = 1, 2, ... \tag{17}$$

Note that in this optimization program the only decision variable is about c_t^d and the data includes not only A_t and k_t but also n_t, which is given by either (11) or (12). The appendix 2 proves that we can write the solution to (14)-(17) in terms of the following equation (see the appendix 2 for details):

$$c_t^d = G_{c2}(k_t, A_t, n_t) \tag{18}$$

Given this adjusted consumption plan, the product market should be cleared if the household demands the quantity $f(k_t, n_t, A_t) - c_t^d$ for investment. Therefore, c_t^d in (18) should also be the realized consumption.

3 Estimation and Calibration

3.1 The Empirically Testable Model

Next we provide an empirical study of our model as presented in the last section. However, the model in the last section is only for illustrative purpose. The above

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however, the marginal cost for firms is rather flat (as empirical literature has argued, see Blanchard and Fisher, 1989) and the marginal disutility is also rather flat the overall loss may not be so high. The departure of the value function – as measuring the welfare of the representative household – from the standard case is studied in Gong and Semmler (2003)

[17]For a similar type of adaptive optimization, where optimizing agents learn about an environment after a first step of optimization has been undertaken, see Sargent (1999) and Zhang and Semmler (2004).

discussed model cannot be directly tested with empirical data, not only because we do not specify the forms of production function, utility function and the stochastic process of A_t, but also we do not introduce the growth factor into the model. For an empirically testable model, we employ the specifications as formulated by King, Plosser and Rebelo (1988).

Let K_t denote the capital stock, N_t the per capita working hours, Y_t the output and C_t the consumption. Assume the capital stock in the economy follows the transition law:

$$K_{t+1} = (1 - \delta)K_t + A_t K_t^{1-\alpha}(N_t X_t)^\alpha - C_t, \tag{19}$$

where δ is the depreciation rate; α is the share of labor in the production function $F(\cdot) = A_t K_t^{1-\alpha}(N_t X_t)^\alpha$; A_t is the temporary shock in technology and X_t the permanent shock that follows a growth rate γ.[18] The model is nonstationary due to X_t. To transform the model into a stationary setting, we divide both sides of equation (19) by X_t:

$$k_{t+1} = \frac{1}{1+\gamma}\left[(1-\delta)k_t + A_t k_t^{1-\alpha}(n_t \bar{N}/0.3)^\alpha - c_t\right], \tag{20}$$

where $k_t \equiv K_t/X_t$, $c_t \equiv C_t/X_t$ and $n_t \equiv 0.3 N_t/\bar{N}$ with $\bar{N}$ to be the sample mean of N_t. Note that n_t is often regarded to be the normalized hours. The sample mean of n_t is equal to 30 %, which, as pointed out by Hansen (1985), is the average percentage of hours attributed to work. Note that the above formulation also indicates that the form of $f(\cdot)$ in the previous section may follow

$$f(\cdot) = A_t k_t^{1-\alpha}(n_t \bar{N}/0.3)^\alpha \tag{21}$$

while $y_t \equiv Y_t/X_t$ with Y_t to be the empirical output.

With regard to the household preference, we shall assume that the utility function take the form

$$\log c_t + \theta \log(1 - n_t) \tag{22}$$

The temporary shock A_t may follow an AR(1) process:

$$A_{t+1} = a_0 + a_1 A_t + \epsilon_t, \tag{23}$$

where ϵ_t is an independently and identically distributed (*i.i.d.*) innovation: $\epsilon_t \sim N(0, \sigma_\epsilon^2)$.

3.2   The Data Generating Processes

For our empirical test, we consider three model variants: the typical RBC model, as a standard for comparison, and two labor market disequilibrium models with the disequilibrium rules as expressed in (11) and (12) respectively. Specifically, we shall call the benchmark model the Model I; the disequilibrium model with short side rule (11) the Model II; and the disequilibrium model with the compromising rule (12) the Model III.

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[18]Note that X_t includes both population and productivity growth.

3.2.1 The Typical RBC Model

For the benchmark RBC model, the data generating process include (20), (23) as well as

$$c_t = G_{11}A_t + G_{12}k_t + g_1 \tag{24}$$
$$n_t = G_{21}A_t + G_{22}k_t + g_2 \tag{25}$$

with y_t implied by (21). Note that here (24) and (25) are the linear approximations to (6) and (7). The coefficients G_{ij} and g_i ($i = 1, 2$ and $j = 1, 2$) are the complicated functions of the model's structural parameters, α, β, among others. They are computed by a numerical algorithm using the method of linear-quadratic approximation.[19] Given these coefficients and the parameters in equation (23), including σ_ε, we can simulate the model to generate stochastically simulated data. These data can then be compared to the sample moments of the observed economy.

Obviously, the typical RBC model does not allow for nonclearing of the labor market. The moments of the labor effort are solely reflected by the decision rule (25), which is quite similar in its structure to the other decision rule given by (24), i.e., they are both determined by k_t and A_t. This structural similarity are expected to produce two possible labor market puzzles as aforementioned:

- the volatility of the labor effort cannot deviate too much from the volatility of consumption, which generally appears to be smooth,

- the moments of the labor effort and consumption are likely to be strongly correlated.

3.2.2 The Disequilibrium Models

To define the data generating process for our disequilibrium models, we shall first modify (25) as

$$n_t^s = G_{21}A_t + G_{22}k_t + g_2 \tag{26}$$

On the other hand, the equilibrium in the product market indicates that c_t^d in (18) should be equal to c_t. This equation can be approximated as

$$c_t = G_{31}A_t + G_{32}k_t + G_{33}n_t + g_3 \tag{27}$$

In appendix 2 we provide the details how to compute the coefficients G_{3j}, $j = 1, 2, 3$, and g_3.

