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Financial and Labor Market Interactions

Specific investment and labor market activity

by

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Abstract

A double-sided matching process is considered where firms have to search for both financial investors and workers. The value of the match is endogenous as all three actors can proceed on investments specific to the match. Financial investors will screen possible debtors to the quality of their investment project, firms will select their technology and workers will decide upon their effort level. In equilibrium, when wages and debt levels are negotiated, this may lead to multiple equilibria whose characteristics are influenced by various policy variables. Moreover, for highly sclerotic markets, these equilibria may disappear altogether.

Keywords: Market liquidity, matching on financial and labor markets, market interaction, institutional complementarity

JEL-Classification: G24, J64, O14

1 Introduction

European economies have fared relatively well, on average, over the last two decades, despite much talking about "Eurosclerosis" and structural rigidities. While this assessment has to be qualified for some economies, the euro area as a whole had a similar per capita performance as the USA, even over the more wobbling 1990, where common wisdom usually sees the latter far ahead of the former. Nevertheless, GDP *per capita* increased at 1.7% between 1991 and 2001 in the euro area and has been only slightly higher in the USA with 1.9% (Vijselaar and Albers, 2002). More importantly, labor productivity per hour worked experienced a stronger increase in Europe (2.0%) than in the USA (1.6%). Similarly, innovative activity of European enterprises have been everything else than disappointing, even though large differences persist across Europe (Bassanini and Ernst, 2002). This barely squares with the common perception of these two regions, giving a bright picture of flexible markets in the USA while painting darker colors for Europe's rigid and aging economies.

The puzzle becomes even more apparent when concentrating solely on the analysis of imperfections of either labor or financial markets alone. Usually, studies that aim at disentangling the contribution of particular forms of characteristics of these markets across different economies only produce very fragile results, leading some researchers to conclude that more fundamental factors should be retained such as the overall financial development and the legal framework to protect property rights. Nevertheless, while this seems to be an interesting road to take, persistent differences in some performance variables, such as sectoral specialization and structural change or firm demographics and firm size distribution suggest that substantial variation at a more disaggregate level continues to characterize economies (Ernst, 2004), with potentially important macroeconomic effects such as the reaction to supply, demand and policy shocks. These differences, however, cannot be explained by referring to one-dimensional global policy indicators anymore.

Recent contributions in this field, therefore, turned to a more encompassing analysis of the different transmission mechanisms that various policy-induced or institutional market imperfections may have on economic performance. In particular, explicitly considering market interactions where imperfections on two different markets could simultaneously affect macroeconomic performance turned out to be a very fruitful approach (Acemoglu, 2000; Amable and Gatti, 2002; Amable, Ernst, Palombarini, 2005; Wasmer and Weil, 2002). In these models, informational asymmetries, coordination problems and contracting problems are considered to generate economy-wide spillovers beyond the frictions on the market on which they are originating. Even though their own-market effect may still be ambiguous - following results of the earlier research - the spillover onto other markets (the market interaction effect) as well as the combined effect with other characteristics of the macroeconomy (the complementary effect) have the potential to explain structural differences between economies (see Nicoletti *et al.*, 2001, for a recent study).

Against this background, the following paper tries to develop a more general framework through which market interaction and complementary effects can be studied and their impact on the macroeconomy be analyzed. The aim of the paper is twofold: On the one hand, we are going to demonstrate the potential importance of market interaction for the functioning of the macroeconomy, possibly affecting the characteristics and the number of arising equilibria. On the other hand, in establishing these different equilibria, we are able to show that it may not necessarily be possible to establish a Pareto-ranking between them but that they nevertheless show persistent differences on a more disaggregate level, potentially affecting their reaction to supply, demand and policy shocks.

In the following paper, market interactions arise as a consequence of contractual imperfections on one market that affect outcomes on others. Given that economic activity implies the exchange of goods and services on different markets if not at the same time then at least in a specific order, the individual decision making process will create interrelations between the contractual shortcomings on one market and the decision to engage in economic relations on others. In particular, when firms are financially constraint to seek for outside funding, the extent to which they have access to finance will affect their possibility to put vacancies on the labor market. Moreover, in general equilibrium, labor market developments will feed back into the financial markets, determining the expected returns of financial funds. However, going beyond the current stance of the literature we argue that the effect of market imperfections may be ambiguous due to a particular combination of search and contractual frictions that interact across different markets.

