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**Endogenous Growth, Skill-Biased Technical
Change and Wage Inequality**

by

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1 Introduction

In this contribution, we present an endogenous growth model which essentially extends the idea of the paper by Murphy, Riddell and Romer (1998).¹ In that paper, the authors assume that technical progress leads to wage differentials between high-skilled workers and unskilled workers. As technical progress occurs, the relative marginal productivity of different input changes. Yet, if there is sufficient complementarity between skills and new technology the demand for more educated employees rises, too, which generates an increase in their wages relative to those of the unskilled workers.

2 The Model

Murphy, Riddell and Romer (1998), however, do not model their idea within an endogenous growth model. To achieve this we start with the endogenous Romer (1990) growth model. Additionally, we assume that the number of capital goods, i.e. the designs A , positively affects the efficiency of both unskilled and skilled labour input. That is we assume that capital is associated with positive externalities. We do this because technical progress is embodied in new capital goods. So, the installment of a new machine does not only raise the capital stock but it also increases the productivity of the labour input.

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¹However, in contrast to Krusell et al. we do not consider substitution between unskilled labour and capital.

That is any worker is expected to produce more output with the new machine compared to the old machine.

The structure of the productive sector is the same as in the basic Romer model (1990). So, we will only briefly sketch the derivation of the equations. To integrate our idea in the Romer (1990) growth model, we first have to introduce the modified Cobb-Douglas production function

$$Y = K^{1-\alpha} A^\alpha \eta^{\alpha-1} \left\{ \gamma_1 [A^\xi (H - H_A)]^{\frac{\sigma_p-1}{\sigma_p}} + (1 - \gamma_1) [A^\epsilon L]^{\frac{\sigma_p-1}{\sigma_p}} \right\}^{\frac{\alpha \sigma_p}{\sigma_p-1}}, \quad (1)$$

with K physical capital, H_Y highly qualified employees producing output and H_A highly qualified employees engaged in $R\&D$. $H = H_Y + H_A$ gives the total number of high qualified work in the economy. L gives the number of low qualified workers who only produce output. $\sigma_p > 1$, finally, gives the elasticity of substitution between H_Y and L . ξ and ϵ measure the impact of the external effect, i.e. the impact of technical progress, on H_Y and L . η gives the units of foregone output which are needed to produce one unit of an intermediate good.

Thus, the capital accumulation equation can be written as

$$\dot{K} = K^{1-\alpha} A^\alpha \eta^{\alpha-1} \{X\}^{\frac{\alpha \sigma_p}{\sigma_p-1}} - C, \quad (2)$$

with

$$X = \gamma_1 [A^\xi (H - H_A)]^{\frac{\sigma_p-1}{\sigma_p}} + (1 - \gamma_1) [A^\epsilon L]^{\frac{\sigma_p-1}{\sigma_p}}. \quad (3)$$

The firms in the final good sector behave competitively. The solution to their optimization problem again gives the inverse demand function for the intermediate good $x(i)$. With the production function (1) it is given by

$$p(i) = X^{\frac{\alpha \sigma_p}{\sigma_p-1}} (1 - \alpha) K^{-\alpha} \eta^{-\alpha} A^\alpha. \quad (4)$$

The intermediate firm which produces $x(i)$ takes this function as given in solving its optimization problem. The solution to this problem gives the interest rate as

$$r = (1 - \alpha)^2 \eta^{\alpha-1} K^{-\alpha} A^\alpha X^{\frac{\alpha \sigma_p}{\sigma_p-1}}. \quad (5)$$

Neglecting depreciation of knowledge, the differential equation describing the evolution of the stock of knowledge or the number of designs, A , is given by

$$\dot{A} = \mu H_A^\gamma A^\phi, \quad (6)$$

with $\gamma, \phi \in (0, 1)$.

The differential equation describing the evolution of H_A over time is also obtained as follows: First, one uses the fact that the price of knowledge at time t , $P(t)$, is equal to the present value of the stream of profits, π , of each intermediate firm because the research sector behaves competitively. This leads to the differential equation

$$\dot{P}_A = rP_A - \pi, \quad (7)$$

with $\pi = rx\eta\alpha/(1-\alpha)$. Second, the rental rate of human capital in the final good sector and in the research sector must be equal. This fact gives rise to the following differential equation

$$\begin{aligned} \frac{\dot{P}_A}{P_A} &= \left(\frac{\alpha \sigma_p}{\sigma_p - 1} - 1 \right) \frac{\dot{X}}{X} + (1 - \alpha) \frac{\dot{K}}{K} + \left(\frac{\sigma_p - 1}{\sigma_p} - 1 \right) \frac{\dot{H} - \dot{H}_A}{H - H_A} + \\ &\quad (1 - \gamma) \frac{\dot{H}_A}{H_A} + \left(\alpha - \phi + \xi \frac{\sigma_p - 1}{\sigma_p} \right) \frac{\dot{A}}{A}. \end{aligned} \quad (8)$$