Next we consider the labor demand derived from the production function $F(\cdot) = A_t K_t^{1-\alpha}(N_t X_t)^\alpha$. Let $X_t = Z_t L_t$, with Z_t to be the permanent shock resulting purely from productivity growth, and L_t from population growth. We shall assume that L_t has a constant growth rate μ and hence Z_t follows the growth rate $(\gamma - \mu)$. The production function can be written as $Y_t = A_t Z_t^\alpha K_t^{1-\alpha} H_t^\alpha$, where H_t equals $N_t L_t$, which can be regarded as total labor hours. Taking the partial derivative with respect to H_t and recognizing that the marginal product of labor is equal to the real wage, we thus obtain

$$w_t = \alpha A_t Z_t k_t^{1-\alpha}(n_t^d \bar{N}/0.3)^{\alpha-1}$$

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[19] The algorithm that we use here is from Gong and Semmler (2002, 2004). It was originally developed in Gong (1997).

This equation is equivalent to (10). It generates the demand for labor as

$$n_t^d = (\alpha A_t Z_t / w_t)^{1/(1-\alpha)} k_t (0.3/\bar{N}) \tag{28}$$

Note that the per capita hours demanded n_t^d should be stationary if the real wage w_t and productivity Z_t grow at the same rate. This seems to be consistent with the U.S. experience that we shall now calibrate.

Thus, for the disequilibrium model with short side rule, Model II, the data generating process includes (20), (23), (11), (26), (27) and (28) with w_t given by the observed wage rate. As mentioned above we thereby do not attempt to give the actually observed sequence of wages a specific theoretical foundation. For our purpose it suffices to take the empirically observed series of wages. For Model III, we use (12) instead of (11).

3.3 The Data and the Parameters

Before we calibrate the models, we shall first specify the parameters. There are altogether 10 parameters in our three variants: $a_0, a_1, \sigma_\varepsilon, \gamma, \mu, \alpha, \beta, \delta, \theta$ and ω. We first specify α and γ respectively at 0.58 and 0.0045, which are standard. This allows us to compute the data series of technology A_t. With this data series, we estimate the parameters a_0, a_1 and σ_ε. The next three parameters β, δ and θ are estimated with the general method of moments (GMM) by matching the moments of the model generated by (20), (24) and (25). The estimation is conducted by a global optimization algorithm called simulated annealing.[20]. We specify μ at 0.001, which is close to the average growth rate of the labor force in U.S. The parameter ω in Model III is set to 0.2465. It is estimated by minimizing the residual sum of square between actual employment and the model generated employment. The estimation is executed by a conventional algorithm, the grid search. Table 1 illustrates these parameters:

Table 1: Parameters Used for Calibration

a_0	0.0333	σ_ε	0.0185	μ	0.0010	β	0.9930	θ	2.0189
a_1	0.9811	γ	0.0045	α	0.5800	δ	0.0208	ω	0.2465

The data set used in this paper is taken from Christiano (1987). The wage series are obtained from Citibase. It is re-scaled to match the model's implication.[21]

3.4 Calibration Results

Table 2 provides our calibration results from 5000 stochastic simulations. The results in this table are confirmed by Figure 1, where a one time simulation with the observed innovation A_t are presented. All time series are detrended by the HP-filter.

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[20]For the detail of this estimation and the simulated annealing, see Semmler and Gong (1996, 1997).

[21]Note that this re-scaling is necessary because we do not exactly know the initial condition of Z_t, which we set equal to 1. We re-scaled the wage series in such a way that the first observation of employment is equal to the demand for labor as specified by equation (28).

Table 2: Calibration of the Model Variants
(numbers in parentheses are the corresponding standard deviations)

	Consumption	Capital	Employment	Output
Standard Deviations				
Sample Economy	0.0081	0.0035	0.0165	0.0156
Model I Economy	0.0091	0.0036	0.0051	0.0158
	(0.0012)	(0.0007)	(0.0006)	(0.0021)
Model II Economy	0.0137	0.0095	0.0545	0.0393
	(0.0098)	(0.0031)	(0.0198)	(0.0115)
Model III Economy	0.0066	0.0052	0.0135	0.0197
	(0.0010)	(0.0010)	(0.0020)	(0.0026)
Correlation Coefficients				
Sample Economy				
Consumption	1.0000			
Capital Stock	0.1741	1.0000		
Employment	0.4604	0.2861	1.0000	
Output	0.7550	0.0954	0.7263	1.0000
Model I Economy				
Consumption	1.0000			
	(0.0000)			
Capital Stock	0.2043	1.0000		
	(0.1190)	(0.0000)		
Employment	0.9288	−0.1593	1.0000	
	(0.0203)	(0.0906)	(0.0000)	
Output	0.9866	0.0566	0.9754	1.0000
	(0.00332)	(0.1044)	(0.0076)	(0.0000)
Model II Economy				
Consumption	1.0000			
	(0.0000)			
Capital Stock	0.4944	1.0000		
	(0.1662)	(0.0000)		
Employment	0.4874	-0.0577	1.0000	
	(0.1362)	(0.0825)	(0.0000)	
Output	0.6869	0.0336	0.9392	1.0000
	(0.1069)	(0.0717)	(0.0407)	(0.0000)
Model III Economy				
Consumption	1.0000			
	(0.0000)			
Capital Stock	0.4525	1.0000		
	(0.1175)	(0.0000)		
Employment	0.6807	-0.0863	1.0000	
	(0.0824)	(0.1045)	(0.0000)	
Output	0.8924	0.0576	0.9056	1.0000
	(0.0268)	(0.0971)	(0.0327)	(0.0000)

First we want to remark that the structural parameters that we have used here for calibration are estimated by matching the Model I Economy to the Sample Economy. The result, reflected in Table 2, is therefore somewhat biased in favor of the Model I Economy. It is not surprising that for most variables the moments generated from the Model I Economy are closer to the moments of the Sample Economy. Yet even in this case, there is an excessive smoothness of the labor effort and the employment series of the data cannot be matched. For our time period, 1955.1 to 1983.4, we find 0.32 in the Model I Economy as the ratio of the standard deviation of labor effort to the standard deviation of output. This ratio is roughly 1 in the Sample Economy. The problem is somewhat resolved in our Model II and Model III Economies representing labor market disequilibrium. There the ratio is 1.38 and 0.69 for the Model II and Model III Economies respectively.

