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Sustainability of US public debt: Estimating smoothing spline regressions

by

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Abstract

In this paper we analyze how the primary surplus to GDP ratio in the US reacts to variations in the public debt-GDP ratio. In contrast to earlier studies we perform non-parametric and semi-parametric estimations. Our results show that the response of the primary surplus to GDP ratio is a positive nonlinear function of the debt-GDP ratio. Further, our estimations demonstrate that the coefficient giving the response of the surplus ratio to a change in the debt ratio declines over time when we assume a linear model with time dependent coefficients.

JEL: H63, E62

Keywords: Public Debt, Sustainability, Non-parametric estimation, smoothing splines

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1 Introduction

Bohn (1998) presented a new sustainability test for public debt because, as he argues, conventional tests are restrictive since they conclude too soon that a given policy is unsustainable. As an alternative, Bohn suggests to investigate how the primary surplus reacts to increases in the debt ratio. If the primary surplus to GDP ratio increases at least linearly with a rising debt ratio, public debt is sustainable. Intuitively, the reasoning behind this test is obvious. If the government raises the primary surplus to GDP ratio as the debt to GDP ratio increases it takes corrective actions which guarantee that the government remains solvent. Formally, this was proven by Bohn (1998) assuming a constant reaction coefficient. Canzonerie et al. (2001) generalized the proof by allowing for a time dependent reaction coefficient.[1]

For the US, Bohn (1998) estimated OLS regressions and found that the primary surplus-GDP ratio is a positive function of the debt ratio implying that US debt policy is sustainable. In addition, Bohn estimated parametric regression where the debt ratio enters the equation to be estimated in a nonlinear way. He found evidence that the marginal effect of the debt ratio on the primary surplus-GDP ratio rises with higher debt ratios suggesting a nonlinear, convex relationship. The latter is also supported by Sarno (2001) who tested for nonlinearities and detected strong evidence for nonlinearities in the process generating the US debt-GDP time series.

However, neither of these authors apply non-parametric estimation techniques and they do not present an explicit estimate for the function giving the relation between the primary surplus ratio and the debt ratio. In this paper we take the next step by exploring functional shapes of the effects. To do so, we take advantage of recent developments in statistics. About a decade ago Hastie & Tibshirani (1990) introduced Generalized

[1] The proof is restrictive since it requires that the reaction coefficient is positive at any moment in time. However, using continuous time it can be shown that a reaction coefficient which is positive on average guarantees sufficiency for dynamically efficient economies, see Greiner (2004).

Additive Models as a new flexible class of regression model. The theoretical achievements were accompanied by numerical developments which led to the success of the software packages S-PLUS and later on R (see also Venables & Ripley, 2003). The main idea behind this class of models is, that the effect of an explanatory variable on some measurement of interest is not modelled as parametric (usually linear) function, but kept flexible by just postulating that the effect is smooth in the sense of differentiability. The functional shape is thereby to be estimated from the data, either using local, that are kernel based methods, or spline smoothing. Available software readily allows to fit such models. With the contributions by Wood (2000, 2001) also the disputable point of choosing the right amount of smoothing has been settled in practice. A motivating overview of the state of art in this field can be found in Ruppert, Wand & Carroll (2003).

In this paper we apply non-parametric estimation to US data. We are focussing the question whether the relation between the primary surplus to GDP ratio and the debt-GDP ratio is characterized by nonlinearities. In particular, we want to visualize the function governing the response of the primary surplus ratio to changes in the debt ratio. To get insight into this question we pursue smoothing spline regression. Furthermore, we estimate a semi-parametric regression where we assume a linear relation between the primary surplus ratio and the debt ratio. However, the coefficients associated with the predictor variables are assumed to be time varying. This leads to a varying coefficient model as introduced in Hastie & Tibshirani (1993).

The rest of the paper is organized as follows. In the next section, we test for nonlinearities in the relationship between the primary surplus to GDP ratio and the debt-GDP ratio. Section 3 estimates a semi-parametric function where the exogenous variables enter the equation to be estimated in a linear way but the coefficients are allowed to be nonlinear functions of time. Section 4, finally, concludes.

2 A nonparametric model

Starting point of Bohn's sustainability test is the assumption that the primary surplus to GDP ratio is a positive function of the debt-GDP ratio and of other variables affecting the primary surplus ratio. Estimating this policy rule allows conclusions as to the sustainability of a given fiscal policy. If the primary surplus-GDP ratio rises at least linearly with the debt-GDP ratio a given fiscal policy can be shown to be sustainable.

