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An economic model of work-related stress

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Abstract

In this paper we present an economic model of optimal consumption and labour supply where we assume that working may generate stress which affects the well-being of the representative individual. As to stress we posit that it is influenced by cumulated past labour and capital. The latter reflects the fact that work-related stress evolves gradually over time and that it is more likely to occur in modern societies. Using optimal control theory we demonstrate that sustained cycles may result. Further, we numerically compute the global optimal value function and give a representation of the limit cycle.

Keywords: Work-related stress, intertemporal optimization, limit cycles

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1 Introduction

According to the European Agency for Safety and Health at Work, work-related stress affects 28% of workers in the European Union (EU) and is the second most common health problem related to work, after back pain ([3], p. 10). On the individual level, the consequences of stress are that the person's general quality of life as well as his well-being are detracted. For some people who experience work-related stress the consequences may be more drastic implying that stress negatively affects their health. Typical health problems caused by work-related stress are for example insomnia, constant tiredness, high blood pressure, nervous twitches, just to mention a few.

In addition to these individual problems work-related stress also causes costs for society. So, the European Commission estimates that costs due to work-related stress in the EU amount to at least 20 billion Euro annually ([7]). For France, [1] have estimated that work-related stress cost between 830 and 1656 million Euro in 2000, which represented 13% to 26% of total spending of social security occupational illnesses and work injuries branch. Besides direct health costs work-related stress leads to costs due to absenteeism and raising individuals' quitting behaviour causing costs for firms. [14] find that individuals experiencing stress are 25% more likely to hold intentions to quit or being absent from work than those without work-related stress.

As concerns stress, one has to point out that stress can also be favourable to a person's well-being and a certain amount of stress is even needed in order to remain healthy and alert. Therefore, in the psychological literature one finds the distinction between two forms of stress, between the so-called eustress and the distress. Eustress is the positive form of stress which is beneficial to an individual. This kind of stress or pressure is stimulating and enhances performance. However, when stress or pressure becomes too large such that the individual perceives himself unable to cope successfully with a situation, he is subject to distress, the negative form of work-related stress (as to the distinction between eustress and distress see [16] or [5]). The latter form of stress, distress, is perceived as negative by

a person and may lead to that sort of individual health problems mentioned above.

In general, work situations are experienced as stressful when the demands made on the person do not match the resources available (in the individual or provided by the organization) or do not meet the person's needs and motivation. This can also serve as a definition for (di-)stress. As concerns the causes of work-related stress, [15] summarizes the factors under four headings, which are then differentiated further: quantitative overload, qualitative underload, lack of control over work and lack of social support.

In this paper, we will focus on the first and the third factor. So, one of our assumptions will be that stress arises as individuals simply have too much work, a fact which seems to be of relevance particularly for Japan (see [14]). But we refrain from modelling the second factor, qualitative under- or overload as a possible source of stress. Qualitative underload means that the individual's work is not demanding so that he may be bored by his work, overload simply means that the work is too difficult. The other factor generating stress is the lack of control over work which is perceived as a threat to individual freedom, autonomy and identity. We take account of this factor because there is strong evidence that machine- and systems-paced work, especially of high rate, is detrimental to psychological and physical health (see e.g. [4] or [2]). It should also be pointed out that workload has to be considered in relation to work pace such that it is in particular the interrelation between these two factors which generates stress.

The rest of the paper is organized as follows. In the next section we present our model and our modelling of work-related stress. Section 3 studies the dynamics of the model and section 4, finally, concludes.

2 The structure of the model

Our model consists of a representative household with a utility function which positively depends on consumption at time t , $C(t)$, and negatively on labour, $L(t)$. The latter models the preference for leisure, as usual in economics. In addition, we make the assumption

that utility depends on work-related stress, $S(t)$. As to the effect of stress on utility we posit that utility rises with stress when stress is below a certain threshold, S^* , and declines when stress exceeds this threshold. Thus, we take into consideration that for small values of stress a rise in stress may well have a positive effect on a person's well-being which raises his work performance (see e.g. [5], pp. 6-8). In this case we speak of eustress. If stress becomes too large, i.e. if it exceeds the threshold S^* , well-being declines with stress. In this case, we speak of distress as already mentioned in the Introduction.