Further evidence on the better fit of the disequilibrium model – as concerns the volatility of the macroeconomic variables– is also demonstrated in the Figure 1 where the horizontal figures show, from top to bottom, actual (solid line) and simulated data (dotted line) for consumption, capital stock, employment and output, the three columns representing the figures for Model I, Model II and Model III Economies. As can be seen, in particular the Model III Economy fits, along most dimensions, best the actual data. As can be seen from the separate figures, the volatility of employment has been greatly increased for both Model II and Model III. In particular, the volatility in the Model III Economy is close to the one in the Sample Economy, although too high a volatility is observable in the Model II Economy. We therefore may conclude that Model III is the best in matching the labor market volatility.

We want to note that the failure of the benchmark RBC model to match the volatility of employment of the data is extensively discussed in the recent paper by Schmidt-Grohe (2001). For her employed time series data 1948.3 - 1997.4, Schmidt-Grohe (2001) finds that the ratio of the standard deviation of employment to the standard deviation of output is roughly 0.95, close to our Sample Economy. Yet for the typical RBC model, the ratio is found to be 0.49, which is too low comparing to her empirical data. For the indeterminacy model, she finds a ratio to be 1.45, which seems too high. As noted above, a similarly high ratio of standard deviations can also be observed in our Model II Economy where the short side rule leads to excessive fluctuations of the labor effort.

Next, let us look at the cross-correlations of the macroeconomic variables. In the Sample Economy, there are three relevant correlations we can observe: a strong correlation between consumption and output, roughly 0.75, and between employment and output, about 0.72, and a weak correlation between consumption and employment, about 0.46. The first two strong correlations can also be found in all of our simulated economies. However, in our Model I Economy – and this only holds for the Model Economy I (the typical RBC model) – in addition to the first two correlations, consumption and employment are, with 0.93, also strongly correlated. Yet, as above discussed, this cannot be found in the actual data.

This result of the standard RBC model is not surprising given that movements of employment as well as consumption reflect the movement in the state variables capital and temporary shock. They, therefore, should be somewhat correlated. We remark here that such an excessive correlation has, to our knowledge, not explicitly been discussed in the RBC literature, including the review by King and Rebelo (1999),
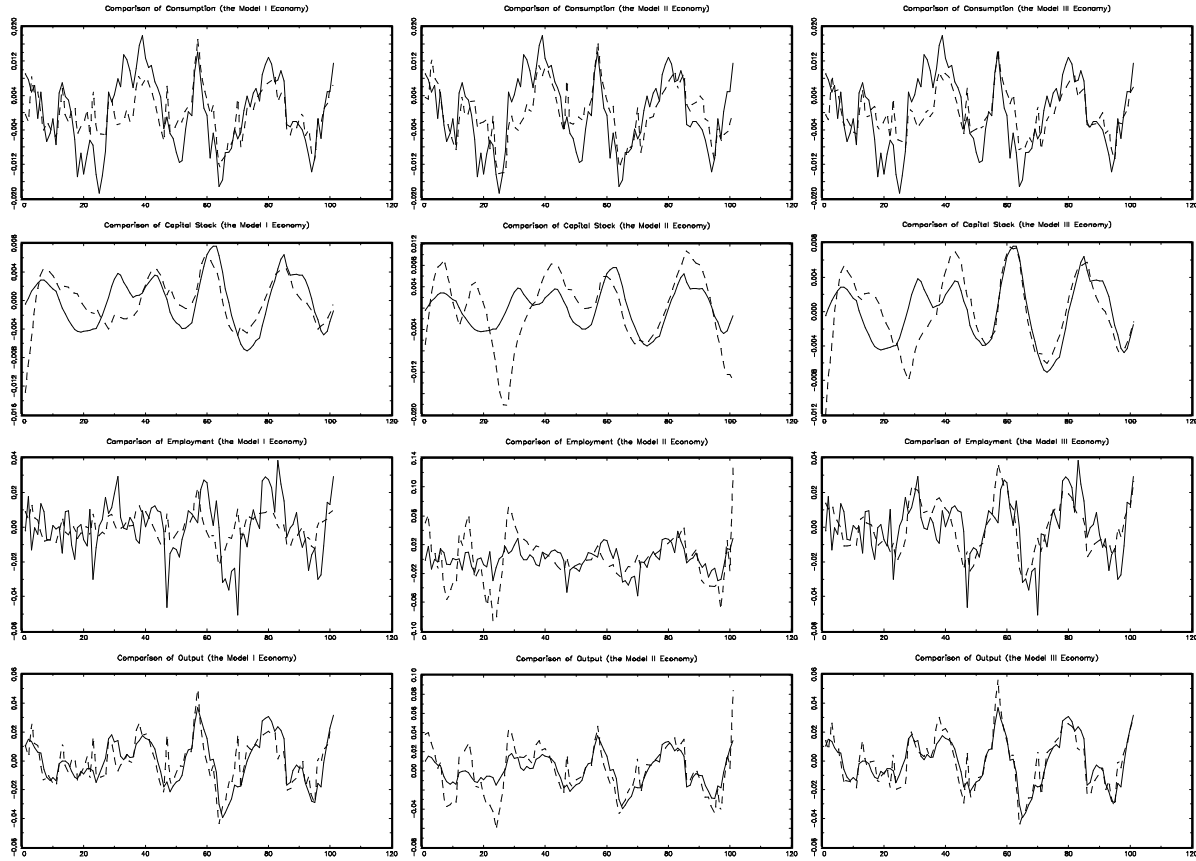
Figure 1: Simulated Economy versus Sample Economy: U.S. Case (solid line for sample economy, dotted line for simulated economy)

and also the recent study by Schmidt-Grohe (2001). Discussions have often focused on the correlation of employment with output.