With s_t denoting the primary surplus to GDP ratio and d_t the debt-GDP ratio, we model s_t to depend on d_t in a flexible, non-parametric manner via

$$s_t = \alpha + f_1(d_t) + f_2(GVAR_t) + f_3(YVAR_t) + \epsilon_t, \;\; \epsilon_t \sim iid(0, \sigma^2). \tag{1}$$

Here, $GVAR$ is the level of temporary government spending and $YVAR$ is a business cycle indicator (cf. Bohn, 1998). In (1), $f_l(\cdot)$ are considered as smooth non-parametric functions, $l = 1, 2, 3$. These functions are kept unspecified and will be estimated from the data (see Appendix for details).

Figure 1 shows the data ranging from year 1916 to 1995. The shaded area indicates World War I and II plus the subsequent year. These data are excluded from the analysis for two reasons. First, from a statistical viewpoint these years show extreme values in the covariates and hence they would act as leverage points in the estimation. By excluding the points leverage biased estimates are avoided. Secondly, from an economic point of view estimating a policy rule such as (1) assumes a reaction which holds in normal times but not necessarily in war times. During wars the response of public surpluses to public debt cannot be expected to be the same as in times of peace. So, from an economic perspective war periods are considered as exogenous shocks which destabilize the system and may make policy rules which hold in peace irrelevant.

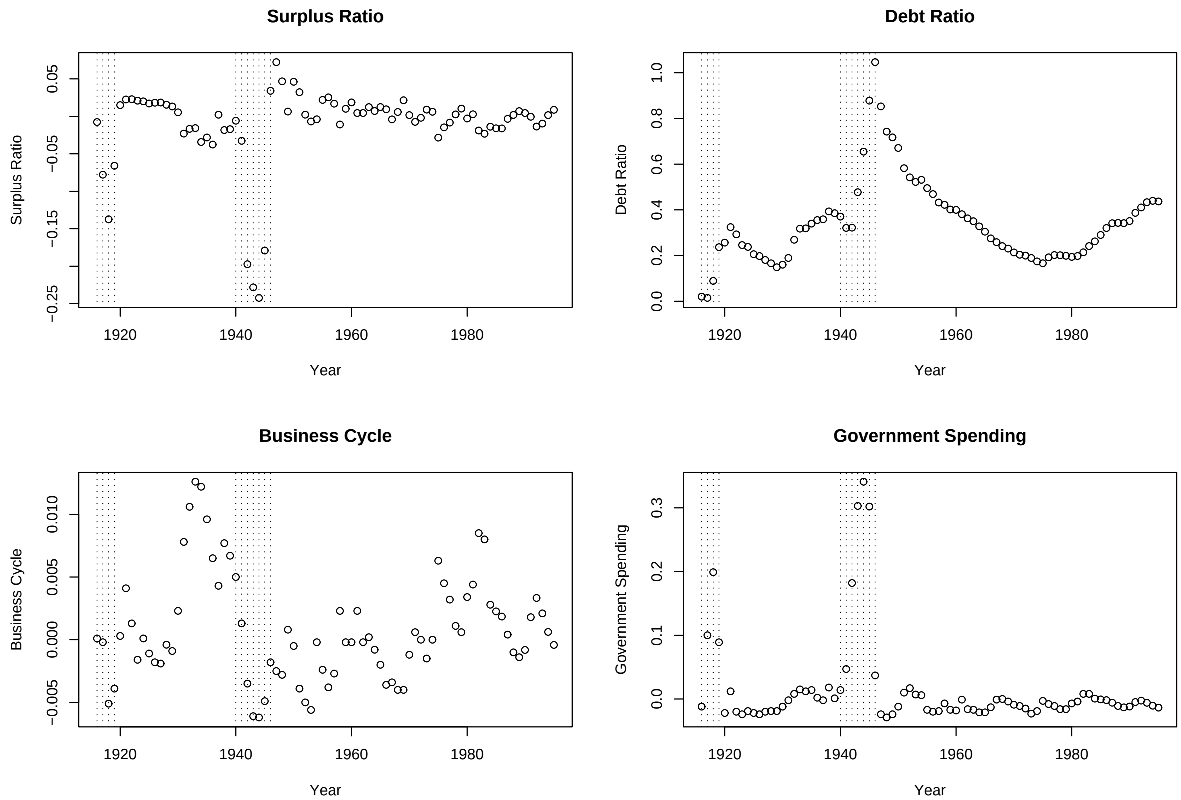

Figure 1: Data used in the estimations.