Dividing (7) by P_A , setting the resulting expression equal to (8) and solving for \dot{H}_A yields

$$\begin{aligned} \dot{H}_A &= Z^{-1} \cdot \\ &\quad \left[\left(\frac{\gamma}{\gamma_1} \right) (1 - \alpha) \mu X H_A^{\gamma-1} (H - H_A)^{\frac{1+\sigma_p}{\sigma_p}} A^{\phi-1+\xi \frac{1-\sigma_p}{\sigma_p}} - (1 - \alpha) (H - H_A) \frac{C}{K} + \right. \\ &\quad \left. \left(\frac{\sigma_p - 1}{\sigma_p} - 1 \right) n_H H + \left(\alpha - \phi + \xi \frac{\sigma_p - 1}{\sigma_p} \right) \mu H_A^\gamma A^{\phi-1} (H - H_A) + \right. \\ &\quad \left. X^{-1} \left(\frac{\alpha \sigma_p}{\sigma_p - 1} - 1 \right) \frac{\sigma_p - 1}{\sigma_p} \gamma_1 (A^\xi (H - H_A))^{\frac{\sigma_p - 1}{\sigma_p}} \cdot \right. \\ &\quad \left. (\xi \mu H_A^\gamma A^{\phi-1} (H - H_A) + H n_H) + (H - H_A) \alpha (1 - \alpha) \eta^{\alpha-1} K^{-\alpha} A^\alpha X^{\alpha \sigma_p / (1 - \sigma_p)} + \right. \\ &\quad \left. X^{-1} \left(\frac{\alpha \sigma_p}{\sigma_p - 1} - 1 \right) \frac{\sigma_p - 1}{\sigma_p} (1 - \gamma_1) (A^\xi L)^{\frac{\sigma_p - 1}{\sigma_p}} (H - H_A) \cdot \right. \\ &\quad \left. (\epsilon \mu H_A^\gamma A^{\phi-1} + n) \right], \end{aligned} \quad (9)$$

with

$$Z = \left(\frac{\sigma_p - 1}{\sigma_p} \right) - (1 - \gamma) \frac{H - H_A}{H_A} + \gamma_1 \left(\frac{\sigma_p - 1}{\sigma_p} \right) X^{-1} \left(\frac{\alpha \sigma_p}{\sigma_p - 1} \right) (A^\xi (H - H_A))^{\frac{\sigma_p - 1}{\sigma_p}}$$

n_H and n give the growth rate of the total stock of human capital or skilled labour H and the growth rate of labour supply L , i.e.

$$\dot{H} = H n_H, \quad (10)$$

$$\dot{L} = L n. \quad (11)$$

The model is completed by modelling the household sector. The household sector consists of two representative households. The first supplies skilled labour H , the second supplies unskilled labour L . The optimization problem of the household gives the growth rate as

$$\max_{C_j} \int_0^{\infty} e^{-\rho_j t} u_j(C_j(t)) dt, \quad j = H, L, \quad (12)$$

subject to their budget constraints. Further, the following identities hold, $K_H + K_L = K$ and $C_H + C_L = C$, i.e. the total aggregate capital stock and consumption equal the sum of the capital stocks and consumption of both households.

The optimization problem gives the growth rates of consumption as

$$\frac{\dot{C}_j}{C_j} = -\frac{\rho_j - r}{\sigma_j}, \quad j = H, L, \quad (13)$$

with $1/\sigma_j$ the constant intertemporal elasticity of substitution of consumption between two points in time of household j , $j = H, L$, and ρ_j the subjective rate of time preference of household j , $j = H, L$.

Our economy is completely described by equations (2), (6), (9), (10), (11) and (14), with the interest rate r given by (5). In the following we will focus on the wage premium, i.e. the ratio of the wages earned by high-skilled workers to those of low-skilled workers, and derive implications of our model for this variable on the BGP.