A prominent feature of our disequilibrium model, see the Model II and III Economies, is that employment is no longer significantly correlated with consumption. This is because we have made a distinction between the demand and supply of labor. Labor supply and consumption reflect the moments of capital and technology. The realized employment is not necessarily the same as the desired labor supply. The correlation of employment with consumption is therefore reduced.

4   Some Remarks on International Differences

In section 2.4 we have introduced rules that might be implemented when there is a nonclearing labor market. In this respect, as our calibration in section 3 has shown,

the most promising route to model, and to match, stylized facts of the labor market, through a microbased labor market behavior, is the compromising model. One hereby may pay attention to the fact that two characteristics of the labor market impact the magnitude of the ω in our compromising rule. The first is the wage stickiness that drives a wedge between the household's desired supply of labor, n^s, as derived through optimal behavior in section 2.3 and the firm's optimal demand of labor, n^d. The case of flexible wages leads to $n^s = n^d$. This is similar to Woodford's (2003, ch.3) idea of a deviation of the efficient and natural level of output where the efficient level of output is achieved only if there are no frictions in the economy. In our case, no frictions in the labor market cause n^s equal n^d.

On the other hand, beside the above mentioned reason for a wedge between the desired labor supply and labor demand, there can be labor market institutions, for example corporatist structures, that our ω measures. Our ω expresses how much weight is given to the desired labor supply or desired labor demand. A small ω means that the agency, representing the household, has a high weight in determining the outcome of the employment compromise. A high ω means that the firm's side is stronger in employment negotiations. As the empirical estimations in Ernst, Gong and Semmler (2004) have shown the former case, a low ω, is very characteristic of Germany, France and Italy whereas a larger ω is found for U.S. and the U.K.

Given the rather corporatist relationship of labor and the firm in some European countries, with some considerable labor market regulations through legislature and union bargaining (rules of employment protection, hiring and firing restrictions, extension of employment even if there is a shortfall of sales etc.)[22], our ω may thus measure differences concerning labor market institutions between the U.S. and European countries. This has already been stated in the 1960s by Meyers. He states: "One of the differences between the United States and Europe lies in our attitude toward layoffs... When business falls off, he [the typical American employer] soon begins to think of reduction in work force... In many other industrial countries, specific laws, collective agreements, or vigorous public opinion protect the workers against layoffs except under the most critical circumstances. Despite falling demand, the employer counts on retraining his permanent employees. He is obliged to find work for them to do... These arrangements are certainly effective in holding down unemployment". (Meyers, 1964:)

Thus, we wish to argue that the major international difference causing employment variation does arise less from real wage stickiness (due to the presence of unions and the extend and duration of contractual agreements between labor and the firm)[23] but rather it seems to be the degree to which compromising rules exist and which side dominates the compromising rule. A lower ω, defining, for example, the compromising rule in Euro-area countries, can show up as difference in the variation of macroeconomic variables. This is demonstrated in table 3 contrasting a European sample economy,

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[22]This could also be realized by firms by demanding the same (or less) hours per worker but employing more workers than being optimal. The case would then correspond to what is discussed in the literature as labor hoarding where firms hesitate to fire workers during a recession because it may be hard to find new workers in the next upswing, see Burnside et al. (1993). Note that in this case firms may be off their marginal product curve and thus this might require wage subsidies for firms as has been suggested by Phelps (1997).

[23]In fact real wage rigidities in the U.S. are almost the same as in European countries, see Flaschel, Gong and Semmler (2001).

the German, with the U.S. [24]

<div align="center">

Table 3: The Standard Deviations (U.S. versus Germany)

	Germany (detrended)	U.S. (detrended)
consumption	0.0146	0.0084
capital stock	0.0203	0.0036
employment	0.0100	0.0166
output	0.0258	0.0164
temporary shock	0.0230	0.0115
efficiency wage	0.0129	0.0273

</div>

We can observe that first, employment and the efficiency wage (defined as real wage divided by productivity) are among the variables with the highest volatility in the U. S. economy. However, in the German economy they are the smoothest variables. Second, in the U. S. economy, the capital stock and temporary shock to technology are both relatively smooth. In contrast, they are both more volatile in Germany. These results are likely to be due to our first remark regarding the difference in employment volatility. The volatility of output must be absorbed by some factors in the production function. If employment is smooth, the other two factors have to be volatile.

Indeed, recent Phillips curve studies do not seem to reveal much difference in real wage stickiness between Germany and the U.S., although the German labor market is often considered less flexible.[25] Yet, there are differences in another sense. In Germany, there are stronger influences of labor unions and various legal restrictions on firms' hiring and firing decisions, shorter work week even for the same pay etc. [26] Such influences and legal restriction will give rise to the smoother employment series in contrast to the U.S.. Such influences and legal restriction, or what Solow (1979) has termed the moral factor in the labor market, may also be viewed as a readiness to compromise as our Model III suggests. Those factors will indeed give rise to a lower ω and a smoother employment series.[27]

So far we only have shown that our model of nonclearing labor market seems to match better the variation in employment than the standard RBC model. Yet, we did not attempt to explain the secular trend of the unemployment rate neither for the U.S. nor for Germany. We want to express a conjecture of how our model can be used to study the trend shift in employment. We want to note that the time series data for the table 3 (U.S. 1955.1-1983.1, Germany 1960.1-1992.1) are from a period where the U.S. had higher – but falling – unemployment rates, whereas Germany had still lower but rising unemployment rates. Yet, since the end of the 1980s the level of the

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[24]The U.S. data used here are the same as employed in section 3. The standard deviations of variables of the Germany economy are computed for the time period 1960.1-1992.