Model (1) is now fitted to the data and the resulting fits including their confidence bands are shown in Figure 2. The smoothing parameter is thereby chosen data driven. As reference line we also include a parametric linear fit resulting from the model

$$s_t = \alpha + \beta_1 \, d_t + \beta_2 \, GVAR_t + \beta_3 \, YVAR_t + \varepsilon_t \tag{2}$$

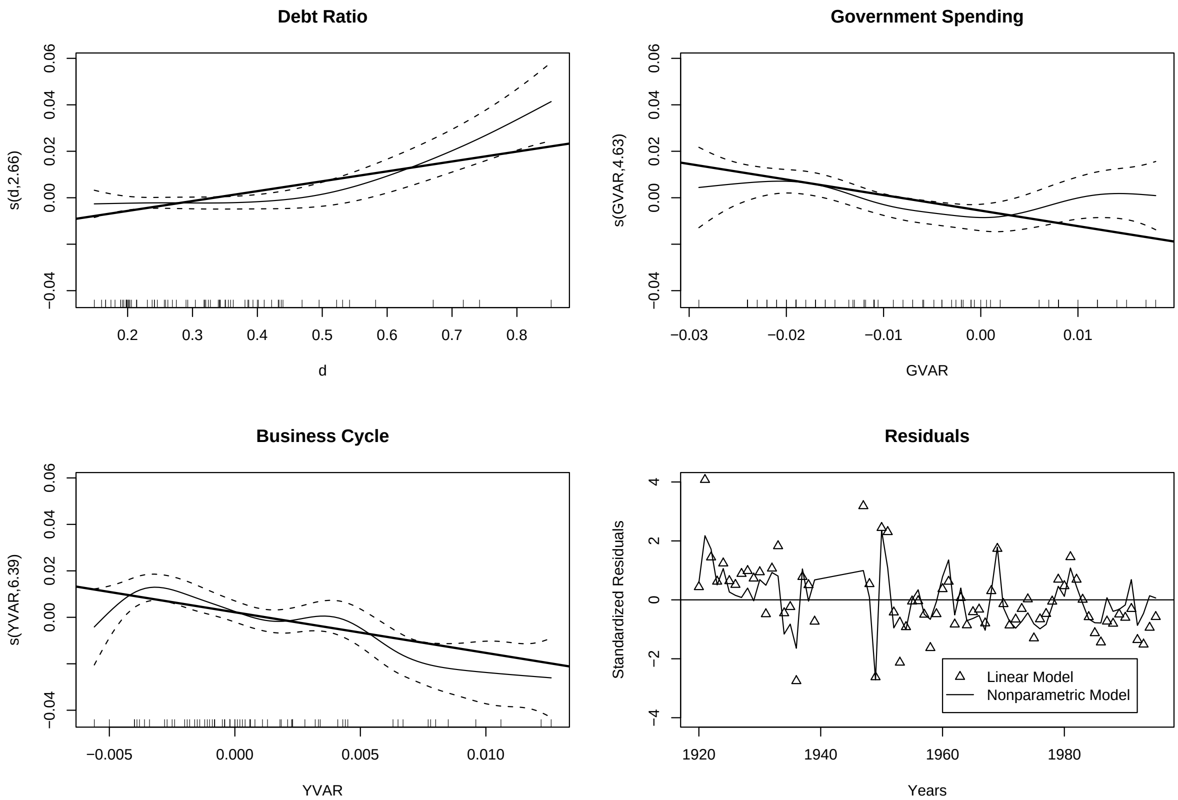

Figure 2: Non-parametric estimates for model (2) using the data as shown in Figure 1 excluding 1916-1919 and 1940-1946.

The resulting estimates in model (2) are also shown in Table 3 in the Appendix. Apparently, as can be seen from Figure 2, there is evidence, that the linear model (2) does not describe the data sufficiently well, as the effects of d_t, $GVAR_t$ and $YVAR_t$ show a nonlinear shape. The numbers in brackets on the vertical axis are the trace of the smoothing matrix of each smooth term and give the degree of complexity of each term (see also the Appendix). If a variable entered the model linearly we would have 1, and the higher the number the stronger the degree of nonlinearity.

Moreover, the adjusted R^2 for model (2) is 0.63 while for the non-parametric model (1) we achieve 0.76. The Durbin Watson statistic for the parametric residuals is 1.47

while the non-parametric residuals lead to 1.58. The improvement of the non-parametric over the parametric model is also mirrored in the bottom right plot of Figure 2 where we compare the fitted adjusted and normalized residuals $\hat{\varepsilon}_i$ in the two models. Note that, due to the standardization, the residuals should behave like $N(0,1)$ variables. For the parametric models this is not the case and we see clear outliers. In contrast, for the non-parametric models, where residuals are connected by a line for better visual impression, outliers are not exposed.