In particular, in the presence of match-specific assets that have to be built up to improve the firm's performance, quasi-rents generated through the search process allow to remunerate this specific investment. These specific assets may arise for various reasons and may interact with each other, determining the global value of the match. For instance, firms and workers may have to invest in match-related capital such as firm-specific skills, technological effort and innovation that are only valuable inside the relation. Financial investors, on the other hand, may proceed at market screening *ex-ante* in order to select good entrepreneurs or monitor the firm *ex-post* monitoring in order to control for good managerial effort. All three types of specific investment may be important to generate high returns to the match and may enter in a complementary way - directly or indirectly - into the firm's production function. For instance, high levels of innovative effort raises the returns to finance and hence increase incentives for financial investors to enter the market. On the other hand, a decrease of screening effort allows for more bad entrepreneurs to enter the market, increasing the risk of early destruction and consequently reducing innovative and workers' effort.

For optimal investment in specific assets to occur, the necessary incentives have to be provided through sustained returns to investment. Incentives to invest in specific assets are, however, usually negatively correlated with the outside option of both the investor and the bargaining partner. Consequently, high market liquidity - i.e. low market frictions - may negatively influence the specific investment provided by either firms, workers or financial investors, as the specific match-value decreases. Given the interaction that exists between markets, the reduced incentives for one investment type will spill over to the other market, decreasing overall investment into the firm's assets, ultimately lowering its productivity. It seems therefore, that there may exist a trade-off between efficiency gains that can be achieved in very liquid markets - and that usually lead search models to show increasing returns to market liquidity - and specific investment that would allow for a higher firm productivity. Consequently, while more flexible, liquid markets allow for a quick reallocation of resources through increased matching, more rigid markets may provide the necessary incentives for specific investments that are related to the success of existing firms.

The paper relies on an extension of a matching model developed by Wasmer and Weil (2000) as it provides a parsimonious way to analyze labor and financial market imperfections, taking into account informational and search frictions. Conversely to their paper - where search friction arise only exogenously through fixed search and set-up costs - ours is based on an endogenous match value, following the specific investments firms, workers and financial investors are able to make. This increases the complexity of the economic mechanisms, such that only numerical simulations provide an insight in their functioning. Nevertheless, making the match value endogenous yields non-trivial and non-monotonous relations between either credit market or labor market liquidity on the one hand and unemployment and GDP growth on the other.

The paper is organized as follows. The paper's model is introduced in the following section 2: agents and their decision variables are presented and match values depending on the stage of the firm discussed. In section 3, the outcome of the wage and debt bargaining are derived and the resulting levels of specific investments analyzed. Section 4 derives the reactions of the specific investments to labor and credit market frictions in the partial equilibrium framework. Section 5 discusses the equilibrium schedules describing the general equilibrium and derives conditions for multiplicity of equilibrium and presents some comparative statics. A final section concludes.

2 Agents and Match Values

2.1 Entrepreneurs, workers and financial investors

Following Wasmer and Weil (2000), three types of agents are considered: entrepreneurs, workers and financiers. Entrepreneurs have ideas but cannot work in production and possess no capital. Worker transform entrepreneurs' ideas into output but have neither entrepreneurial skills nor capital; financiers (or bankers) have access to the financial resources required to implement production but cannot be entrepreneurs nor workers. A productive firm is thus a relationship between an entrepreneur, a financier and a worker. Each agent may invest in a specific asset, improving his ability and lost when the relationship is dissolved.

Producing output in a firm requires a team of one entrepreneur and one worker. Labor market frictions are present under the form of a matching process à la Pissarides (2000), with a constant returns matching function[1] $h(\mathcal{U},\mathcal{V})$. Matches between workers and firms depend on job vacancies $\mathcal{V}$ and unemployed workers $\mathcal{U}$. From the point of view of the firms, labor market tightness is measured by $\theta \equiv \mathcal{V}/\mathcal{U}$. Labor market liquidity will be $1/\theta$. The instantaneous probability of finding a worker is thus $h(\mathcal{U},\mathcal{V})/\mathcal{V} = h(1/\theta,1) \equiv q(\theta)$, $q'(\theta) < 0$.