As to the utility function $U(\cdot)$ we assume¹ that it is separable in C , L and S and a root function of C , linear in L and quadratic in S . Thus, the utility function is given by

$$U(C, L, S) = \sqrt{C} + (\bar{L} - L) - a(S - S^*)^2, \quad (1)$$

with $S^* > 0$, \bar{L} the maximum available labour supply and $a > 0$ a constant.

As to the constraints the first is the usual budget constraint stating that production is spent for consumption and saving, i.e.

$$\dot{K} = K^\alpha L^\beta - C - \delta K. \quad (2)$$

K is capital and equals cumulated past investment and $\delta > 0$ is the depreciation rate. $K^\alpha L^\beta$ is the production function with $\alpha \in (0, 1)$ the capital share and $\beta \in (0, 1)$ the labour share.

The second constraint describes the stress variable S . Stress is a function of cumulated past labour. Thus, we take into account that work-related stress is caused by labour and, second, that it is the cumulative effect of labour which leads to stress. The latter seems to be important because working overtime one or two times a month does not necessarily lead to stress. However, if this occurs more often stress is likely to occur. In addition, we assume that the effect of labour on stress is the stronger the higher the capital stock is. We do this because, as mentioned in the Introduction, machine- and system-paced work is detrimental to health since it forces people to work in accordance with the machines

¹In the following we delete the time argument t .

depriving them of their control over work, which implies that their personal needs may be left behind. Further, work-related stress is a phenomenon which occurs primarily in developed countries which dispose of higher capital stocks compared to countries say 100 hundred years ago. With these assumptions the change in stress can be described by the following differential equation,

$$\dot{S} = L K - \eta S, \quad (3)$$

with $\eta > 0$ reflecting the ability to recover.

The intertemporal optimization problem, then, is to choose consumption and labour such that the discounted stream of utility over an infinite time horizon is maximized subject to the two constraints (2) and (3). Denoting by $\rho > 0$ the subjective discount rate, the formal problem is to

$$\max_{C,L} \int_0^{\infty} e^{-\rho t} \left(\sqrt{C} + (\bar{L} - L) - a(S - S^*)^2 \right) dt, \quad (4)$$

subject to (2) and (3).

Optimality conditions are derived from the current-value Hamiltonian which is given by

$$H = \sqrt{C} + (\bar{L} - L) - a(S - S^*)^2 + \lambda_1(K^\alpha L^\beta - C - \delta K) + \lambda_2(L K - \eta S), \quad (5)$$

with λ_1 and λ_2 denoting costate variables or shadow prices of K and S .

The necessary optimality conditions are given by

$$\frac{\partial H}{\partial C} = 0.5 C^{-0.5} - \lambda_1 = 0 \quad (6)$$

$$\frac{\partial H}{\partial L} = -1 + \lambda_1 \beta L^{\beta-1} K^\alpha + \lambda_2 K = 0 \quad (7)$$

$$\dot{\lambda}_1 = (\rho + \delta)\lambda_1 - \lambda_1 \alpha K^{\alpha-1} L^\beta - \lambda_2 L \quad (8)$$

$$\dot{\lambda}_2 = (\rho + \eta)\lambda_2 + 2a(S - S^*). \quad (9)$$

In addition we require that the transversality condition $\lim_{t \rightarrow \infty} e^{-\rho t} (\lambda_1 K + \lambda_2 S) = 0$ must be fulfilled.