[25]See Flaschel, Gong and Semmler (2001).

[26]See,for example, Nickell (1997) and Nickell et al. (2003), and see already Meyers (1964).

[27]It might reasonably be argued that, due to intertemporal optimization subject to the budget constraints, the supply specified by the decision rule may only approximate the decisions of those households for which unemployment is not expected to pose a problem on their budgets. Such households are more likely to be currently employed and protected by labor unions and legal restrictions. In other words, currently employed labor decides, through the optimal decision rule, about labor supply and not those who are currently unemployed. Such a feature could presumably be better studied by an intertemporal model with heterogenous households, see, for example, Uhlig and Xu (1996).

unemployment rate in Germany has considerably moved up, partly, of course due to the unification of Germany after 1989.

One recent attempt to better fit the RBC model's predictions with labor market data has employed search and matching theory.[28] Informational or institutional search frictions may then explain the equilibrium unemployment rate and its rise. Yet, those models usually observe that there has been a shift in matching functions due to evolution of unemployment rates such as, for example, experienced in Europe since the 1980s, and that the model itself fails to explain such a shift.[29]

In contrast to the literature on institutional frictions in the search and matching process we think that the essential impact on the trend in the rate of unemployment seems to stem from both changes of preferences of households as well as a changing trend in the technology shock.[30] Concerning the latter, as shown in Gong and Semmler (2003, ch. 9), the Solow residual, as it used in RBC models as the technology shock, greatly depends on endogenous variables (such as capacity utilization). Thus exogenous technology shocks constitute only a small fraction of the Solow residual. We thus might conclude that cyclical fluctuations in output and employment are not likely to sufficiently be explained by productivity shocks alone. Gali (1999) and Francis and Ramey (2001, 2003) have argued that other shocks, for example demand shocks, are important as well.

Yet, in the long run, the change in the trend of the unemployment rate is likely to be related to the long-run trend in the true technology shock. Empirical evidence on the role of lagging implementation and diffusion of new technology for low employment growth in Germany can be found in Heckman (2003) and Greiner, Semmler and Gong (2004). In the context of our model this would have the effect that labor demand, given by equ. (25) may fall short of labor supply given by equ. (23). This is likely to occur in the long-run if the productivity Z_t in equ. (25) starts tending to grow at a lower rate which many researchers recently have maintained to have happened in Germany, and other European countries, since the 1980s.[31] Yet, as recent research has stressed, for example, the work by Phelps, see Phelps (1997) and Phelps and Zoega (1998), there have also been secular changes on the supply side of labor due to changes in preferences of households.[32] Some of those factors affecting the households' supply of labor have been discussed above.

5   Conclusions

Market clearing is a prominent feature in the standard RBC model. This assumption presumes wage and price flexibility. In this paper, we have introduced an adaptive optimization behavior and a multiple stage decision process that, given wage stickiness,

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[28]See Merz (1999) and Ljungqvist and Sargent (1998, 2003).

[29]For an evaluation of the search and matching theory as well as the role of shocks to explain the evolution of unemployment in Europe, see Blanchard and Wolfers (2000) and Blanchard (2003).

[30]See Campbell (1994) for a modelling of a trend in technology shocks.

[31]Of course, the trend in the wage rate is also important in the equation for labor demand (in equ. 25). For an account of the technology trend, see Flaschel, Gong and Semmler (2001), and for an additional account of the wage rate, see Heckman (2003).

[32]Phelps and his co-authors have pointed out that an important change in the households' preferences in Europe is that households now rely more an assets instead of labor income.

results in a nonclearing labor market in an otherwise standard stochastic dynamic model. Nonclearing labor market is then a result of different employment rules derived on the basis of a multiple stage decision process. Calibration for the U. S. economy shows that such model variants will produce a higher volatility in employment, and thus fit the data significantly better than the standard model.[33]

As concerning international aspects of our study we presume that different labor market institutions result in different weights defining the compromising rule. The results for Euro-area economies, for example, for Germany in contrast to the U.S., are consistent with what has been found in many other empirical studies with regard to the institutions of the labor market.

Finally, with respect to the trend of lower employment growth in some European countries as compared to the U.S. since the 1980s, our model suggests that one has to study more carefully the secular forces affecting the supply and the demand of labor as modeled in our multiple stage decision process of section 2. In particular, on the demand side for labor, the slow down of technology seems to have been a major factor for the low employment growth in Germany and other countries in Europe.[34] On the other hand there has also been changes in the preferences of households. Our study has provided a framework that allows to also follow up such issues.[35]

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[33]Gong and Semmler (2003) have also computed the welfare loss of our different model variants of nonclearing labor market and found, similarly to Sargent and Ljungqvist (1998), that the welfare losses are very small.

[34]See Blanchard and Wolfers (2000), Greiner, Semmler and Gong (2004) and Heckman (2003)

[35]This is undertaken in Gong and Semmler (2004, ch. 9).

