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**Externalities of Investment and Endogenous
Growth:
Theory and Time Series Evidence**

by

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Abstract

In the paper we present and estimate an endogenous growth model in which sustained per capita growth is the result of positive externalities of investment in physical capital. In contrast to the usual assumption that investment raises physical capital and, as a by-product, a stock of knowledge one for one, we suppose a different framework. So, we treat physical and human capital as two distinct variables and underline the importance of the stock of knowledge per physical capital as to the growth performance of countries. Estimation of that model for France, Germany and Japan shows that it is compatible with empirical data. For Great Britain the model performs poor and for the USA it does not produce reasonable outcomes at all. One conclusion we draw from our studies is that an endogenous growth model with positive externalities of investment is of empirical relevance. However, the growth process is also determined by country specific factors such that cross-countries studies should be considered with some care.

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1 Introduction

One strand in endogenous growth theory assumes that investment in capital shows positive external effects. That approach goes back to Romer (1986) who presented an endogenous growth model in which capital shows decreasing returns to scale on the microeconomic level of an individual firm but increasing returns on the macroeconomic level, due to spillovers. Because of increasing returns to capital on the economy-wide level, positive sustained per capita growth can be observed in such an economy. However, Romer did not focus on pure physical capital but on knowledge as a broader and more general concept of capital.

Applying the concept of positive externalities of investment to physical capital alone makes sense, too. That seems reasonable because DeLong and Summers (1991) have demonstrated that investment, particularly in machinery, is associated with very strong positive externalities.¹ An empirical study by Romer (1987) also seems to confirm the presence of positive externalities associated with physical capital. Romer used aggregate long run economic data and made conventional growth accounting studies with capital and labour as the only input factors. The capital share he obtained is between 0.7 and 1 which is clearly larger than the share of capital income as a fraction of total national income.

Further, Levine and Renelt (1992) and Sala-i-Martin (1997) have shown that the investment share is a robust variable in explaining economic growth. This positive and statistically significant effect of investment on the growth rate of countries suggests that investment not only affects the stock of physical capital but also raises a sort of intangible capital stock, like knowledge for example, such that the social return to investment is larger than the private return. Therefore, building a theoretical model which contains positive externalities of investment into physical capital seems to be a reasonable approach.

However, assuming that physical capital and knowledge capital evolve at the same pace such that they can be taken together in one state variable implies that, in the long run, all economies converge to the same balanced growth rate (cf. Xie, 1994). That is, asymptotically

¹See also the paper by Hamilton and Monteagudo (1998) who find that capital is associated with positive external effects by estimating the Solow growth model.

the economies all grow at the same rate. Looking at the real world, however, this does not seem to be realistic. Instead, it seems that there are so-called convergence clubs, meaning that some countries converge to a situation with a relatively high growth rate, whereas, other countries are characterized by low growth rates. This observation is considered as a stylized fact in economic growth theory (see Durlauf and Quah, 1999).

In this paper, we present a theoretical growth model with externalities of investment which is more general than the growth model presented by Romer (1986) and which may give rise global indeterminacy. That is, in our version two balanced growth paths can be observed depending on the parameter values. Moreover, we will apply time series techniques in order to test the empirical relevance of our endogenous growth model.

The rest of the paper is organized as follows. In section 2, we present our growth model with positive externalities of investment where we treat physical and knowledge capital as two separate state variables. In section 3, we use calibration technique combined with empirical estimations in order to test that model for France, Germany and Japan as well as Great Britain and the US. The last section finally concludes our paper.

2 The Model

We consider a decentralized economy which consists of a representative household and a representative firm which behaves competitively. Further, there is a positive externality associated with investment which consists in building up knowledge capital in our economy.

The Productive Sector

The productive sector consists of many firms which can be represented by one firm which produces a homogeneous good $Y_a(t)$ with a Cobb-Douglas production function:

$$Y_a(t) = (A(t)L(t))^\alpha K_a(t)^{1-\alpha},$$

with $Y_a(t) = L(t)Y(t)$ aggregate output, $K_a(t) = L(t)K(t)$ the aggregate stock of physical capital and $A(t)$ individual stock of knowledge. $\alpha \in (0, 1)$ denotes the labour share in the

production function. In per capita terms the production function can be written as

$$Y(t) = A(t)^\alpha K(t)^{1-\alpha}.$$

It should be noted that our specification of the production function implies that knowledge is a non-excludable but rival public good just as in the Lucas (1988) model. This holds because none of the firms can be excluded from the use of the stock of knowledge but its use is subject to congestion. The latter is modelled by assuming that the per capita stock of knowledge, $A(t) = A_a(t)/L(t)$, affects the productivity of labour input. This assumption also eliminates a scale effect which would be present if $A_a(t)$ replaced $A(t)$ in $Y_a(t)$.