From (6) and (7) we get optimal consumption and labour supply as functions of the costate variables and of state variables as

$$C = 0.25 \lambda_1^{-2}, \quad L = \left(\frac{\beta \lambda_1 K^\alpha}{1 - \lambda_2 K} \right)^{1/(1-\beta)}. \quad (10)$$

Inserting (10) in \dot{K} , \dot{S} , $\dot{\lambda}_1$ and $\dot{\lambda}_2$ gives an autonomous system of differential equations in the state variables K and S and in the costate variables λ_1 and λ_2 . This system is given by

$$\dot{K} = K^\alpha L(K, \lambda_1, \lambda_2, \cdot)^\beta - C(\lambda_1) - \delta K \quad (11)$$

$$\dot{S} = L(K, \lambda_1, \lambda_2, \cdot) K - \eta S \quad (12)$$

$$\dot{\lambda}_1 = (\rho + \delta)\lambda_1 - \lambda_1 \alpha K^{\alpha-1} L(K, \lambda_1, \lambda_2, \cdot)^\beta - \lambda_2 L(K, \lambda_1, \lambda_2, \cdot) \quad (13)$$

$$\dot{\lambda}_2 = (\rho + \eta)\lambda_2 + 2a(S - S^*). \quad (14)$$

3 The dynamics of the model

3.1 The analytical model

Equations (11)-(14) completely describe the dynamic behaviour of our model. We are interested in the dynamics around a rest point or stationary point of this system, in particular in the question of whether the model converges to the rest point or whether it may generate cycles for example. To do so we first assume that a unique rest point exists for the analytical model and compute the Jacobian matrix evaluated at the rest point.

The Jacobian is given by

$$J = \begin{pmatrix} \alpha K^{\alpha-1} L^\beta + K^\alpha \beta L^{\beta-1} L_K - \delta & 0 & \beta K^\alpha L^{\beta-1} L_{\lambda_1} - C_{\lambda_1} & K^\alpha \beta L^{\beta-1} L_{\lambda_2} \\ L + K L_K & -\eta & K L_{\lambda_1} & K L_{\lambda_2} \\ -a_{31} & 0 & a_{33} & -a_{34} \\ 0 & 2a & 0 & \rho + \eta \end{pmatrix},$$

with

$$\begin{aligned}
a_{31} &= \lambda_2 L_K + \lambda_1 ((\alpha - 1)\alpha K^{\alpha-2} L^\beta + \alpha\beta K^{\alpha-1} L^{\beta-1} L_K) \\
a_{33} &= \rho + \delta - \alpha K^{\alpha-1} L^\beta - \lambda_2 L_{\lambda_1} - \alpha\beta K^{\alpha-1} \lambda_1 L^{\beta-1} L_{\lambda_1} \\
a_{34} &= \lambda_2 L_{\lambda_2} + \alpha\beta K^{\alpha-1} \lambda_1 L^{\beta-1} L_{\lambda_2} + L.
\end{aligned} \tag{15}$$

L_k, C_k denote the derivative of L and C with respect to variable k , $k = K, \lambda_1, \lambda_2$. From (10) it can easily be seen that the derivatives have the following signs:

$$C_{\lambda_1} < 0, \quad L_{\lambda_1} > 0, \quad L_{\lambda_2} > 0, \quad L_K > = < 0 \text{ for } \lambda_2 > = < -\alpha K^{-2\alpha-1}(1 - \lambda_2 K). \tag{16}$$

The eigenvalues of that matrix are given by

$$\mu_{1,2,3,4} = \frac{r}{2} \pm \sqrt{\left(\frac{r}{2}\right)^2 - \frac{W}{2}} \pm \sqrt{\left(\frac{W}{2}\right)^2 - \det J},$$

with W defined as

$$W = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{24} \\ a_{42} & a_{44} \end{vmatrix} + 2 \begin{vmatrix} 0 & a_{14} \\ 0 & a_{34} \end{vmatrix}$$

where a_{ij} is the element of the i -th row and j -th column (see [6]).