Appendix 1: Wage Setting

Suppose that at the beginning of period t the household is allowed to change (or set) the wage rate given the data (A_t, k_t), and the sequence of expectation on $\{A_{t+i}\}_{i=1}^{\infty}$, where A_t and k_t are referred to as the technology and capital stock respectively. If changing the wage rate implies no adjustment cost for the household while the household also knows the production function $f(A_t, k_t, n_t)$, where n_t refers to labor, so that it may also know the firm's demand for labor, the decision problem of the household with regard to wage setting may be expressed as follows:

$$\max_{\{c_{t+i}, w_{t+i}\}_{i=0}^{\infty}} E_t \left[\sum_{i=0}^{\infty} \beta^i U(c_{t+i}, n(w_{t+i}^*, k_{t+i}, A_{t+i})) \right] \tag{29}$$

subject to

$$k_{t+i+1} = (1 - \delta)k_{t+i} + f(A_{t+i}, k_{t+i}, n(w_{t+i}^*, k_{t+i}, A_{t+i})) - c_{t+i} \tag{30}$$

Above, δ is the depreciation rate; β is the discounted factor; $U(\cdot)$ is the household's utility function, which depends on consumption c_{t+i} and the labor effort $n(w_{t+i}^*, k_{t+i}, A_{t+i})$. Note that here $n(w_{t+i}^*, k_{t+i}, A_{t+i})$ is the function of the firm's demand for labor, which is derived from the following condition of marginal product equal to the wage rate:

$$w_{t+i}^* = f_n(A_{t+i}, k_{t+i}, n_{t+i})$$

This decision problem gives rise to the solution

$$c_{t+i} = g_c(k_{t+i}, A_{t+i}) \tag{31}$$
$$w_{t+i}^* = g_w(k_{t+i}, A_{t+i}) \tag{32}$$

where $i = 0, 1, 2, ...$ Even if the wage rate follows the determination as expressed in (32) while the household's labor supply follows $n(w_{t+i}^*, k_{t+i}, A_{t+i})$, then still the household's labor supply could be equal the labor demand. In this sense, the labor market could be "cleared".[36]

However, as in recent New Keynesian literature, see Woodford(2003:441) and Erceg et al. (2003), applying the Calvo (1983) price setting scheme to wage setting, only the fraction λ of labor, due to the adjustment cost, is allowed to optimally set the wage in period t. Therefore, the observed wage rate w_t could be written as

$$w_t = \lambda w_t^* + (1 - \lambda)w_{t-1} \tag{33}$$

Such a wage dynamic indicates that there exists a gap between optimum wage w_t^* and observed wage w_t. Obviously, in this latter case, it is very unlikely that the observed wage rate will clear the labor market.[37]

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[36]Note that this sense of market clearing is not strict, since what the household supplies is not necessarily its desired supply. We discuss the decision regarding the household's desired supply of labor effort in section 2.2.

[37]Indeed, even if the observed wage w_t is equal to the optimum wage w_t^*, indicating no adjustment cost for the wage change, the labor market still can not be cleared in a strict sense, see footnote 12 for explaining this point.

Appendix 2: Adaptive Optimization

For the problem (14) - (16), we define the Lagrange:

$$
\begin{aligned}
L \;=\; & E_t \left\{ \left[\log c_t^d + \theta \log(1 - n_t) \right] + \right. \\
& \lambda_t \left[k_{t+1}^s - \frac{1}{1+\gamma} \left[(1-\delta)k_t^s + f(k_t^s, n_t, A_t) - c_t^d \right] \right] \Bigg\} + \\
& E_t \left\{ \sum_{i=1}^{\infty} \beta^i \left[\log(c_{t+i}^d) + \theta \log(1 - n_{t+i}^s) \right] + \right. \\
& \beta^i \lambda_{t+i} \left[k_{t+1+i}^s - \frac{1}{1+\gamma} \left[(1-\delta)k_{t+i}^s + f(k_{t+i}^s, n_{t+i}^s, A_{t+i}) - c_{t+i}^d \right] \right] \Bigg\}
\end{aligned}
$$

Since the decision is only about c_t^d, we thus take the partial derivatives of L with respect to c_t^d, k_{t+1}^s and λ_t. This gives us the following first-order condition:

$$
\frac{1}{c_t^d} - \frac{\lambda_t}{1+\gamma} = 0; \tag{34}
$$

$$
\frac{\beta}{1+\gamma} E_t \left\{ \lambda_{t+1} \left[(1-\delta) + (1-\alpha)A_{t+1} \left(k_{t+1}^s \right)^{-\alpha} \left(n_{t+1}^s \bar{N}/0.3 \right)^{\alpha} \right] \right\} = \lambda_t \tag{35}
$$

$$
k_{t+1}^s = \frac{1}{1+\gamma} \left[(1-\delta)k_t^s + A_t(k_t^s)^{1-\alpha} \left(n_t \bar{N}/0.3 \right)^{\alpha} - c_t^d \right], \tag{36}
$$

Recall that in deriving the decision rule as expressed in (24) and (25) we have postulated

$$
\lambda_{t+1} = H k_{t+1}^s + Q A_{t+1} + h \tag{37}
$$

$$
n_{t+1}^s = G_{21} k_{t+1}^s + G_{22} A_{t+1} + g_2 \tag{38}
$$

where H, Q, h, G_{21}, G_{22} and g_2 have all been resolved preliminarily in the household optimization program. Taking expectation for both sides of equation (37) and (38), we obtain

$$
E_t \lambda_{t+1} = H k_{t+1}^s + Q(a_0 + a_1 A_t) + h \tag{39}
$$

$$
E_t n_{t+1}^s = G_2 k_{t+1}^s + D_2(a_0 + a_1 A_t) + g_2 \tag{40}
$$

Our next step is to linearize (34), (35) and (36) around the steady states. Suppose they can be written as

$$
F_{c1} c_t + F_{c2} \lambda_t + f_c = 0 \tag{41}
$$

$$
F_{k1} E_t \lambda_{t+1} + F_{k2} E_t A_{t+1} + F_{k3} k_{t+1}^s + F_{k4} E_t n_{t+1}^s + f_k = \lambda_t \tag{42}
$$