A BGP for this model is derived as for the semi-endogenous growth model with R&D in Jones (1995). We define a BGP as a path with a constant output/capital ratio, Y/K , and where all variables grow at constant but not necessarily equal rates. This implies

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{C}}{C} = \frac{\dot{C}_H}{C_H} = \frac{\dot{C}_L}{C_L}.$$

It should be noted that a common growth rate of consumption of the two households implies that aggregate consumption grows at the same rate. That holds because of

$$\frac{\dot{C}}{C} = \frac{\dot{C}_H}{C_H} \frac{C_H}{C} + \frac{\dot{C}_L}{C_L} \frac{C_L}{C} = g \left(\frac{C_H}{C} + \frac{C_L}{C} \right) = -\frac{\rho_j - r}{\sigma_j}, \quad j = H, L, \quad (14)$$

with $g = \dot{C}_H/C_H = \dot{C}_L/C_L$.

The intertemporal elasticity of substitution of the two households and the subjective rate of time preference need not necessarily be equal but may differ. However, for a common growth rate of consumption to exist those parameters cannot take arbitrary values. That holds because a common growth rate for C_H and C_L implies that

$$\frac{\sigma_L}{\sigma_H} = \frac{\rho_L - r}{\rho_H - r}$$

must always be fulfilled.

Further, using $d/dt(\dot{A}/A) = 0$ and $d/dt(\dot{K}/K) = 0$ yields

$$\frac{\dot{A}}{A} = \frac{\gamma}{1-\phi} n_{H_A}, \quad \frac{\dot{K}}{K} = \frac{\gamma}{1-\phi} n_{H_A} + \frac{\sigma_p}{\sigma_p-1} \frac{\dot{X}}{X},$$

where we define $n_{H_A} = \dot{H}_A/H_A$.

If the growth rates of A and H_Y are constant it can easily be shown that the following relation holds concerning the growth rate of X ,

$$\lim_{t \rightarrow \infty} \frac{\dot{X}}{X} = \frac{\sigma_p - 1}{\sigma_p} (\xi n_A + n_{H_Y}), \quad \text{for } (\epsilon n_A + n) < (\xi n_A + n_{H_Y}),$$

with n and n_i denoting the growth rate of L and of variable i , $i = A, H_Y$ respectively. The condition $(\epsilon n_A + n) < (\xi n_A + n_{H_Y})$ does not pose a sincere limitation of our model. This holds because, on the one hand, the growth rate of high skilled labour in the final goods sector, n_{H_Y} , has been larger than the growth rate of unskilled labour, n , in the industrialized countries in the recent decades. Further, on the other hand, it is to be expected that the external effect associated with new machines, i.e. with a rise in A , is higher for skilled labour than for unskilled labour, implying $\xi > \epsilon$.

Moreover, dividing (7) by P_A and setting it equal to (8) shows that on a BGP H_A and H_Y must grow at the same rate.² This fact together with the constraint $H_A + H_Y = H$ implies that H also grows at the rate with which H_A and H_Y grow on the BGP, i.e. we have $n_{H_A} = n_{H_Y} = n_H$. Thus, on a BGP we have

$$\frac{\dot{K}}{K} - n_H - n = n_H \frac{\gamma}{1-\phi} (1 + \xi) - n.$$

This equation demonstrates that a sufficient and necessary condition for the growth rate of aggregate variables to exceed the growth rate of skilled and unskilled labour is

$$n_H \frac{\gamma}{1-\phi} (1 + \xi) > n. \quad (15)$$

If the growth rate of human equals zero this model does not yield sustained per capita growth in the long run, just as the modified Romer model. Then on the BGP aggregate output, aggregate physical capital and aggregate consumption grow at the same rate as labour input implying that per capita variables are constant. Looking at condition (15) one realizes that it is more likely to be fulfilled the more productive human capital in

²It can be shown that for $(\epsilon n_A + n) > (\xi n_A + n_{H_Y})$ no BGP with constant and positive per capita growth rate exists.

the research process is, i.e. the higher γ , and the larger the positive externality of new designs, i.e. the higher ξ .

However, sustained per capita growth only occurs if the growth rate of human capital is positive, that is if $n_H > 0$. Thus, we again get a semi-endogenous growth model where conventional government policies cannot affect the long run balanced growth rate. In the next section we will discuss the implications of this model as to the wage premium.

3 The Wage Premium

The wage premium is defined as the ratio of the wages earned by high-skill workers and the wages earned by low-skill workers. This variable is of potential interest to economists because it can be seen as a measure for the flexibility of the labour market which has repercussions for the unemployment rate in an economy. For example, a low rate of change of the wage premium may be an indicator for a relatively high increase in low-skilled wages which has negative effects on overall employment³. Further, the wage premium reflects the inequality between high-skilled and low-skilled workers. If the wage premium rises, the gap between employees getting high wages and employees getting low wages widens which tends to make the income distribution more unequal. If the wage premium falls the reverse holds.