Looking at the formula for the eigenvalues, one immediately realizes that the eigenvalues are symmetric around $\rho/2$. Since $\rho > 0$ holds this implies that the system is never completely stable (in the sense that all eigenvalues have negative real parts) but it can only be saddle point stable. From an economic point of view, convergence to the stationary state means that all variables are constant in the long run. That is there are constant levels of consumption and labour supply and, as a consequence, a constant capital stock and a constant level of stress. The transitional behaviour of the variables in case of saddle point stability is characterized by unimodal time paths if the eigenvalues are real. If the eigenvalues are complex conjugate, however, the variables are characterized by cyclical oscillations until the stationary point is reached. This means that both the capital stock as

well as the level of stress shows oscillations over time, however, with declining amplitudes until the stationary point is reached asymptotically.

Besides convergence to the stationary state in the long run, the system may show persistent endogenous cycles. This behaviour can be observed if the dynamic system (11)-(14) undergoes a Hopf bifurcation. A Hopf bifurcation states the following (for a complete statement of the Hopf bifurcation theorem see e.g. [12]): Assume that we continuously vary a parameter, say the discount rate, and that for a certain critical value of that (bifurcation) parameter two eigenvalues become purely imaginary. Assume in addition that the crossing speed of the eigenvalues is non-zero as the bifurcation parameter is varied. Then, there exist stable or unstable limit cycles which occur for values of the bifurcation parameter which are larger or smaller than the critical parameter value for which two eigenvalues are purely imaginary.

Let us find out whether persistent cycles may occur in our model. From the formula of the eigenvalues (see, e.g. [6]) we know that $W > 0$ is a necessary condition for two purely imaginary eigenvalues and, thus, for the emergence of a Hopf bifurcation which leads to stable limit cycles. Looking at the constant W we see that only the expression $a_{11}a_{33} - a_{31}a_{13}$ may become positive. Using the fact that $a_{11} + a_{33} = \rho$ holds (cf. [8], p. 134) we may write $a_{11}a_{33} - a_{31}a_{13}$ as

$$\begin{aligned} a_{11}a_{33} - a_{31}a_{13} = & \\ & (\alpha K^{\alpha-1}L^\beta - \lambda_2 L_{\lambda_1} + \alpha\beta K^{\alpha-1}\lambda_1 L^{\beta-1}L_{\lambda_1} - \delta) \cdot \\ & (\rho + \delta - \alpha K^{\alpha-1}L^\beta - \lambda_2 L_{\lambda_1} - \alpha\beta K^{\alpha-1}\lambda_1 L^{\beta-1}L_{\lambda_1}) + \\ & (\lambda_2 L_K + \lambda_1 ((\alpha - 1)\alpha K^{\alpha-2}L^\beta + \alpha\beta K^{\alpha-1}L^{\beta-1}L_K)) (\beta K^\alpha L^{\beta-1}L_{\lambda_1} - C_{\lambda_1}). \end{aligned} \quad (17)$$

For $\delta \geq \alpha K^{\alpha-1}L^\beta - \lambda_2 L_{\lambda_1} + \alpha\beta K^{\alpha-1}\lambda_1 L^{\beta-1}L_{\lambda_1}$ the first term in equation (17), $a_{11}a_{33}$, is negative while it is difficult to make a clear statement for the second term, $-a_{31}a_{13}$. However, it is seen that a positive λ_2 makes a positive sign of the second term more likely.²

²Recall that the signs of L_i and C_i are given in (16) and that $\lambda_2 > 0$ implies $L_K > 0$, since $1 - \lambda_2 K > 0$ must hold for L to be real.

Further, a positive λ_2 together with a high subjective discount rate make it more likely that the first term is positive, too. So, a Hopf bifurcation leading to limit cycles is more likely for a positive value of λ_2 together with a high discount rate. It should be noted that λ_2 at the stationary point is positive (negative) if the level of stress is lower (higher) than S^* .