$$
k_{t+1}^s = A k_t + W A_t + C_1 c_t^d + C_2 n_t + b \tag{43}
$$

Expressing $E_t \lambda_{t+1}, E_t n_{t+1}^s$ and $E_t A_{t+1}$ in (42) in terms of (39), (40) and $a_0 + a_1 A_t$ respectively, we obtain

$$
\kappa_1 k_{t+1}^s + \kappa_2 A_t + \kappa_0 = \lambda_t \tag{44}
$$

where, in particular,

$$\kappa_0 = F_{k1}(Qa_0 + h) + F_{k2}a_0 + F_{k4}(G_{22}a_0 + g_2) + f_k$$
$$\kappa_1 = F_{k1}H + F_{k3} + F_{k4}G_{21}$$
$$\kappa_2 = F_{k1}Qa_1 + F_{k2}a_1 + F_{k4}G_{22}a_1$$

Using (41) to express λ_t in (44), we further obtain

$$\kappa_1 k_{t+1}^s + \kappa_2 A_t + \kappa_0 = -\frac{F_{c1}}{F_{c2}}c_t^d - \frac{f_c}{F_{c2}}$$

which is equivalent to

$$k_{t+1}^s = -\frac{\kappa_2}{\kappa_1}A_t - \frac{F_{c1}}{F_{c2}\kappa_1}c_t^d - \frac{\kappa_0}{\kappa_1} - \frac{f_c}{F_{c2}\kappa_1} \tag{45}$$

Substituting equation (45) into the right side of (43), we resolve c_t^d as

$$c_t^d = -\left(\frac{F_{c1}}{F_{c2}\kappa_1} + C_1\right)^{-1}\left[Ak_t + \left(\frac{\kappa_2}{\kappa_1} + W\right)A_t + C_2n_t + \left(b + \frac{\kappa_0}{\kappa_1} + \frac{f_c}{F_{c2}\kappa_1}\right)\right]$$

References

[1] Benassy, J.-P. (1984): The Economics of Market Disequilibrium, New York: Academic Press.

[2] Benassy, J.-P. (1995): Money and Wage Contract in an Optimizing Model of the Business Cycle, Journal of Monetary Economics, vol. 35: 303-315.

[3] Benassy, J.-P. (2002): The Macroeconomics of Imperfect Competition and Non-clearing Markets, Cambridge: MIT Press.

[4] Blanchard, O. (2003): "Comments on Ljungqvist and Sargent" in: Knowledge, Information, And Expectations in Modern Macroceconomics", edited by P. Aghion, R. Frydman, J. Stiglitz and M. Woodford, Princeton Unviersity Press, Princeton: 351-356.

[5] Blanchard, O. and S. Fischer (1989): Lectures on Macroeconomics, Cambridge: MIT-Press

[6] Blanchard, O. and J. Wolfers (2000): "The Role of Shocks and Institutions in the Rise of Unemployment: The Aggregate Evidence", Economic Journal 110:C1-C33.

[7] Burnside,A. C. , M. S. Eichenbaum and S. T. Rebelo (1993): Labor Hoarding and the Business Cycle, Journal of Political Economy, vol. 101: 245-273.

[8] Calvo, G. A. (1983): Staggered Contracts in a Utility Maximization Framework, Journal of Monetary Economics, vol. 12: 383-98.

[9] Campbell, J. (1994): Inspecting the Mechanism: An Analytical Approach to the Stochastic Growth Model. Journal of Monetary Economics 33(3): 443-506.

[10] Christiano, L. J., M. Eichenbaum and C. Evans (2001): Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, NBER Working Paper Series, wp8403.

[11] Danthine, J.P. and J.B. Donaldson (1990): "Efficiency Wages and the Business Cycle Puzzle", European Economic Review 34: 1275-1301.

[12] Danthine, J.P. and J. B. Donaldson (1995): "Non-Walrian Economies," in T.F. Cooly (ed), Frontiers of Business Cycle Research, Princeton:Princeton University Press.

[13] Erceg, C. J., D. W. Henderson and A. T. Levin (2000): Optimal Monetary Policy with Staggered Wage and Price Contracts, Journal of Monetary Economics, Vol. 46: 281 - 313.

[14] Ernst, E., G. Gong and W. Semmler (2004): Labor Market and Employment in Euro-Area countries, Bielefeld University, mimeo.

[15] Flaschel, P. G. Gong and W. Semmler (2001): A Keynesian Macroeconometric Framework for the Analysis of Monetary Policy Rules. Journal of Economic Behavior and Organization, vol. 46:101-136.

[16] Francis, N. and V.A. Ramey (2001): "Is the Technology-Driven Real Business Cycle Hypothesis Dead? Shocks and Aggregate Fluctuations Revisited", University of California, San Diego.

[17] Francis, N. and V.A. Ramey (2003): "The Source of Historical Economic Fluctuations: An Analysis using Long-Run Restrictions", University of California, San Diego.

[18] Gali, J. (1999): Technology, Employment, and the Business Cycles: Do Technology Shocks Explain Aggregate Fluctuations? American Economic Review, vol. 89: 249-271.

[19] Gong, G. (1997): Essays in Economic Fluctuation. Ph. D. Dissertation, New School University, U.S.A.

[20] Gong, G. and W. Semmler (2002): Solving Stochastic Dynamic Optimization Models with Approximation: Some Examples, Paper presented in the 8th International Conference of the Society for Computational Economics and Finance, Aix-En-Provence, France.

[21] Gong, G. and W. Semmler (2003): Business Cycles, Wage Stickiness and Nonclearing Labor Market, Paper presented in the 9th International Conference of the Society for Computational Economics and Finance, Seattle, U.S.A.