As concerns the determination of the wage premium it is affected by two factors. First, an increase in the supply of high-skilled workers reduces the wage rate for this kind of work and tends to reduce the wage premium. Second, a rise in the number of skilled workers implies that the profitability of technologies increases which are complementary to skilled labour (cf. Acemoglu, 1998). This has a positive effect on the wages of high-skilled workers and, consequently, raises the wage premium. It should be noted that the increase in the supply of high-skilled workers may be the result of government policies. If a government decides to raise the expenditures for education the number of high-skilled workers is expected to rise over time. If it reduces public spending for education, the converse holds.

To derive the wage premium in our model we recall that the production function is given by

$$Y = K^{1-\alpha} A^\alpha \eta^{\alpha-1} \{X\}^{\frac{\alpha \sigma_p}{\sigma_p-1}}, \quad (16)$$

with

$$X = \gamma_1 \left[A^\xi (H - H_A) \right]^{\frac{\sigma_p-1}{\sigma_p}} + (1 - \gamma_1) \left[A^\epsilon L \right]^{\frac{\sigma_p-1}{\sigma_p}}$$

and $H - H_A = H_Y$. Assuming competitive markets, the wage rates of the highly and low qualified employees are equal to the marginal products of high and low qualified work in

³Of course, in our model we do not explicitly take into consideration unemployment.

the production sector. This gives

$$w_H = \alpha \gamma_1 \eta^{\alpha-1} K^{1-\alpha} A^\alpha X^{\frac{\alpha \sigma_p}{\sigma_p-1}-1} A^{\frac{\xi(\sigma_p-1)}{\sigma_p}} H_Y^{-\frac{1}{\sigma_p}}, \quad (17)$$

$$w_L = \eta^{\alpha-1} \alpha (1 - \gamma_1) K^{1-\alpha} A^\alpha X^{\frac{\alpha \sigma_p}{\sigma_p-1}-1} A^{\frac{\epsilon(\sigma_p-1)}{\sigma_p}} L^{-\frac{1}{\sigma_p}}. \quad (18)$$

The ratio of the marginal products of the two types of labour, the wage premium w_p , is given by:

$$w_p \equiv \frac{w_H}{w_L} = \frac{\gamma_1}{1 - \gamma_1} \left[\frac{A^\xi}{A^\epsilon} \right]^{\frac{\sigma_p-1}{\sigma_p}} \left[\frac{H_Y}{L} \right]^{-\frac{1}{\sigma_p}} \quad (19)$$

This result is similar to the one obtained in the paper by Murphy et al. (1998). The major difference is that, in our model, the external effect of technical change, ξ and ϵ , appears in the wage premium.

Defining

$$A_{w_p} \equiv \frac{A^\xi}{A^\epsilon}, \quad L_{w_p} \equiv \frac{H_Y}{L}. \quad (20)$$

we can derive a differential equation describing the evolution of the ratio w_p which is

$$\frac{\dot{w}_p}{w_p} = \left(\frac{\sigma_p - 1}{\sigma_p} \right) \frac{\dot{A}_{w_p}}{A_{w_p}} - \left(\frac{1}{\sigma_p} \right) \frac{\dot{L}_{w_p}}{L_{w_p}}. \quad (21)$$

From the definitions of A_{w_p} and L_{w_p} we get

$$\frac{\dot{A}_{w_p}}{A_{w_p}} = (\xi - \epsilon) \frac{\dot{A}}{A}, \quad (22)$$

$$\frac{\dot{L}_{w_p}}{L_{w_p}} = \frac{\dot{H}_Y}{H_Y} - \frac{\dot{L}}{L}. \quad (23)$$

Looking at the wage premium, equation (19), we see that four main factors determine this variable.

First, the quotient of the productivity parameters $\gamma_1/(1 - \gamma_1)$. If γ_1 is very small and close to zero the wage premium will have a small value, too. A small value for γ_1 means that the productivity of the high-skilled workers relative to the low-skilled workers is small, i.e. low-skilled workers contribute more to the output than high-skilled workers. Consequently, the wage of the low-skilled workers is relatively high and the wage premium is relatively low. If γ_1 is large, say near to one, the reverse holds. That is the productivity of the high-skilled workers is relatively high and, as a consequence, their wage rate and the wage premium are high, too.

Second, the ratio A^ξ/A^ϵ affects the wage premium. A high (low) value for ξ relative to ϵ means that the positive external effect of technical change affects high-skilled workers to a greater (lower) degree compared to low-skilled workers. That is, technical change, an increase in A , leads to a stronger (smaller) increase in the productivity of high-skilled workers compared to low-skilled workers. As a consequence, for a given level of A the wage rate and the wage premium of the high-skilled workers will be relatively high (low).