From an economic point of view, the conditions leading to persistent cycles can be interpreted such that these oscillations may occur when the individual's stress level is smaller than S^* , that is when the individual experiences eustress in his work, and when he is impatient, the latter being reflected by a high subjective discount rate. These conditions state that a rise in the level of stress raises the individual's well-being suggesting that he identifies himself with his work and may even be enthusiastic in his work. Since the individual is impatient, he works a lot at nearby time periods, thus raising the level of stress and his well-being as he is in the eustress range. However, with a rising level of stress the marginal value of additional stress, its shadow price, declines. A declining shadow price of stress leads the individual to reduce his work supply generating a decline in the stress level. This goes on until the shadow price of stress rises again, due to the decline in the level of stress, thus, leading to persistent cycles. It should be mentioned that cyclical labour supply implies oscillations in the income and also in consumption.

In order to gain additional insight into our model and to prove the existence of persistent cycles we next present a numerical example.

3.2 A numerical example

To study our model numerically, we assume a constant returns to scale production function with a capital share of 30% and a labour share of 70%, i.e. $\alpha = 0.3$ and $\beta = 0.7$. δ and n are set to $\delta = 0.075$ and $n = 0.05$. S^* is set to $S^* = 3$, $a = 0.1$ and the subjective discount rate ρ serves as bifurcation parameter.

Before we study the time paths of the variables of our model we address the question

of existence and uniqueness of a stationary state. To do so, we first set $\rho = 0.05$. With this parameter value we solve (12) = 0 with respect to S and insert the resulting³ $\bar{S}(K, \lambda_2, \lambda_1, \cdot)$ in (11), (13) and (14) and, then, solve (14) = 0 with respect to λ_1 yielding $\bar{\lambda}_1(K, \lambda_2, \cdot)$. Inserting $\bar{\lambda}_1(K, \lambda_2, \cdot)$ in (11) and (13) and solving (11) = 0 and (13) = 0 with respect to K and λ_2 gives the rest point of the dynamic system. Figure (1) shows the $\dot{K} = 0$ and $\dot{\lambda}_1 = 0$ curves in the $(K - \lambda_2)$ plane demonstrating that there exists a unique rest point for our model. Varying the discount rate with $\rho = 0$ as lower bound and $\rho = 0.35$ as upper bound does not change the qualitative outcome, i.e. there always exists a unique rest point. Further, it should be noted that for about $\rho = 0.054$ we get $\bar{\lambda}_2 = 0$ and $\bar{S} = S^* = 3$ whereas for $\rho < (>) 0.054$, $\bar{\lambda}_2$ is negative (positive) and \bar{S} is larger (smaller) than $S^* = 3$.

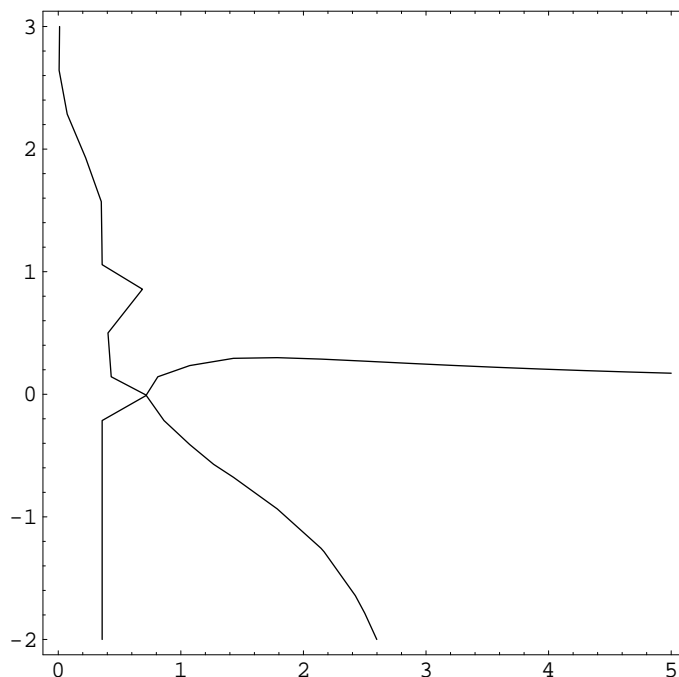


Figure 1: $\dot{K} = 0$ (monotonously falling) and $\dot{\lambda}_1 = 0$ (first rising, then declining) curves in the $(K - \lambda_2)$ plane.