The firm behaves competitively yielding²

$$r = (1 - \alpha)K^{-\alpha}A^\alpha \tag{1}$$

$$w = \alpha A^\alpha K^{1-\alpha}. \tag{2}$$

The External Effect

The stock of knowledge capital A is assumed to be a by-product of cumulated past gross investment (cf. Arrow, 1962, Levhari, 1966, or Sheshinski, 1966). In contrast to the usual assumption, however, we assume that investment at certain dates shows different weights concerning its contribution to the current stock of knowledge capital (as to the use of weighting functions in growth models see e.g. Ryder and Heal (1973) or Wan (1970)). Formally, this stock can be expressed as

$$A(t) = \varphi \int_{-\infty}^t e^{\eta(s-t)} I(s) ds,$$

with I gross investment per capita, $\eta \geq 0$ depreciation rate of per capita knowledge and $\varphi > 0$ gives the contribution of one unit of investment to the formation of knowledge capital, which is assumed to be given exogenously. In reality, the parameter φ is expected to be an endogenous function which may depend on the time spent for education for example. For simplicity, however, we take this parameter as an exogenous function. The effects of the parameters φ and η

²In the following we omit the time argument.

becomes clearer by differentiating A with respect to time leading to

$$\dot{A} = \varphi I - \eta A. \quad (3)$$

Thus, this formulation implies that the stock of knowledge may be subject to depreciation which can be justified by adopting a more Schumpeterian perspective in which new investment raises the stock of knowledge but, at the same time, makes a fraction obsolete. That can be justified by supposing that any new capital good requires new knowledge in order to be operated efficiently. Consequently, a certain fraction of the current stock becomes irrelevant for the production process although physically still present.

With that assumption we intend to formalize in a way what Abramovitz (1986, 1994) has summarized under the rubric social capability, which is a necessary condition to achieve economic growth and prosperity. According to that concept countries must be able to adopt existing technologies and to produce with them in order to achieve economic growth. That is more important than to develop new products or methods of production. That approach seems of particular relevance for less developed countries which intend to catch up with highly developed economies. A prerequisite for the ability to adopt modern technologies and to achieve economic growth is that economies dispose of a sufficiently high social capability. With social capability Abramovitz refers to technical competence, which determines the ability to adopt modern methods of production, but also to other factors such as the stability of governments and of the monetary sector and the attitude towards wealth and capitalism for example.

However, it must be underlined that our model is only a very modest attempt to put Abramovitz's ideas in a formal framework. That holds because φ is given exogenously in our framework, whereas in reality it is expected to be also determined by endogenous variables like education for example, as already mentioned above, or other factors such as institutional or cultural ones as mentioned above. But our approach can be justified as a first approximation to integrate this effect in a formal model.

The Household Sector

The household maximizes the discounted stream of utility resulting from consumption C over an infinite time horizon:

$$\max_C \int_0^{\infty} e^{-(\rho-n)t} U(C) dt. \quad (4)$$

$\rho > 0$ gives the rate of time preference and n is the growth rate of labour supply, which is normalized to one at $t = 0$, i.e. $L(0) = 1$. $U(\cdot)$ stands for the utility function, with $U'(\cdot) > 0$ and $U''(\cdot) < 0$. For a CRRA utility function the intertemporal elasticity of substitution of consumption between two points in time, $1/\sigma \equiv -U'/U''C$, is constant. The household's budget constraint in per capita terms is written as

$$C + \dot{K} + (\delta + n)K = w + rK, \quad (5)$$

with K physical capital and δ the depreciation rate. To derive optimality conditions for problem (4) subject to (5) we first note that a solution to the household's optimization problem exists if the growth rate of K and A , g , is bounded by $g < \rho - n$ (cf. Greiner and Semmler, 1996). The necessary conditions for a maximum of (4) subject to (5) are derived by formulating the current value Hamiltonian

$$H(\cdot) = U(C) + \gamma(-C - (\delta + n)K + w + rK),$$

with γ the current value co-state variable. The maximum principle gives $U'(C) = \gamma$.

The evolution of γ is described by

$$\dot{\gamma} = (\rho + \delta)\gamma - r\gamma.$$

Furthermore, we need the transversality condition $\lim_{t \rightarrow \infty} e^{-(\rho-n)t} \gamma(t) K(t) = 0$ to hold which is automatically fulfilled for $g < \rho - n$.³ Combining the condition $U'(C) = \gamma$ with the equation giving $\dot{\gamma}$ yields the growth rate of private consumption as

$$\frac{\dot{C}}{C} = -\frac{\rho + \delta}{\sigma} + \frac{r}{\sigma}. \quad (6)$$

³The assumption $g < \rho - n$ is also sufficient for (4) to take on a finite value.