An entrepreneur incurs search costs before production starts. These costs must be financed by external funding. Wasmer and Weil (2000) consider credit market frictions modelled in the same way as labor market frictions: a matching function formalizes at the aggregate level the relationship between a banker and a firm. Den Haan, Ramey and Watson (1999) also modelled credit market imperfections with the help of a matching function between borrowers and lenders. In addition to search costs, financial investors can decide to monitor projects closely to increase the realised outcome; in order to do so, they have to invest in a monitoring technology, spending η.

If $\mathcal{B}$ is the number of bankers looking for borrowers and $\mathcal{F}$ the number of entrepreneurs looking for financing, the flow of loan contracts successfully signed is given by $m(\mathcal{B},\mathcal{F})$, with m a constant returns functions with positive and decreasing marginal returns to each input. From the point of view of firms, credit market tightness is measured by $\phi \equiv \mathcal{F}/\mathcal{B}$ and $1/\phi$ is an index of credit market liquidity, i.e. the ease with which entrepreneurs can find financing. The instantaneous probability than an entrepreneur will find a banker is $m(\mathcal{B},\mathcal{F})/\mathcal{F} = m(1/\phi,1) \equiv p(\phi)$. This probability is increasing in credit market liquidity, i.e. decreasing in credit market tightness. The probability that a banker will find a borrower is $m(\mathcal{B},\mathcal{F})/\mathcal{B} = m(1,\phi) = \phi \cdot p(\phi)$. This probability is increasing in credit market tightness, thus decreasing in credit market liquidity.

Workers, firms and banks have the possibility to choose the level of match-specific investment they want to expose. In particular, we assume that workers choose the effort level, $e \in \{0,1\}$, firms the technology, $T \in R^+$, and banks the level of firm monitoring, $\eta \in R^+$.

2.2 The life cycle of a firm

During the course of its life, the firm passes through four stages: fund raising, recruitment, production and destruction. In each stage a particular interaction between different market participants is taking place, while the market interaction process runs through the intertemporal linkages that exists between the different stages given the presence of the entrepreneur on different markets over the firm's life cycle.

1. *Fund raising:* In stage 0, entrepreneurs are looking (at a flow search cost c) for a financial investor willing to finance the posting of a job vacancy, while financiers are searching for clients at a flow search cost k; in addition, they have to pay η in order to committ to monitoring the firm during the production stage. The probability that

[1] h has positive and decreasing marginal returns on each input.