Figure 2 shows the function $f_1(d_t)$ obtained from estimating (1). This function indicates that the primary surplus to GDP ratio rises with a higher debt ratio. Further, the function shows that the increase in the surplus ratio is larger for higher debt ratios, suggesting a convex function. Since the surplus ratio rises to a stronger degree when the debt ratio is higher the debt ratio should display mean reversion. The latter fact implies that the debt ratio remains bounded and, thus, guarantees sustainability of public debt.

Our result derived above suggests a nonlinear relationship between the primary surplus ratio and the debt ratio among others. However, our inference should be based on terms of statistical significance. Therefore, the next step is to assess which of the non-parametric components are actually required in the sense of statistical significance. We pursue this question by significance tests based on bootstrapping.

Let H_1 denote model (1) with all components entering non-parametrically. We denote this by defining with $\mathcal{N}$ the index set of non-parametric components. For model (1) we have

$$\mathcal{N}_1 = \{d, GVAR, YVAR\} \tag{3}$$

With H_0 we denote a simplified model where one (or more) of the components are modelled linearly instead of non-parametrically. For instance, $\mathcal{N}_0 = \{d, GVAR\}$ stands for the model

$$s_t = \alpha + f_1(d_t) + f_2(GVAR_t) + \beta_3 YVAR_t + \varepsilon_t \tag{4}$$

and $\mathcal{N}_0 = \emptyset$ represents model (2). Moreover we define with $\hat{\mu}_{1t}$ and $\hat{\mu}_{0t}$ the fitted mean values for the t-th observation s_t under model H_1 and H_0, respectively. In order to test model H_0 against H_1 we calculate the F type statistic

$$F = \frac{RSS_0 - RSS_1}{RSS_1} \tag{5}$$

where $RSS_l = \sum_{t=1}^{n}(s_t - \hat{\mu}_{lt})^2, l = 0, 1$. If both models H_0 and H_1 are parametric (and nested) we can use classical asymptotic theory for assessing the significance of F with reference distribution given by the F distribution. This asymptotic theory does not hold for non-parametric tests (see e.g. Härdle & Mammen, 1993). We therefore assess F by bootstrapping. To do so we generate new observations

$$s_t^* = \hat{\mu}_{0t} + \varepsilon_t^* \tag{6}$$

from the fitted H_0 model, where ε_t^* is a bootstrapped residual. In principle, there are several options to draw ε_t^* (see for instance Shao & Tu, 1995, or Efron & Tibshirani, 1993). We decided for a residual bootstrap, that is we draw ε_t^* from the fitted residuals $\hat{\varepsilon}_{11}, \ldots, \hat{\varepsilon}_{1n}$ in the H_1 model with $\hat{\varepsilon}_{1t} = s_t - \hat{\mu}_{1t}$. Note that bootstrap residuals must not be drawn from the fitted H_0 residuals $\hat{\varepsilon}_{0t}$. This would be wrong, because if the H_0 model is not correct, this results in biased residuals. Hence, bootstrapping with residuals drawn from the H_0 model would mean simulating from a false model.

With (6) we now run the standard ideas of bootstrapping by refitting models H_1 and H_0 to the bootstrapped data s_t^* to obtain a bootstrapped F statistic F^*. Repeating this B times provides a simulated reference distribution of F under H_0 which finally allows to assess significance.

We have now gathered the prerequisites to run a model selection which is pursued in a stepwise backward selection by replacing one non-parametric function by a linear function after the other. The results are listed in Table 1. The final model shows a non-parametric structure in the debt ratio d and in government spending $GVAR$ yielding (4) as the final model. It should be noted that the business cycle variable $YVAR$ does not enter the equation non-parametrically and the shape shown in Figure 2 is not significantly different from a straight line. The fact that $YVAR$ does not enter non-parametrically holds because we cannot reject $H_0 : \mathcal{N}_0 = \{d, GVAR\}$ against $H_1 : \mathcal{N}_1 = \{d, GVAR, YVAR\}$ while $H_0 : \mathcal{N}_0 = \{d, YVAR\}$ and $H_0 : \mathcal{N}_0 = \{GVAR, YVAR\}$ are both rejected in favour of $H_1 : \mathcal{N}_1 = \{d, GVAR, YVAR\}$. Further, both the debt ratio d as well as temporary government spending $GVAR$ enter the equation in a non-parametric way as the last two lines in Table 1 clearly show. Now, both $H_0 : \mathcal{N}_0 = \{d\}$ and $H_0 : \mathcal{N}_0 = \{GVAR\}$ are rejected in favour of $H_1 : \mathcal{N}_1 = \{d, GVAR\}$.