Equilibrium Conditions

The use of equilibrium conditions can be justified by supposing a theoretical dichotomy between growth and business cycles and by arguing that growth theory is primarily concerned with the long-run behaviour of economies. Since components, which are fixed in the short run, become flexible in the long-run adjustment mechanisms may take effect such that the economy attains an equilibrium.

The household's budget constraint, (5), together with (1) and (2), which give the return to capital and the wage rate respectively describe the evolution of the physical capital stock. The growth rate of consumption is given by (6) and the growth rate of knowledge capital, finally, is described by (3) with $I = Y - C$. This leads to the following differential equation system, which completely describes our competitive economy.

$$\frac{\dot{C}}{C} = -\frac{\rho + \delta}{\sigma} + \frac{(1 - \alpha)K^{-\alpha}A^\alpha}{\sigma} \quad (7)$$

$$\frac{\dot{K}}{K} = -(\delta + n) - \frac{C}{K} + \left(\frac{A}{K}\right)^\alpha \quad (8)$$

$$\frac{\dot{A}}{A} = -\eta + \varphi\left(\frac{I}{A}\right). \quad (9)$$

The initial conditions are $K(0) = K_0 > 0$, $A(0) = A_0 > 0$ and $C(0) > 0$ can be chosen freely. Further, the transversality condition $\lim_{t \rightarrow \infty} e^{-(\rho-n)t} \gamma(t) K(t) = 0$ must be fulfilled, with $\gamma = \gamma(C)$ determined by the maximum principle.

Looking at this system, we realize that for a constant level of knowledge capital $A(t)$ the growth rates \dot{C}/C and \dot{K}/K become negative for $K \rightarrow \infty$ implying that in this case sustained per capita growth is not feasible and our model is equal to the conventional neoclassical Ramsey type growth model and does not reveal sustained per capita growth. Only if the external effect of investment concerning the formation of knowledge capital is strong enough, so that the marginal product of physical capital does not necessarily converge to $\rho + \delta$ in the long-run, endogenous growth is feasible. This, for its part, is only possible if there is a sufficiently high social capability so that any unit of investment raises the knowledge capital to a great degree.

Let us in the following assume that the parameters in our economy are such that sustained per capita growth can be observed. This is the more interesting and also more relevant case

if one looks at the long-run behaviour of market economies. Analyzing the dynamics of this model, it can be shown that multiple balanced growth paths (BGP) may be the outcome, i.e. in the long run two economies may reveal different growth rates and do not necessarily converge to the same growth path. We do not go into the details of this analysis but only refer to Greiner and Semmler (1996) for an explicit study of the local dynamics of that model.

As to the empirical verification of that model we first state that on a balanced growth path all variables grow at the same constant growth rate, implying that the ratio $c = C/K$ is constant over time. Differentiating c with respect to time gives $\dot{c}/c = \dot{C}/C - \dot{K}/K$ or explicitly:

$$\frac{\dot{c}}{c} = (\delta + n) - \frac{\delta + \rho}{\sigma} + \frac{1 - \alpha}{\sigma} \left(\frac{A}{K} \right)^\alpha - \frac{I}{K}, \quad (10)$$

with $I = Y - C$ and with the growth rate of A given by (9).

Consequently, (9) and (10) completely describe the dynamics of our model in the neighborhood of a BGP. For $\dot{c}/c = 0 \leftrightarrow \dot{C}/C = \dot{K}/K$ and $\dot{K}/K = \dot{A}/A$ the economy has attained the BGP.

3 Time Series Evidence

In this section we will analyze the empirical relevance of our theoretical model by looking at the time series of different countries. To find whether our model is compatible with real world economies we resort to a mixture of calibration and econometrics. The reason for that procedure is twofold: First, it is not possible to estimate (9) because the stock of knowledge A , which is formed as a by-product of investment, is not observable. Therefore, we first have to construct the series of $A(t)$. We do that by using different plausible values for the parameter.⁴ φ . Second, in estimating equation (10) the constant contains δ, n, ρ and σ , i.e. values for those parameters cannot be obtained from our empirical estimation. Therefore, we will take values which are considered as realistic in calibration studies and test whether those are compatible with the coefficients obtained by our empirical estimation.

⁴We tried to evaluate the parameters η and φ by estimating a regression with the change of labour productivity as a proxy for \dot{A} which is explained by investment. The results gave about the values we then used but were not statistically significant.