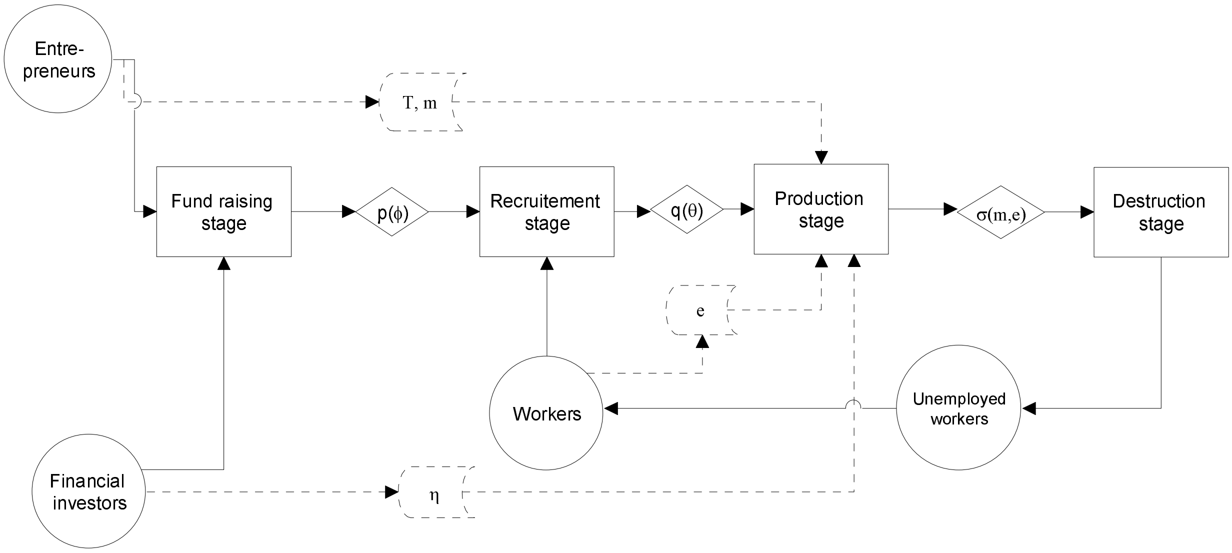

Figure 1: The timing of market interactions

a entrepreneur meets a financier (equivalently, the probability of transition to the recruitment stage) is $p(\phi)$.

2. *Recruitment:* In stage 1, entrepreneurs invest in productive technology and start looking for the worker that will enable them to take up production. The investment consist of two parts: first, entrepreneurs will invest T in dedicated capital which is not contractible; second, they have to invest m in organizational capital to make sure to obtain the optimal amount of effort from their worker. The probability that an entrepreneur will meet a worker, and that the recruitment stage will end, is $q(\theta)$.

3. *Production:* In stage 2, the firm starts production and is generating flow profits $\psi(e) \cdot y(T, \eta)$, depending on the installed technology as well as on the worker's effort[2] and the bank's monitoring committment. It uses these profits to pay its workers a wage w and by paying back to its financiers a flow amount ρ for the entire duration of the match. Both factor payments are negotiated before production starts and contingent on the production technology and the specific investments the three actors have undertaken.

4. *Destruction:* In the final stage 3, the match between firm and worker is destroyed. We assume that destruction depends partly on the organizational technology that allows to extract effort but also exogenous factors such as the degree of product market competition; transition from stage 2 to 3 occurs with probability $\sigma(m, e)$.

[2]The variable e actually denotes any kind of specific investment by the worker, i.e. effort, specific human capital investments, specific side payments necessary for taking up the job such as moving expenses, etc.

The above flow diagram describes the different stages of the matching and production process. Using the notation introduced here, we can then formalize the different stages of the firm's life cycle referring to the value of the firm's and the financial investor's assets as well as the job value.

2.2.1 The value of a firm

Let F_i, $i \in \{0, 1, 2, 3\}$ denote the different stages of the firm's life cycle and r the given risk-less interest rate. Then the Bellman equations for the firm values can be written as follows:

$$
\begin{aligned}
r \cdot F_0 &= -c + p(\phi) \cdot (F_1 - F_0) + \dot{F}_0 && (1)\\
r \cdot F_1 &= -T - m + q(\theta) \cdot (F_2 - F_1) + \dot{F}_1 && (2)\\
r \cdot F_2 &= \psi(e) \cdot y(T, \eta) - w - \rho + \sigma(m, e) \cdot (F_3 - F_2) + \dot{F}_2 && (3)\\
F_3 &= 0 && (4)
\end{aligned}
$$

where $\sigma(m, e = 1) = \overline{\sigma}$ and $y_T > 0$, $y_\eta > 0$, $y_{TT} < 0$, $y_{\eta\eta} < 0$. Moreover, for convenience, we want to assume that $\psi(e = 0) = 0$; nothing substantially is changed using this assumption. Finally, as the value of a firm is destroyed with the end of the match, we have $F_3 = 0$.

In the fund raising stage, firms spend c to match with an appropriate financial investor which will happen with probability $p(\phi)$. After installing the productive technology, T, and organizing the production process, m, the firm finds a suitable worker and will switch to the production stage with probability $q(\theta)$. There, it receives a stream of gross profits of $y(T, \eta)$- depending on the monitoring committment by financial investors - that have to be used to pay wages, w, and make debt reimbursements, ρ.