The corresponding fitted smooth functions $f_1(\cdot)$ and $f_2(\cdot)$ of (4) are basically the same as those shown in Figure 2 and are therefore not reproduced again. Especially, the primary surplus ratio again turns out to be a convex function of the debt ratio.

H_0	H_1	p-value	selected model
$\mathcal{N}_0 = \{d, GVAR\}$	$\mathcal{N}_1 = \{d, GVAR, YVAR\}$	0.122	H_0
$\mathcal{N}_0 = \{d, YVAR\}$	$\mathcal{N}_1 = \{d, GVAR, YVAR\}$	0.036	H_1
$\mathcal{N}_0 = \{GVAR, YVAR\}$	$\mathcal{N}_1 = \{d, GVAR, YVAR\}$	0.044	H_1
$\mathcal{N}_0 = \{d\}$	$\mathcal{N}_1 = \{d, GVAR\}$	0.008	H_1
$\mathcal{N}_0 = \{GVAR\}$	$\mathcal{N}_1 = \{d, GVAR\}$	0.022	H_1

Table 1: Test results (p-value) for backward selection routine.

3 A time varying coefficient model

The previous section demonstrated that there is evidence for nonlinearities in the process determining the response of the primary surplus ratio to variations in the debt ratio. In this section we pursue the question of whether the functional relationship had been stable over time. To do so, we assume a linear relationship but let the coefficients depend non-parametrically on time. Allowing for time dependent coefficients implies that the marginal effect of the variables may change over time giving sufficient flexibility. Therefore, a linear structure can be justified in the estimation of the functional relationship between the surplus ratio and the debt ratio.

The equation to be estimated, then, is given by

$$s_t = \alpha + \beta_1(t)\, d_t + \beta_2\, GVAR_t + \beta_3\, YVAR_t + \varepsilon_t \tag{7}$$

Now, $\beta_1(t)$ gives the effect of the debt ratio depending on time t. In principle, one could also allow β_2 as well as β_3 to depend on t. However there is no evidence in the data for such dynamic structure, tested in an analogous way as in the previous section. From an economic point of view this implies that the effects of temporary government spending and of business cycles on the primary surplus ratio are constant over time. Thus, there is no indication that the fiscal behaviour of the government as to variations in public spending and business cycles has changed. This can be justified by resorting to the tax smoothing model stating that public deficits should be used to compensate declining tax revenues such that tax rates are smoothed over time. Therefore, there should be no systematic time trend in the coefficients of $GVAR$ and $YVAR$ giving policy makers' response to temporary government spending and to business cycle variations.

As database we now consider post World War II years starting with 1947. The fitted coefficient $\beta_1(t)$ is shown in Figure 3 together with the estimated residuals $\hat{\varepsilon}_t$. The Durbin Watson statistic for $\hat{\varepsilon}_t$ takes the value 1.65 and R^2 is calculated as 0.72. The parametric estimates β_2 and β_3 are listed in Table 2.

coefficient	estimate	std dev	t-value
Intercept	-0.007	0.008	-0.834
GVAR	-1.968	0.533	-3.689
YVAR	-0.626	0.146	-4.29

Table 2: Parametric estimates for model (7) using the data as shown in Figure 1 from 1947-1995.

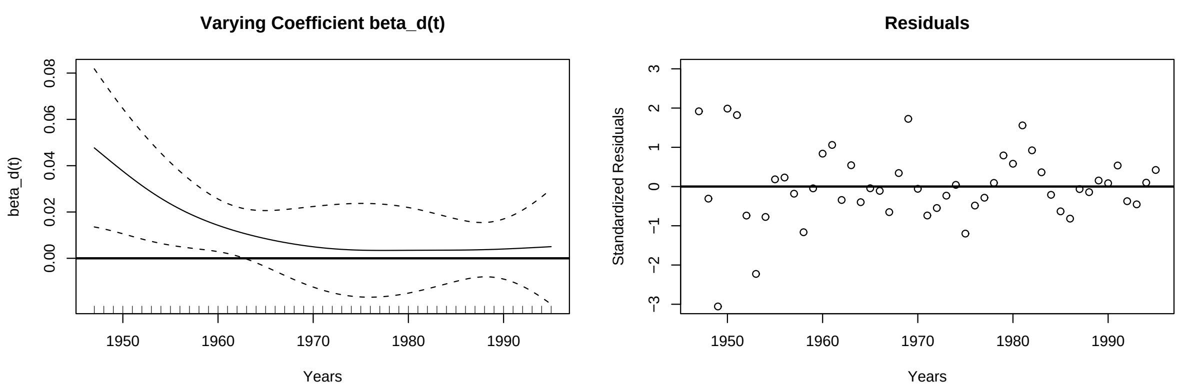

Figure 3: The reaction coefficient $\beta_1(t)$ as a function of time obtained from estimating (7) and the residuals of the estimation.