To estimate equation (10) we replace the differential operator by first differences and consider one period to comprise one year. The equation to be estimated by non-linear least squares then is

$$D(\ln(C_t/K_t)) = c_1 + \frac{c_2}{\sigma} \left(\frac{A_{at}}{K_t} \right)^{1-c_2} - c_3 \frac{I_t}{K_{t-1}} + u_t, \quad (11)$$

with $c_3 = 1$ and u_t a stochastic residual which is NID with zero mean and finite variance.

We start with Germany.

Germany⁵

In estimating (11) for Germany with data from 1950-1994 we use different values for φ in constructing the variable $A(t)$. η is left unchanged throughout and set to $\eta = 0.06$.⁶ σ is set to one which is the value obtained by most of the empirical studies (see Blanchard and Fischer, 1989, p. 44).

As to the magnitude of the external effect of investment we resort to the study by DeLong and Summers (1991) who have found that the external effect of investment is up to 30 percent. Other studies where the whole aggregate production is associated with externalities, and not only investment, suggest that the externality is in the range of 40-60 percent (see Benhabib and Farmer, 1995).

To obtain an initial value for $A(1950)$ we follow the approach by Park (1995, p. 590) and set $A(0) = \varphi I(0)(1 + g)/(g + \eta)$,⁷ with g the average growth rate of investment for the period 1950-1994. In addition, we raised $A(1950)$ by values between 20 and 75 percent in order to obtain statistically significant results. In general, it turns out that the increase in $A(1950)$ had to be the higher the lower the external effect of investment φ is set. We suppose that the necessity for this increase is due to the fact that the capital stock in most of the countries we considered was almost completely destroyed after World War II so that the ratio of knowledge to physical capital, A/K , was very high at the beginning of the period we consider. However, building $A(1950)$ according to the procedure described above does not take account of that fact, which is only captured if $A(1950)$ is raised by a certain percentage.

⁵Data for Germany are for West Germany only.

⁶For lower or higher values of η the results did not change basically.

⁷Note that we use in our estimation aggregate total variables. Consequently, we employ $AL = A_a$.

The data⁸ for consumption and investment are from Statistisches Bundesamt (1974) and Sachverstaendigenrat (1995) and for private capital from Statistisches Bundesamt (1991, 1995).

Table 1 gives the outcome for different values of φ .

Table 1: Estimation of equation (11) for Germany

$\varphi = 0.3$				$\varphi = 0.4$			
Par.	Est.	Std. Err.	t-Stat.	Par.	Est.	Std. Err.	t-Stat.
c_1	-0.106	0.044	-2.417	c_1	-0.08	0.034	-2.516
c_2	0.405	0.065	6.214	c_2	0.327	0.053	6.175
R^2	0.38			R^2	0.38		
$\varphi = 0.5$				$\varphi = 0.6$			
Par.	Est.	Std. Err.	t-Stat.	Par.	Est.	Std. Err.	t-Stat.
c_1	-0.073	0.029	-2.549	c_1	-0.08	0.029	-2.729
c_2	0.272	0.044	6.124	c_2	0.255	0.043	5.997
R^2	0.38			R^2	0.38		
$\varphi = 0.7$				$\varphi = 0.8$			
Par.	Est.	Std. Err.	t-Stat.	Par.	Est.	Std. Err.	t-Stat.
c_1	-0.268	0.149	-1.8	c_1	-0.294	0.187	-1.571
c_2	0.474	0.161	2.954	c_2	0.478	0.197	2.423
R^2	0.4			R^2	0.4		

The table⁹ shows that the capital share $c_2 = 1 - \alpha$ varies between 26 and 48 percent, which seem to be plausible values, depending on the value for φ . The coefficient $c_1 = n - \rho$, which gives the sum of the population growth rate and the discount rate of the household sector, lies between -7.3 and -29.4 percent.

⁸The computations were done with EViews, Version 2.0. The data are available on request.

⁹We should like to point out that the Durbin-Watson and the Breusch-Godfrey test did not suggest that the residuals are serially correlated. That holds for all our estimations unless stated otherwise. Further, the hypotheses of non-significance of the complete regression could be rejected at the 1% percent significance level for all of our regressions unless stated otherwise.

As to the discount rate values around 6.5 percent are often used in calibration exercises and can be seen as realistic (see Benhabib and Farmer, 1994, or Benhabib and Perli, 1994). Concerning the labour supply it is true that the later, in number of persons, increased between the 1950's and the 1990's, but total labour input has decreased because of a decline in the number of hours worked by an employee. For example, the total amount of effective labour input in Germany between 1960 and 1994 has decreased from 56 million hours to 45 million which corresponds to an annual decline of about 0.6 percent. Therefore, n can be regarded as zero or even slightly negative.