2.2.2 Financial intermediaries

Let B_i, $i \in \{0, 1, 2, 3\}$ denote the values of the financial investor over the four different stages of the its life cycle. Then the Bellman equations for the financial investor values can be written as follows:

$$
\begin{aligned}
r \cdot B_0 &= -k - \eta + \phi \cdot p(\phi) \cdot (B_1 - B_0) + \dot{B}_0 && (5)\\
r \cdot B_1 &= -\gamma + q(\theta) \cdot (B_2 - B_1) + \dot{B}_1 && (6)\\
r \cdot B_2 &= \rho + \sigma(m, e) \cdot (B_3 - B_2) + \dot{B}_2 && (7)\\
B_3 &= 0 && (8)
\end{aligned}
$$

During the fund raising stage, the financial investor spends k as general search costs and commits η to monitor the firm's realisation of the investment. Having match with probability $\phi \cdot p(\phi)$, the financial investor finance the recruitment period before the firm finds its labor force, spending γ. After this period, he expects to recover his negotiated debt ρ before the firm quits the market with exit probability $\sigma(m, e)$.

2.2.3 Workers

Workers expect wages w in exchange for their work effort $e \in \{0,1\}$. When the firm quits the market, the work relation terminates as well, which happens with probability $\sigma(m,e)$. The effort of the worker, e, improves the firm's productivity but constitutes a specific investment as it is linked to the relationship between the worker and the firm. The higher the investment, the more specific it is and the more costly the loss of the job.

More generally, e can be interpreted as any kind of match-specific investment that is valuable for the firm, such as specific human capital investment or social capital that strengthen any implicit components in the labor contract. Once unemployed, workers benefit from a revenue b waiting to get a chance for a new match, leading to a value of U for unemployed workers.

$$r \cdot W = w - e + \sigma(m,e) \cdot (U - W) + \dot{W} \tag{9}$$
$$r \cdot U = b + \theta \cdot q(\theta) \cdot (W - U) + \dot{U} \tag{10}$$

In equilibrium - when $\dot{W} = \dot{U} = 0$ - the value of the job is then determined by the expected net return a worker gets:

$$W - U = \frac{w - e - b}{r + \theta \cdot q(\theta) + \sigma(m,e)}.$$

3 Bargaining and specific investments

3.1 Wage and Debt negotiations

As a first step, the two factor payments - wages and debt repayments - have to be determined. Given the search framework on both the financial and the labor market - wages and debt repayments can be expected to be negotiated to split the match rent.

3.1.1 Splitting profits between workers and employers

Wage bargaining takes place at the second stage. The firm and the union share the surplus of their relationship according to a generalized Nash bargaining rule:

$$w_u^* = \arg\max (F_2 - F_0)^{1-\chi} \cdot (W - U)^{\chi}$$

where $\chi \in (0,1)$ measures the bargaining power of the union in the relationship and w_u^* denotes the bargained level of wages. This bargaining leads to the following wage:

Proposition 1 *The wage schedule in any firm is the following:*

$$w_u^* = \chi\left(\psi\left(e^*\right)\cdot y(T^*,\eta^*) - m^* - \rho^*\right) + (1-\chi)\left(b+e^*\right) \tag{11}$$

P roof. The first order condition of the surplus sharing rule yields:

$$\chi\cdot(F_2 - F_0) = (1-\chi)\cdot(W-U)$$

or

$$\chi\cdot F_2 = (1-\chi)\cdot(W-U)$$

since free entry requires that $F_0 = 0$. (3) and (4) together with $e^* = 1$ give:

$$F_2 = \frac{\psi\left(e^*\right)\cdot y(T^*,\eta^*) - m - w - \rho}{r+\sigma\left(m,e\right)} \tag{12}$$

(9) implies that $W - U = \frac{w-e^*-r\cdot U}{r+\sigma(m,e)}$, which results in the wage schedule given in the proposition. ■

The bargained wage is a weighted sum of the firm's output net of the repayment to the bank and a term expressing the annuity value of the utility of an unemployed plus the specific investment cost. The larger the worker's bargaining power, the larger the share of the firm's net surplus that he can extract. If workers have no bargaining power, they are paid their opportunity cost of working, i.e. $e^* + r\cdot U$.