Figure 3 clearly demonstrates that the reaction coefficient $\beta_1(t)$ is characterized by a negative trend over time. This implies that the reaction of the primary surplus to GDP ratio has declined over the last 50 years. But it should be noted that the coefficient has remained positive over the whole time period we consider. The result that the reaction coefficient $\beta_1(t)$ has declined over time is also consistent with the last section where we found that the primary surplus reacts the stronger to variations in public debt the higher the debt ratio is. Since debt ratios were particularly high after World War II we can expect that the reaction coefficient $\beta_1(t)$ is high in those periods when we assume a linear but time varying relationship.

We should also like to point out that $GVAR$ and $YVAR$ have the expected signs as in the paper by Bohn (1998). However, in contrast to Bohn, the $GVAR$ coefficient is larger than one and the $YVAR$ coefficient is smaller than one in absolute values. This implies that the primary surplus falls by less than GDP in a recession and by more than GDP as temporary government spending rises.

4 Conclusion

This paper has tested for sustainability of US public debt by analyzing how the primary surplus reacts to variations in the debt ratio. Estimating smoothing spline regressions demonstrated that the primary surplus to GDP ratio is a nonlinearly increasing function of the debt ratio. In addition, we could provide an estimate for the function giving the response of the primary surplus ratio to the debt ratio. This function showed that the response of primary surplus-GDP ratio seems to increase with a rising debt ratio confirming Bohn's conjecture.

Further, we performed a semi-parametric estimation where we assumed a linear relationship, but with the parameter giving the response of the primary surplus ratio to changes in the debt ratio being time dependent. With the parameter being time dependent the linear model is sufficiently flexible to reflect the situation in the US. This estimation demonstrated that the sustainability coefficient declined over time but seems to remain positive over the time period under consideration. Thus, our estimations provide additional evidence that US public debt was sustainable over the time period considered although there was a negative trend.

From a methodological point of view this paper has provided an example for smoothing spline regressions which allow to perform non-parametric and semi-parametric estimations. These are more flexible than OLS estimations and contain the latter as a special case. Surprisingly, there are only few applications of smoothing spline estimations in economics although these estimations may yield insights into economic structure which are difficult to detect with ordinary OLS estimation.

A Nonparametric estimation

The subsequent algorithm is based on Wood (2000) and implemented in the public domain software package R (see Ihaka & Gentleman, 1996). The program and more information about it can be downloaded from http://www.r-project.org/. We exemplify the fit with the simplified model

$$s_t = \alpha + f(d_t) + \varepsilon_t$$

Let s_t and d_t be the observed values for $t = 1, 2, \ldots$. For fitting we replace $f(d_t)$ by the parametric form

$$f(d_t) = d_t\,\beta_d + Z(d_t)\,\gamma \tag{8}$$

where $Z(d_t)$ is a high dimensional basis in d_t, for instance a cubic spline basis, and γ as corresponding coefficient. Conventionally, $Z(d_t)$ is 10 to 40 dimensional. In practice, if a larger basis is used this is reduced to a smaller basis using only those basis functions corresponding to the largest Eigenvalues of $Z(d_t)Z(d_t)^T$, see Wood (2000) for more details.

In principle, with replacement (8) one ends up with a parametric model. However, fitting the model in a standard OLS fashion is unsatisfactory due to the large dimensionality of $Z(d_t)$ which will lead to highly variable estimates. This can be avoided by imposing an additional penalty term on γ, shrinking its values to zero. To be more specific, we obtain an estimate by minimizing the penalized OLS criterion

$$\sum_t \{s_t - d_t\beta_d - Z(d_t)\gamma\}^2 + \lambda\gamma^T P\gamma$$

with λ called the smoothing or penalty parameter and $\gamma^T P\gamma$ as penalty. Matrix P is thereby chosen in accordance to the basis (see Ruppert, Wand & Carroll, 2003, for more details). It is easy to see that choosing $\lambda = 0$ yields an unpenalized OLS fit, while $\lambda \to \infty$ typically implies $\gamma = 0$ depending on the choice of P. Hence, λ steers the amount of smoothness of the function with a simple linear fit as one extreme and a high dimensional parametric as the other extreme.