Looking at table 1 and keeping that in mind we see that supposing $\varphi = 0.3$, $\varphi = 0.4$, $\varphi = 0.5$ and $\varphi = 0.6$ yield plausible outcomes whereas higher and lower values for that coefficient do not seem to be compatible with empirical observations because they yield too low values for $n - \rho$. However, those values for the external effect of investment seem very high if one takes the results obtained by DeLong and Summers. But it must be underlined that our model considers positive externalities of investment as the only source for economic growth whereas others, such as intentional investment in the formation of human capital or R&D investment, are neglected. Consequently, those aspects can be expected to be included in the positive externalities of investment giving a higher value for φ .

In Figure 1 we show the actual time series compared to the fitted one for $\varphi = 0.5$.

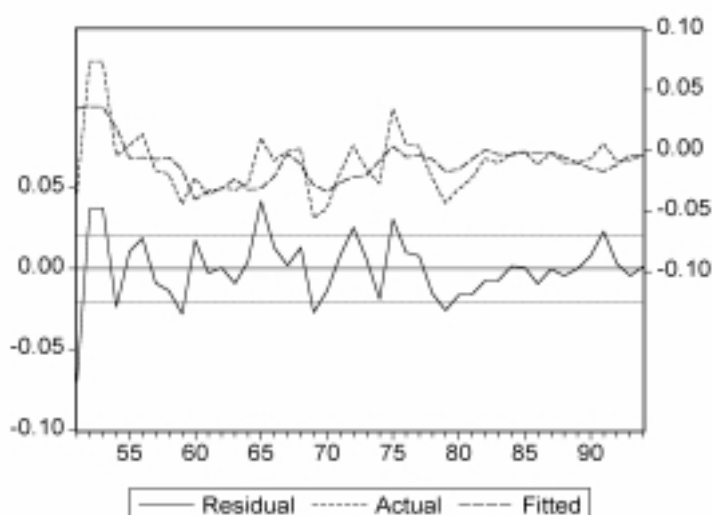


Figure 1:

Setting $\varphi = 0.2$ (0.8) the capital share is $1 - \alpha = 0.42$ (0.49) and $n - \rho = -0.08$ (insignificant). For values lower than 0.2 and higher 0.8 no statistically significant results are obtained or the capital share is not plausible, i.e. 0.6 or larger.

We also estimated equation (11) assuming $\sigma = 1.5$. The estimated coefficients then are less plausible. The estimations yield $1 - \alpha = 0.49$ (0.47) for $\varphi = 0.5$ (0.6). The coefficient $c_1 = (\delta + n) - (\rho + \delta)/1.5$, however, is not statistically significant any longer. For all other values of φ our estimations do not produce statistically significant results.

Thus, the estimates for Germany indicate that our generalized growth model with positive externalities of investment yield plausible estimates for a different quantitative external effects of investment.¹⁰

Another aspect, which might give rise to criticism, concerns our assumption of constant returns to scale in the aggregate production function, $Y = A^\alpha K^{1-\alpha}$. A more general formulation would be $Y = A^\alpha K^\beta$, with $\alpha, \beta \in (0, 1)$. If $\alpha + \beta < 1$ we have decreasing returns to scale and sustained per-capita growth is not feasible. For $\alpha + \beta > 1$ we have increasing returns and we can again observe sustained per-capita growth. However, it is difficult to completely characterize the dynamics of our model for the latter case. Further, increasing returns to scale would imply that the growth rate of market economies are increasing over time, which does not seem to hold in reality. Because of those two reasons, we assume $\alpha + \beta = 1$. Nevertheless, from an empirical point of view that assumption may seem very restrictive. Therefore, we next estimate equation (11) without the assumption of constant returns.

Equation (11) then becomes

$$D(\ln(C_t/K_t)) = c_1 + \frac{c_2}{\sigma} \left(\frac{A_{at}^{c_4}}{K_t^{1-c_2}} \right) - c_3 \frac{I_t}{K_{t-1}} + u_t, \quad (12)$$

with $c_3 = 1$. The results for $\sigma = 1$ and $\varphi = 0.4$ and $\varphi = 0.5$ are shown in table 2.¹¹

¹⁰We estimated (11) also without the restriction $c_3 = 1$. c_3 was significant and took on values around 0.9. However, the hypothesis that $c_3 = 1$ could not be rejected.

¹¹We also made the estimation for $\varphi = 0.3$ and $\varphi = 0.6$. For those values of φ , the results are the same from a qualitative point of view, in comparison to the estimation with $\alpha + \beta = 1$.