3.1.2 Determining the optimal debt level

The contract between the bank and the entrepreneur stipulates that the bank will finance the recruitment costs (γ) for as long as it takes to find a worker and that the firm will pay a constant amount ρ for as long as the firm exists. Although we refer to the financial intermediary as a 'bank', it can be noted that the financial contract is more similar to an equity contract than to a debt contract. This specification is kept for simplicity's sake. Financier and entrepreneur share the surplus of the relationship according to a generalized Nash bargaining rule:

$$\rho^* = \arg\max\left(F_1 - F_0\right)^{1-\lambda}\cdot\left(B_1 - B_0\right)^{\lambda}$$

where $\lambda\in(0,1)$ measures the bank's bargaining power. This program leads to the following repayment schedule:

Proposition 2 *When the bank screens to accept only good entrepreneurs, the repayment made by the firm to the bank is given by:*

$$\rho^* = \lambda\left(\psi\left(e^*\right)\cdot y(T^*,\eta^*) - m^* - w^*\right) + \frac{\left(\gamma\left(1-\lambda\right) - T^*\lambda\right)\left(r+\sigma\right)}{q(\theta)}. \tag{13}$$

P roof. The following proof assumes that the bank has undertaken the necessary screening to single out good entrepreneurs, i.e. $\delta\left(\eta^*\right) = 0$. Then, the negotiated debt ρ^* must satisfy the first-order condition:

$$(1-\lambda)\cdot\frac{\partial\left(F_1 - F_0\right)/\partial\rho}{\left(F_1 - F_0\right)} + \lambda\cdot\frac{\partial\left(B_1 - B_0\right)/\partial\rho}{\left(B_1 - B_0\right)} = 0 \tag{14}$$

Firm values at stage 0 and 1 - taken from (1) and (2) - give $[r + p(\phi)] \cdot (F_1 - F_0) = -T - m + q(\theta) \cdot (F_2 - F_1) + c$. Moreover, (12) and (2) lead to $[r + q(\theta)] \cdot (F_2 - F_1) = r \cdot \frac{\psi(e)y(T,\eta) - w - \rho}{r + \sigma(m,e)} + T + m$. Therefore:

$$[r + p(\phi)] \cdot (F_1 - F_0) = -T - m + q(\theta) \cdot \left[\frac{r \cdot \frac{\psi(e)y(T,\eta) - w - \rho}{r + \sigma(m,e)} + T + m}{[r + q(\theta)]} \right] + c \tag{15}$$

Furthermore, (7) and (8) imply $B_2 = \frac{\rho}{r + \sigma(m,e)}$, which - plugged into (6) - gives

$$B_1 = \frac{\rho \cdot q(\theta) - \gamma \cdot [r + \sigma(m,e)]}{[r + \sigma(m,e)] \cdot [r + q(\theta)]} \tag{16}$$

while (6) and (5) yield

$$[r + \phi \cdot p(\phi)] \cdot (B_1 - B_0) = k - \gamma + q(\theta) \cdot (B_2 - B_1). \tag{17}$$

Taking account of these results, this can be re-expressed as:

$$[r + \phi \cdot p(\phi)] \cdot (B_1 - B_0) = k - \gamma + q(\theta) \cdot \frac{\rho \cdot r + \gamma \cdot [r + \sigma(m,e)]}{[r + q(\theta)] \cdot [r + \sigma(m,e)]} \tag{18}$$

Finally, (14), (15), (18) together with the optimal wage (11) and the free entry conditions $F_0 = B_0 = 0$ lead to:

$$(1 - \lambda) \cdot (1 - \chi) \cdot B_1 = \lambda \cdot F_1.$$

(16), (12) and (2) give, after rearranging terms, the expression for ρ given in the proposition. ■

3.2 Specific investments

Given these two factor payments, the size of the three different types of specific investments can be determined. Workers will fix their effort level as a function of wages, triggering firms to select an appropriate level of monitoring; financial investors will invest a certain amount (of time and money) to screen possible applicants for funds; and firms will select the technology.