Next, we analyze the local dynamics at the rest point for different values of the discount

³The $\bar{\cdot}$ denotes values at the rest point.

rate by computing the eigenvalues of the Jacobian matrix. It turns out that for $\rho \in (0, 0.3083)$ the eigenvalues are complex conjugate with two having negative real parts and two having positive real parts. This implies that the model is characterized by saddle point stability with a two dimensional stable manifold. For $\rho_{crit} = 0.3083432$ the differential equation system undergoes a supercritical Hopf bifurcation giving rise to stable limit cycles.⁴ The limit cycles occur for an interval with strictly positive measure of the discount rate where the discount rates are slightly larger than the critical value ρ_{crit} . If ρ is increased further all real parts of the eigenvalues become positive implying that the system becomes unstable.

Our analysis so far has used necessary optimality conditions and characterized the local dynamics around the stationary state. To get an idea about the global dynamics of the optimally controlled system we numerically compute the optimal value function by solving the Hamilton-Jacobi-Bellman equation. This method gives the full global information about the optimal value function which, for its part, yields the optimal control in feedback form (a detailed description of the algorithm we use is given in [10], [11]).

In particular, we are interested in the question of whether persistent cycles may turn out to be the optimal solution. Therefore, we set the subjective discount rate ρ to $\rho = 0.3084$ which is slightly larger than the critical value ρ_{crit} where a Hopf bifurcation was detected. The calculation of the optimal controls confirm that persistent cycles turn out to be the optimal solution. Figure (2) shows the convergence of the optimal path to the limit cycle in the $(K - S)$ plane.

⁴The bifurcation analysis was done with LOCBIF (see [13]).

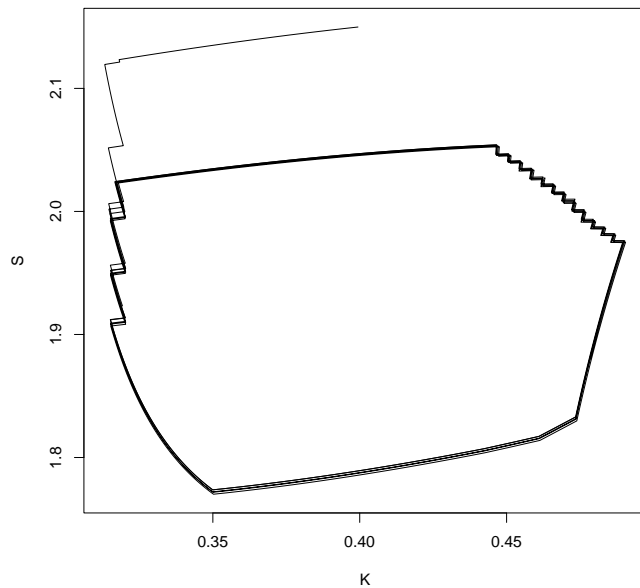


Figure 2: Limit cycle in the $(K - S)$ plane.

4 Conclusion

This paper has presented a simple model of work-related stress. Assuming that the change in stress depends on labour and capital reflecting the fact that stress is built up gradually and that it is more likely to occur in modern societies where machine- and systems-paced work occurs, we could show that persistent cycles may turn out to be the optimal solution.

While there exist quite a lot of empirical papers dealing with work-related stress there are no formal economic models investigating this phenomenon. The reason may be that it is difficult to model the variable stress and the forces affecting it. So, our model is just one possibility to formalize stress and other formulations may be relevant and feasible as well. In particular, since lack of social support is also a factor which can lead to stress (cf. [15] and [9]), possible interactions between working individuals, e.g. between subordinate and superior, should be taken into account in future studies.

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