Table 2: Estimation of equation (12) for Germany

$\varphi = 0.4$				$\varphi = 0.5$			
Par.	Est.	Std. Err.	t-Stat.	Par.	Est.	Std. Err.	t-Stat.
c_1	-0.086	0.037	-2.346	c_1	-0.074	0.034	-2.186
c_2	0.305	0.269	1.135	c_2	0.255	0.271	0.94
c_4	0.709	0.439	1.613	c_4	0.757	0.458	1.652
R^2	0.38			R^2	0.38		

Table 2 demonstrates that the basic results do not change very much. As to $\alpha = c_4$, $\beta = c_2$, we see that $\alpha + \beta$ is slightly larger than one but only marginally. However, both coefficients are not statistically significant. We also made the Wald coefficient test testing whether the hypothesis $\alpha + \beta = 1$ can be rejected. It turns out that the hypothesis cannot be rejected. Thus, we conclude that the assumption of constant returns in our theoretical model is compatible with empirical data for Germany.

Next, we will analyze that model for France.

France

In estimating equation (11) for France we proceed as for Germany. η is again set to 0.06 and φ takes on values between 0.3 and 0.6. The inverse of the intertemporal elasticity of substitution, however, is chosen higher and we set $\sigma = 1.5$. The data for France are taken from the Summers and Heston (1991) database and comprise the period 1950-1992.¹²

¹²Investment in that sample comprises both private and public investment. The capital stock was computed with the perpetual inventory method assuming a depreciation rate of 5 percent.

Table 3: Estimation of equation (11) for France

$\varphi = 0.3$				$\varphi = 0.4$			
Par.	Est.	Std. Err.	t-Stat.	Par.	Est.	Std. Err.	t-Stat.
c_1	-0.095	0.06	-1.595	c_1	-0.08	0.045	-1.759
c_2	0.398	0.125	3.196	c_2	0.317	0.097	3.259
c_3	0.423	0.115	3.674	c_3	0.422	0.115	3.666
R^2	0.42			R^2	0.42		
$\varphi = 0.5$				$\varphi = 0.6$			
Par.	Est.	Std. Err.	t-Stat.	Par.	Est.	Std. Err.	t-Stat.
c_1	-0.071	0.038	-1.851	c_1	-0.065	0.034	-1.907
c_2	0.261	0.079	3.283	c_2	0.22	0.067	3.305
c_3	0.421	0.115	3.66	c_3	0.42	0.115	3.655
R^2	0.42			R^2	0.42		

Estimating (11) with the restriction $c_3 = 1$ generates almost the same coefficients, the residuals, however, then are serially correlated and R^2 takes on very low values (around 0.05). But the capital coefficient is nevertheless statistically significant which is seen by calculating the Newey-West autocorrelation consistent standard errors. We decided to take the coefficient c_3 into the regression because then the autocorrelation vanishes and the fit is considerably better.

Table 3 shows that the capital share $c_2 = 1 - \alpha$ lies between 22 and 40 percent which are plausible values. The coefficient $c_1 = (\delta + n) - (\rho + \delta)/1.5$ is relatively high, in absolute values. For example, if $c_1 = -0.07$ the subjective discount rate is about 13 percent for $n = 0$, which seems to be high. So, $\varphi = 0.5$ or $\varphi = 0.6$ yield the most plausible results, whereas for $\varphi = 0.3$ (0.4) the discount rate is too high.

Setting φ equal to 0.2 and to 0.7 gives a capital share of $1 - \alpha = 0.53$ and 0.19 respectively, which is unrealistically high and low respectively. Setting φ still lower or higher yields still larger and lower values for the capital coefficient which are not plausible any longer.

We also made the estimations with $\sigma = 1$. Then, the capital share turns out to be lower by 10 to 20 percent. For example, setting $\varphi = 0.2$ (0.3, 0.4) yields $1 - \alpha = 0.34$ (0.25, 0.2).

Supposing again $n = 0$, the discount rate implied by those values is $\rho = 8.3$ (6.9, 6.3) percent which are plausible values.

For France we also estimated (12) with $\varphi = 0.5$ and $\varphi = 0.6$. For $\sigma = 1.5$ the results are given in table 4.

Table 4: Estimation of equation (12) for France

$\varphi = 0.5$				$\varphi = 0.6$			
Par.	Est.	Std. Err.	t-Stat.	Par.	Est.	Std. Err.	t-Stat.
c_1	-0.084	0.127	-0.661	c_1	-0.078	0.117	-0.665
c_2	0.26	0.084	3.092	c_2	0.217	0.071	3.047
c_4	0.758	0.159	4.777	c_4	0.802	0.179	4.488
c_3	0.403	0.201	2.008	c_3	0.402	0.2	2.012
R^2	0.42			R^2	0.42		

Both α and β are statistically significant and the sum is again slightly larger one. The hypothesis $\alpha + \beta = 1$, however, cannot be rejected using the Wald coefficient test. Therefore, a theoretical model with constant returns to scale also reflects the growth rate of the ratio C/K for France pretty well.