The fitted function itself can be written as $\hat{f}_1(\mathbf{d}) = H(\lambda)\mathbf{s}$ where $\mathbf{s} = (s_1, s_2, \ldots)$ is the vector of observed surplus values and analogous definition for $\mathbf{d}$. The matrix $H(\lambda)$ results thereby as

$$H(\lambda) = \begin{pmatrix} \mathbf{d} & Z(\mathbf{d}) \end{pmatrix} \left(\begin{pmatrix} \mathbf{d}^T \\ Z^T(\mathbf{d}) \end{pmatrix} \begin{pmatrix} \mathbf{d} & Z(\mathbf{d}) \end{pmatrix} + \lambda \begin{pmatrix} 0 & 0 \\ 0 & P \end{pmatrix} \right)^{-1} \begin{pmatrix} \mathbf{d}^T \\ Z^T(\mathbf{d}) \end{pmatrix}.$$

Matrix $H(\lambda)$ is also called the smoothing matrix and its trace is conventionally defined as the degree of complexity of the fit. Note that as special case we get trace of $H(\infty)$ equals 1 while trace of $H(0)$ is $p+1$ with p as dimension of $Z(d)$. The parameter λ steers the compromise between goodness of fit and complexity of the fit.

To obtain a reliable fit, λ should be chosen data driven. One possibility is to use a generalized cross validation criterion defined through

$$GCV(\lambda) = \sum_t \left(\frac{s_t - \hat{f}(d_t)}{1 - tr(H)/n} \right)^2$$

with n as overall sample size. A suitable choice for λ is achieved by minimizing $GCV(\lambda)$. This can be done iteratively using a Newton-Raphson algorithm, as has been pointed out and implemented by Wood (2000, 2001).

If there is more than one smooth function involved, estimation can be carried out using a backfitting strategy. This means, in order to fit one smooth component all other terms in the model are kept fixed and smoothing like demonstrated above is pursued. One then cycles over the different smooth components by always fitting just one of the smooth functions. This is the major idea behind fitting additive models and we refer to Hastie & Tibshirani (1990) for further technical details.

B Result of parametric linear estimation of (2)

coefficient	estimate	std dev	t-value
Intercept	-0.014	0.004	-3.378
d	0.043	0.009	4.282
GVAR	-0.669	0.145	-4.621
YVAR	-1.750	0.433	-4.043

Table 3: Parametric estimates for model (2) using the data as shown in Figure 1 excluding 1916-1919 and 1940-1946.

C Data

The data are from the appendix in Bohn (1998).

Year	s	GVAR	YVAR	d
1916	-0.007614879647	-0.012000000000	0.000100000000	0.0200000000000
1917	-0.077863727740	0.100000000000	-0.000200000000	0.0146533523546
1918	-0.137360425379	0.199000000000	-0.005100000000	0.0890883902169
1919	-0.065962237772	0.089000000000	-0.003900000000	0.2365673158331
1920	0.014944984665	-0.022000000000	0.000300000000	0.2565480609257
1921	0.022525373998	0.012000000000	0.004100000000	0.3244166239106
1922	0.022840755078	-0.020000000000	0.001300000000	0.2927053847553
1923	0.020893495830	-0.024000000000	-0.001600000000	0.2457790599749
1924	0.020021875754	-0.019000000000	0.000100000000	0.2377833885848
1925	0.017135177572	-0.022000000000	-0.001100000000	0.2058835405765
1926	0.018167760447	-0.024000000000	-0.001800000000	0.1976818469274
1927	0.018432965562	-0.020000000000	-0.001900000000	0.1805893142338
1928	0.015424837091	-0.019000000000	-0.000400000000	0.1660725353409
1929	0.013170324315	-0.019000000000	-0.000900000000	0.1486366234356
1930	0.005364582970	-0.012000000000	0.002303225806	0.1598645725003
1931	-0.023028398500	-0.002000000000	0.007800000000	0.1893445046937
1932	-0.016708169013	0.008000000000	0.010600000000	0.2690012521009
1933	-0.015708646765	0.015000000000	0.012600000000	0.3176637456875
1934	-0.034269561169	0.012000000000	0.012200000000	0.3188557783299
1935	-0.028194997460	0.014000000000	0.009600000000	0.3396827502056
1936	-0.037638597656	0.002000000000	0.006500000000	0.3552141543723
1937	0.002141121861	-0.002000000000	0.004300000000	0.3586397049831
1938	-0.018312450103	0.018000000000	0.007700000000	0.3932026395606
1939	-0.017129071242	0.001000000000	0.006700000000	0.3853396280944
1940	-0.005841196672	0.014000000000	0.005000000000	0.3704486929483
1941	-0.032710701364	0.047000000000	0.001300000000	0.3208764038572
1942	-0.197329522205	0.182000000000	-0.003500000000	0.3221205665027
1943	-0.228252267352	0.303000000000	-0.006100000000	0.4773008417899
1944	-0.242276116610	0.341000000000	-0.006200000000	0.6547003331541
1945	-0.179088060794	0.302000000000	-0.004900000000	0.8780353261957
1946	0.034053479909	0.037000000000	-0.001800000000	1.0464542339468
1947	0.072309639682	-0.024000000000	-0.002500000000	0.8527551100874
1948	0.046704244233	-0.029000000000	-0.002800000000	0.7424113196481
1949	0.006381012416	-0.024000000000	0.000800000000	0.7173055076606
1950	0.046108239406	-0.012000000000	-0.000500000000	0.6709431829330
1951	0.032248647969	0.010000000000	-0.003900000000	0.5822346805978
1952	0.002223980445	0.017000000000	-0.005000000000	0.5420884573986
1953	-0.006838836455	0.007000000000	-0.005600000000	0.5221186593345
1954	-0.003935923672	0.006000000000	-0.000200000000	0.5313528838552
1955	0.021913244871	-0.017000000000	-0.002400000000	0.4950949055579
1956	0.025337313689	-0.020000000000	-0.003800000000	0.4688544775211
1957	0.016904500879	-0.019000000000	-0.002700000000	0.4321483942698