[22] Gong, G. and W. Semmler (2004): Stochastic Dynamic Macroeconomics: Theory, Numerics and Time Series Evidence, forthcoming in Oxford University Press.

[23] Greiner, A., G. Gong and W. Semmler (2004): The Forces of Economic Growth: A Time Series Perspective, forthcoming, Princeton University Press.

[24] Grüne, L. and W. Semmler (2004): Default Risk, Asset Pricing and Debt Control. Revision under review at Journal of Financial Economics.

[25] Hansen, G. H.(1985): Indivisible Labor and BusinessCycle, Journal of Monetary Economics, vol.16, 309-27.

[26] Heckman, J. (2003): "Flexibility and Creation: Job Lessons from the German Experience", in: Knowledge, Information, And Expectations in Modern Macroceconomics", edited by P. Aghion, R. Frydman, J. Stiglitz and M. Woodford, Princeton Unversity Press, Princeton: 357-393.

[27] King, R. G., C. I. Plosser, and S. T. Rebelo (1988): Production, Growth and Business Cycles I: the Basic Neo-classical Model, Journal of Monetary Economics, vol. 21, 195-232.:

[28] King, R. G. and S. T. Rebelo (1999): Resusciting Real Business Cycles, in Handbook of Macroeconomics, Volume I, edited by J. B. Taylor and M. Woodford, Elsevier Science.

[29] King, R.G. and A.L. Wolman (1999): What should the Monetary Authority do when Prices are sticky? in Monetary Policy Rules, edited by J. Taylor, Chicago: The University of Chicago Press.

[30] Ljungqvist, L. and T. Sargent (1998): "The European Unemployment Dilemma", Journal of Political Economy, vol. 106, no.3: 514-550.

[31] Ljungqvist, L. and T. Sargent (2003): "European Unemployment: From a Worker's Perspective", in: Knowledge, Information, And Expectations in Modern Macroceconomics", edited by P. Aghion, R. Frydman, J. Stiglitz and M. Woodford, Princeton Unviersity Press, Princeton: 326-350.

[32] Merz, M. (1999): Heterogenous Job-Matches and the Cyclical Behavior of Labor Turnover", Journal of Monetary Economics, vol. 43: 91-124

[33] Meyers, R.J. (1964): "What can we learn from European Experience, in Unemployment and the American Economy?", ed. by A.M. Ross, New York: John Wiley & Sons, Inc.

[34] Nickell, S. (1997): "Unemployment and Labor Market Rigidities – Europe versus North America", Journal of Economic Perspectives 3, 55-74.

[35] Nickell, S., L. Nunziata, W. Ochel and G. Quintini (2003): "The Beveridge Curve, Unemployment, and Wages in the OECD from the 1960s to the 1990s", in: Knowledge, Information, And Expectations in Modern Macroceconomics", edited by P. Aghion, R. Frydman, J. Stiglitz and M. Woodford, Princeton Unviersity Press, Princeton.

[36] Phelps, E. (1997): Rewarding Work, Cambridge: MIT-Press

[37] Phelps, E. and G. Zoega (1998): "Natural Rate Theory and OECD Unemployment", Economic Journal, 108 (May): 782-801.

[38] Rotemberg, J. (1982): Sticky Prices in the United States, Journal of Political Economy, vol. 90: 1187-1211.

[39] Rotemberg, J. and M. Woodford (1995): Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets, in Frontiers of Business Cycle Research, edited by T. F. Cooley, Princeton: Princeton University Press.

[40] Rotemberg, J. and M. Woodford (1999): Interest Rate Rules in an Estimated Sticky Price Model, in Monetary Policy Rules, edited by J. Taylor, Chicago: the University of Chicago Press.

[41] Sargent, T. (1999): Contested Inflation. Princeton, Princeton University Press.

[42] Schmidt-Grohe, S. (2001): Endogenous Business Cycles and the Dynamics of Output, Hours and Consumption, American Economic Review, vol. 90: 1136-1159.

[43] Semmler, W. and G. Gong (1996): Estimating Parameters in Real Business Cycle Model, Journal of Economic Behavior and Organization, vol. 30: 301-325.

[44] Semmler, W. and G. Gong (1997): Numerical Procedure to Estimate Real Business Cycle Models Using Simulated Annealing, in Computational Approaches to Economic Problems, edited by Hans Amman, Berc Rustem and Andy Whinston, Kluwer Inc.

[45] Solow, R. (1979): "Another Possible Source of Wage Stickiness", Journal of Macroeconomics, vol. 1: 79-82

[46] Taylor, J. B. (1980): Aggregate Dynamics and Staggered Contracts, Journal of Political Economy, Vol. 88: 1 - 24.

[47] Taylor, J. B. (1999): Staggered Price and Wage Setting in Macroeconomics, in Handbook of Macroeconomics, Volume I, edited by J. B. Tayor and M. Woodford, Elsevier Science.

[48] Uhlig, H. (2003): "How well do We understand Business Cycles and Growth", mimeo, Humbold University, Berlin.

[49] Uhlig, H. and Y. Xu (1996): Effort and the Cycle: Cyclical Implications of Efficiency Wages, mimeo, Tilburg Unversity.

[50] Woodford, M. (2003): Interest Rate and Prices, Princeton University Press.

[51] Zbaracki, M. J., M. Ritson, D. Levy, S. Dutta and M. Bergen (2000): The Managerial and Customer Costs of Price Adjustment: Direct Evidence from Industrial Markets, Manuscript, Wharton School, University of Pennsylvania

[52] Zhang, W. and W. Semmler (2004): Monetary Policy Rules under Uncertainty: Adaptive Learning and Robust Control, revised version under review at Macroeconomic Dynamics.