Next, we will test our model for the Japanese economy.

Japan

For Japan, our results are similar to the ones obtained for Germany and France.¹³ Again, we set $\eta = 0.06$ and try different values for φ . The inverse of the intertemporal elasticity of substitution is set as for Germany, i.e. $\sigma = 1$.

¹³The data for Japan are again from the Summers and Heston database. The variables were computed as for France.

Table 5: Estimation of equation (11) for Japan

$\varphi = 0.3$				$\varphi = 0.4$			
Par.	Est.	Std. Err.	t-Stat.	Par.	Est.	Std. Err.	t-Stat.
c_1	-0.135	0.059	-2.281	c_1	-0.114	0.045	-2.543
c_2	0.37	0.091	4.071	c_2	0.292	0.071	4.118
c_3	0.474	0.078	6.066	c_3	0.473	0.078	6.071
R^2	0.55			R^2	0.55		
$\varphi = 0.5$				$\varphi = 0.6$			
Par.	Est.	Std. Err.	t-Stat.	Par.	Est.	Std. Err.	t-Stat.
c_1	-0.23	0.122	-1.886	c_1	-0.411	50.221	-0.008
c_2	0.412	0.148	2.79	c_2	0.566	48.627	0.012
c_3	0.485	0.079	6.134	c_3	0.413	0.132	3.135
R^2	0.55			R^2	0.38		

Table 5 shows that the capital share falls in the range 0.29 to 0.57.¹⁴ While the capital coefficient can be considered as plausible for all values of φ in table 5, except for $\varphi = 0.6$, the coefficient c_1 is reasonable only for $\varphi = 0.4$. In that case the subjective discount rate is about 11 percent, for $n = 0$, which still seems to be relatively high. Choosing $\varphi = 0.2$ the capital coefficient is 0.49 and c_1 is still smaller namely -0.18 . For values of φ larger 0.7 no statistically significant results are obtained.

Performing the above estimations for $\sigma = 1.5$ does not yield interpretable results. In that case no statistically significant outcomes can be obtained.

¹⁴Note that we must again introduce the coefficient c_3 in order to get uncorrelated residuals.

Figure 2 shows the actual and fitted time series for $\varphi = 0.4$.

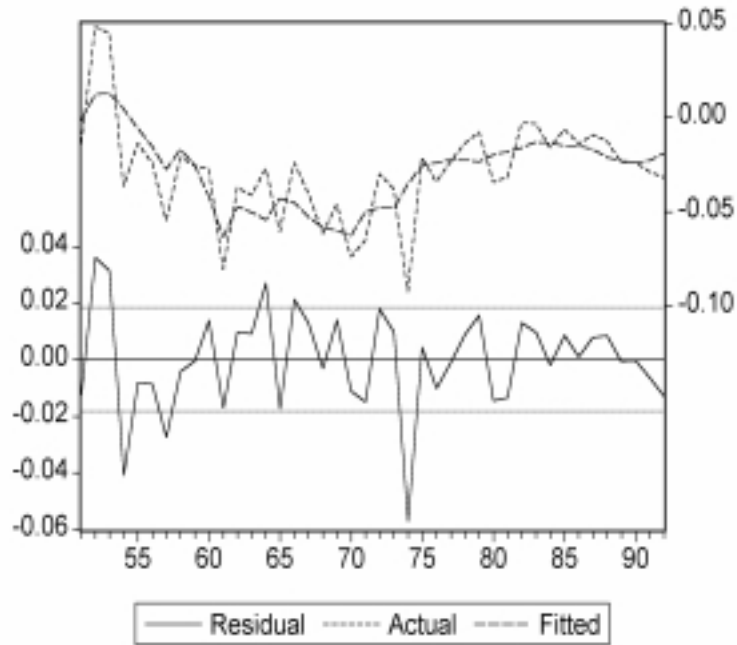


Figure 2:

Estimating (12) for Japan with $\sigma = 1$ and $\varphi = 0.3$ and $\varphi = 0.4$ gives the results shown in table 6.

Table 6: Estimation of equation (12) for Japan

$\varphi = 0.3$				$\varphi = 0.4$			
Par.	Est.	Std. Err.	t-Stat.	Par.	Est.	Std. Err.	t-Stat.
c_1	-0.125	0.138	-0.905	c_1	-0.106	0.112	-0.948
c_2	0.364	0.123	2.947	c_2	0.288	0.085	3.397
c_4	0.632	0.093	6.832	c_4	0.707	0.07	10.027
c_3	0.481	0.12	4.01	c_3	0.481	0.119	4.032
R^2	0.55			R^2	0.55		

As for France both α and β are statistically significant for Japan, but the sum is now slightly smaller one. But again, the hypothesis $\alpha + \beta = 1$ cannot be rejected using the Wald coefficient test.