3.2.1 Effort decision by workers

As wages and debt repayments are fixed through negotiations, firms have to fix the firing probability endogenously and at a sufficiently high level such as to make workers indifferent between the high and low effort choices. In our set-up this boils down to saying that firms will choose the lowest value of the organizational technology m such as to make the two job values equal. In order to keep the model tractable we make a couple of simplifying assumption in the following set-up. In particular, we consider monitoring and effort as additive separate inputs in the destruction probability: $\sigma(m,e) \equiv \widetilde{\sigma_1}(m) + \widetilde{\sigma_2}(e)$. Then the amount of monitoring can be determined by:

$$m^* = \min \{ m | W(e = 1, m) = W(e = 0, m) \} \tag{19}$$

which makes workers effectively choosing the high effort level, i.e. $e^* = 1$. Using (9) and (11) we can derive the optimal monitoring decision as:

$$m^* = \widetilde{\sigma_1}^{-1}\left[\frac{r + \widetilde{\sigma_2}(e^*) + \theta q(\theta)}{\chi(\psi(e^*)y(T^*) - e^* - \rho^* - b)}\right]. \tag{20}$$

3.2.2 Selecting good entrepreneurs

Financial investors will select η such as to maximise its return, ρ. Hence in equilibrium, financial investors determine η^* by maximising their entry value B_0:

$$\eta^* = \arg\max B_0(\eta)$$

which results in the following FOC:

$$E\eta = \lambda\phi p(\phi)(1-\chi)\psi(e^*)q(\theta)\cdot\frac{\partial y(T^*,\eta)}{\partial\eta} - (1-\lambda\chi)(r+q(\theta))(r+\sigma) \stackrel{!}{=} 0. \tag{21}$$

3.2.3 Technology choice by firms

Firms select the appropriate technology in the second period such as to maximize the firm's value:

$$T^* = \arg\max(F_1 - F_0)$$

which - taking into account (11) - results in the following FOC:

$$ET = -r - \sigma + q(\theta)(1-\chi)\cdot\frac{\partial y}{\partial T} \stackrel{!}{=} 0 \Leftrightarrow \frac{\partial y(T,\eta^*)}{\partial T} = \frac{r+\sigma}{(1-\chi)q(\theta)} \tag{22}$$

where, using the implicit function theorem, we can show that $ET = 0$ implies $\frac{dT^*}{d\theta} < 0$.

4 Partial Equilibrium

Having determined all the expressions for the determination of specific investments and agents' income, we can now proceed to the model's equilibrium. It is however useful to consider first the partial equilibrium effects of markets' liquidity on the agents' specific investment levels. Using the above optimality conditions, the following propositions indicate how specific investments on labor and financial markets react to either liquidity changes on both markets.

4.1 Reaction to labor market liquidity

As we have shown above, increasing difficulties for firms to fill a vacancy reduces their incentive to invest in match-specific assets. Similarly, increasing labor market tightness, θ, makes it easier for workers to find an alternative job, hence lowering their effort, which triggers an increased monitoring by firms to keep the worker's incentive constraint (19) in balance.

Proposition 3 (Optimal Monitoring) *Optimal monitoring increases with labor market liquidity, i.e.* $\frac{dm^*}{d\theta} > 0$*; consequently the destruction rate raises with labor market liquidity:* $\frac{d\sigma}{d\theta} > 0$.

P roof. Using (20) and noticing that in partial equilibrium financial market variables are fixed before transaction on the labor market occur, the proposition follows straightforwardly from the above equation. ■

Similarly, given the reduced incentives for firms to invest in the match when labor market tightness raises, financial investors - under certain conditions - will screen more closely to single out good entrepreneurs. However, the optimal screening condition (21) does not yield unambiguous results without further specifications. Nevertheless, under mild specific conditions the following relation between η and θ can be established.

Proposition 4 (Optimal FI monitoring) *Suppose the specification*[3] $y(T,\eta) = y_1(T) + y_2(\eta)$ *with* y_2 *monotonously increasing in* η *and twice continuously differentiable. Then, the reaction of the optimal monitoring of financial investors depends on the curvature of* y_2*. With* $y_2'' < 0$*, the reaction decreases with labor market tightness, i.e.* $\frac{d\eta^*}{d\theta} < 0$*, while with* $y_2'' > 0$*, it increases, i.e.* $\frac{d\eta^*}{d\theta} > 0$.