Third, the number of high-skilled workers relative to the number of low-skilled workers determines the wage premium. If this ratio is high the supply of high-skilled workers is relatively large. As a consequence, the wage premium will take on a low value.

The fourth factor which affects the wage premium is the elasticity of substitution between high-skilled and low-skilled workers, σ_p . To find the effect of σ_p on the wage differential we rewrite (19) and get

$$w_p = \frac{w_H}{w_L} = \frac{\gamma_1}{1 - \gamma_1} A^{\xi - \epsilon} \left[A^{\epsilon - \xi} \left(\frac{L}{H_Y} \right) \right]^{\frac{1}{\sigma_p}}. \quad (24)$$

Differentiating that expression with respect to σ_p gives

$$\frac{\partial(w_H/w_L)}{\partial\sigma_p} = -\frac{\gamma_1}{1 - \gamma_1} A^{\xi - \epsilon} \left[A^{\epsilon - \xi} \frac{L}{H_Y} \right]^{\frac{1}{\sigma_p}} \sigma_p^{-2} \ln \left[A^{\epsilon - \xi} \left(\frac{L}{H_Y} \right) \right]. \quad (25)$$

This expression is positive (negative) for $A^{\epsilon - \xi}(L/H_Y) < (>) 1$. This implies that a higher elasticity of substitution raises (reduces) the wage differential if the ratio (L/H_Y) is relatively large (small), i.e. if it is larger (smaller) than the threshold level $A^{\xi - \epsilon}$. That means if the supply of unskilled workers is relatively high an increase in the elasticity of substitution between high-skilled and low-skilled workers raises the wage differential. If the supply of unskilled workers is low a higher elasticity of substitution between high-skilled and low-skilled workers reduces the wage differential.

The growth rate of the wage differential is given by equation (21) together with (22) and (23). It crucially depends on the elasticity of substitution between high-skilled and low-skilled workers, i.e. on σ_p . The effect of σ_p on the growth rate of the wage differential is obtained by differentiating (21) with respect to σ_p . This gives

$$\frac{\partial(\dot{w}_p/w_p)}{\partial\sigma_p} = \frac{1}{\sigma_p^2} \left[\frac{\dot{A}_{w_p}}{A_{w_p}} + \frac{\dot{L}_{w_p}}{L_{w_p}} \right]. \quad (26)$$

If the sum in brackets is positive an increase in the elasticity of substitution raises the growth rate of the wage differential. This means the difference between high-skilled and low-skilled wages rises with a higher elasticity of substitution provided the term in brackets is positive. This latter expression is composed of two parts. First, the difference

between the growth rates of the labour productivity of high-skilled and low-skilled workers. A positive difference tends to make the expressions in brackets positive. Second, the difference between the growth rates of high-skilled workers and low-skilled workers. If this expression is positive the term in brackets tends to be positive, too.

Thus, we can state that an increase in the elasticity of substitution between high- and low-skilled workers raises the wage differential if the growth rate of productivity of high-skilled workers (as a result of positive externalities of physical capital) is larger than the one of low-skilled workers and if the growth rate of high-skilled labour exceeds the growth rate of low-skilled labour. For example, if high-skilled and low-skilled labour grows at the same rate only the growth rate of the labour productivity of the two groups affects equation (26). Then the growth rate of the wage differential is the higher the higher the elasticity of substitution between high-skilled and low-skilled workers, provided the labour productivity of high-skilled workers grows faster than the labour productivity of low-skilled workers. If the labour productivity of the low-skilled workers grows faster than the one of the high-skilled the reverse holds. Then, an increase in the elasticity of substitution reduces the growth rate of the wage differential, i.e. the difference between high-skilled and low-skilled wages becomes smaller over time.

Further, it is easily seen that $\xi > \epsilon$ implies that the wage differential rises and vice versa, if skilled and unskilled labour grow at the same rate. This makes sense from the economic point of view, because $\xi > (<)\epsilon$ means that the labour productivity of skilled labour grows faster (slower) than that of the unskilled workers. If $\xi = \epsilon$ only the growth rates of skilled and unskilled labour supply affect the growth rate of the wage premium. If skilled labour grows faster (slower) than unskilled labour the wage premium rises (falls) which is reasonable from an economic point of view.

4 Conclusion

In this paper we have considered the connection between income distribution and economic growth. We have presented a model in which technical progress affects the productivity of skilled and unskilled labour differently which gives rise to wage differences between these two kinds of labour. Thus, technical progress may drive a wedge between the revenue of well qualified and less qualified workers making the income distribution more unequal.

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