Year	s	GVAR	YVAR	d
1958	-0.010912608193	-0.007000000000	0.002300000000	0.4219541834609
1959	0.010054194184	-0.017000000000	-0.000200000000	0.4015771315919
1960	0.018610832265	-0.018000000000	-0.000200000000	0.3999430280587
1961	0.004405284508	-0.001000000000	0.002300000000	0.3808737151248
1962	0.004442733991	-0.016000000000	-0.000200000000	0.3627664573455
1963	0.012309684159	-0.017000000000	0.000200000000	0.3501781665047
1964	0.007088722145	-0.021000000000	-0.000800000000	0.3275894574111
1965	0.012376151573	-0.021000000000	-0.002000000000	0.3043977055449
1966	0.009393247017	-0.013000000000	-0.003600000000	0.2750698146738
1967	-0.004078696017	-0.001000000000	-0.003400000000	0.2585172744722
1968	0.005820498037	0.000000000000	-0.004000000000	0.2411662960217
1969	0.021481839701	-0.004000000000	-0.004000000000	0.2303952760315
1970	0.001641520821	-0.009000000000	-0.001200000000	0.2135908267954
1971	-0.007375319338	-0.011000000000	0.000600000000	0.2035765855826
1972	-0.001939746621	-0.015000000000	0.000000000000	0.1996322563698
1973	0.008968268031	-0.023000000000	-0.001500000000	0.1892707975482
1974	0.006079366670	-0.019000000000	0.000000000000	0.1742992951866
1975	-0.028394517356	-0.003000000000	0.006300000000	0.1661964920888
1976	-0.014733835455	-0.008000000000	0.004500000000	0.1920887874873
1977	-0.008338066458	-0.011000000000	0.003200000000	0.2020376446210
1978	0.002574829524	-0.016000000000	0.001100000000	0.2013201680214
1979	0.010322619746	-0.016000000000	0.000600000000	0.1988661192571
1980	-0.002873318907	-0.007000000000	0.003400000000	0.1941259394277
1981	0.002760098206	-0.004000000000	0.004400000000	0.1978240315800
1982	-0.018907708798	0.008000000000	0.008500000000	0.2142146619063
1983	-0.023246387492	0.008000000000	0.008000000000	0.2413981690271
1984	-0.013837524104	0.000591724334	0.002789975557	0.2620421917141
1985	-0.015978257100	-0.000657689466	0.002258186619	0.2900249361646
1986	-0.016123310362	-0.001813603144	0.001846372121	0.3204757839751
1987	-0.003367200695	-0.005867188418	0.000407955866	0.3414085767716
1988	0.001881317332	-0.010948483464	-0.001008184274	0.3428786438530
1989	0.006931835419	-0.013151494506	-0.001398702245	0.3417192606655
1990	0.004387320296	-0.011847536863	-0.000816633639	0.3509508036892
1991	-0.000566196386	-0.004822348258	0.001796193032	0.3867543848530
1992	-0.013624152744	-0.002554864756	0.003329827579	0.4104781465067
1993	-0.009659771553	-0.005968603896	0.002101056719	0.4333773338725
1994	0.001604019782	-0.010566045628	0.000622337870	0.4394221203339
1995	0.008729937171	-0.013572905255	-0.000414530310	0.4367381122243

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