Great Britain and the US

For Great Britain the model with positive externalities does not perform well. In the case of Great Britain we estimated equation (11) for $\eta = 0.06$ and with $\varphi = 0.1$, $\varphi = 0.5$ and $\varphi = 1$. As to the initial conditions concerning knowledge, formed as a by-product of investment, we proceeded as above. In addition, we multiplied the initial value of $A(t)$, $A(1950)$, by 0.1, 0.5, 1, 1.5 and 1.75 and reestimated (11) for each value of $A(1950)$. In all estimations the capital coefficients are either implausible or plausible but not statistically significant, which is seen by calculating the Newey-West autocorrelation consistent (AC) standard errors. The serial correlation was present independent of the choice of σ . We also made the computations using the average growth rate of real investment from 1950-1960 in constructing $A(1950)$, instead of the average from 1950-1992. The outcome, however, was the same from the qualitative point of view.

Table 7 gives the result of the estimation for Great Britain with $\varphi = 0.5$ and $\sigma = 1$. The coefficients are reasonable and the value of R^2 is not too bad. However, c_1 and c_2 are not statistically significant and the residuals are autocorrelated, which may indicate that the model is misspecified for Great Britain.

Table 7: Estimation of equation (11) for Great Britain

$\varphi = 0.5$			
Par.	Est.	Std. Err. (Newey-West AC)	t-Stat. (Newey-West AC)
c_1	-0.053	0.110	-0.483
c_2	0.185	0.181	1.024
c_3	0.576	0.263	2.193
R^2	0.18		

For the US we also estimated equation (11) for $\eta = 0.06$ and with $\varphi = 0.1$, $\varphi = 0.5$ and $\varphi = 1$. In those computations serial correlation of the residuals could be observed, too. We tried to remove the autocorrelation by resorting to different starting values for $A(1950)$, as we did for Great Britain, but did not succeed. Further, the estimated coefficients are implausible so that the model completely fails in explaining the growth performance of that country.

4 Conclusion

First, we should like to point out that the goal of this paper is not to find the magnitude of the external effect of investment. Instead, we only try to test whether our endogenous growth model with positive externalities of investment is compatible with time series data.

As to Germany, France and Japan the estimation results suggest that our endogenous growth model yields plausible outcomes so that it can be used to describe the growth process of those countries for the last forty years. The assumption of constant returns to scale in the aggregate production function also seems to be compatible with empirical data for those countries. That holds because in all of our estimations the Wald coefficient test did not allow to reject hypothesis of constant returns.

However, we must be aware that our model is a highly stylized one which contains only one source of economic growth, namely positive externalities of investment. All other sources, like R&D expenditures or time spent for education, are not included. So, it cannot be expected to yield high values for R^2 . Therefore, estimating other endogenous growth models which explicitly consider education or R&D spending as major sources of economic growth are worth estimating. Probably, such models fit actual time series better or are compatible with time series where our model failed, as in the case of Great Britain or the US.

Further, we must also be aware that the actual time series are influenced by business cycles movements, which are not captured by our growth model. That is clearly seen if one compares the actual time series with the fitted in Figures 1 and 2. While the fitted time series reflects the general evolution of the growth rate of C/K pretty well, it does not follow the peaks and troughs of the actual series.

Another aspect we should like to emphasize is that the growth process in different countries is different. So, we think that time series analyses are preferable to cross-section studies, in which different countries are summarized in one sample. For example, we saw that for France $\sigma = 1$ and $\sigma = 1.5$ yield reasonable outcomes whereas for Japan only $\sigma = 1$ gives plausible results. In addition, the poor performance of our model for Great Britain and the rejection using US data confirms that view. One possible explanation why that model does not succeed in explaining the evolution in those two countries, could be that the capital stock in Great Britain and the US was not completely destroyed after World War II, in contrast to Germany, France and Japan. Since the model with positive externalities of investment seems of particular relevance for countries with a lower initial stock of physical capital, the economic evolution in the latter three countries is reflected by our model in contrast to the evolution in Great Britain and the US.

Finally, it would be interesting to extend this model to allow for international trade. Thus, it would become more realistic since the countries we considered are highly involved in foreign trade. Starting point could be the approach by Lucas (1993) where abilities arise only in the sector where production takes place. Assuming that different countries produce different goods this assumption leads to differences in learning and, thus, in growth rates.

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