P roof. Using the suggested specification, η^* writes as:

$$\eta^* = (y_2')^{-1}\left(\frac{(1-\lambda\chi)(r+q(\theta))(r+\sigma)}{\lambda\phi p(\phi)(1-\chi)\psi(e^*)q(\theta)}\right).$$

As $\frac{(1-\lambda\chi)(r+q(\theta))(r+\sigma)}{\lambda\phi p(\phi)(1-\chi)\psi(e^*)q(\theta)}$ is unambiguously increasing with θ, the reaction of η^* wrt. θ depends on the shape of $(y_2')^{-1}$. As y_2 is monotonous and continuously differentiable, y_2' and $(y_2')^{-1}$ will be monotonous. However, for $y_2'' < 0$, $(y_2')^{-1}$ will decrease with θ, while for for $y_2'' > 0$, it will increase with θ. ■

4.2 Reaction to financial market liquidity

Given the sequencing of the different types of investment, financial market tightness does not play a role in determining technological choice or effort in partial equilibrium. This has simply to do with the fact that at the time, firms and workers meet, the financial structure has already been decided. Nevertheless, financial market tightness will affect incentives for financial investors to screen the market.

[3]Notice that this specification is more restrictive than any specification with positive cross-derivatives.

Proposition 5 (Reaction to financial market liquidity) *An increase in financial market liquidity (increase in ϕ) leads to an increase only of the screening activity by financial intermediaries:*

$$\frac{\partial T^*}{\partial \phi} = 0, \frac{\partial m^*}{\partial \phi} = 0, \frac{\partial \eta^*}{\partial \phi} > 0.$$

P roof. By inspection we can easily see that neither m^* nor T^* depend on ϕ. Regarding η^*, applying the implicit function theorem upon (21) the result easily obtains. ■

Again, notice that all these relations are valid only in partial equilibrium; in general equilibrium, financial market liquidity will affect T and m through the interaction with labor market tightness as we will see in the next section.

5 Equilibrium Relations in general equilibrium

In general equilibrium, the procedure of firm creation, production and destruction is not only run once but multiple times. Hence, new entrepreneurs will be able to react with their investment decisions on changing market conditions on both the financial and the labor market. Given the strategic complementarities between the different investment variables and the reaction in partial equilibrium of all types of investment to either or both types of liquidity we are expecting to see interesting interlinkages between the two markets.

5.1 Multiplicity of equilibria

Proposition 6 *The simultaneous equilibrium on the financial and the labor market is determined by the following two relations:*

$$\frac{c}{p(\phi)} = \frac{1-\lambda}{(r+\sigma)(1-\lambda\chi)} \frac{q(\theta)}{r+q(\theta)} Y(\theta) \quad \text{(FF)}$$

$$\frac{k+\eta^*}{\phi \cdot p(\phi)} = \frac{\lambda}{(r+\sigma)(1-\lambda\chi)} \frac{q(\theta)}{r+q(\theta)} Y(\theta) \quad \text{(BB)}$$

where $Y(\theta) = (1-\chi)\,(\psi(e^*)\,y(T^*,\eta^*) - m^* - e^* - b - \gamma(r+\sigma)) - (r+\sigma)\,\frac{T^*}{q(\theta)} > 0$ *in equilibrium.*

P roof. In equilibrium, no entry opportunities will be missed, hence $B_0 = 0$ and $F_0 = 0$. Together with $B_3 = F_3 = 0$ this yields:

$$B_0 = 0 \Leftrightarrow B_1^b = \frac{k+\eta^*}{\phi \cdot p(\phi)}$$

$$F_0 = 0 \Leftrightarrow F_1^b = \frac{c}{p(\phi)}$$

which defines the backward-looking relations of firm and bank values. Moreover, the forward-looking values for B_1 and F_1 can be obtained by plugging B_2 and F_2 into (6) and (2). This yields:

$$
\begin{aligned}
B_1^f &= \lambda \frac{\frac{1-\chi}{r+\sigma} q(\theta) \left(\psi\left(e^*\right) y\left(T^*\right) - m^* - e^* - b\right) - \left(T^* + \gamma\left(1-\chi\right)\right)}{\left(1-\lambda\chi\right)\left(r+q(\theta)\right)} \\
F_1^f &= \left(1-\lambda\right) \frac{\frac{1-\chi}{r+\sigma} q(\theta) \left(\psi\left(e^*\right) y\left(T^*\right) - m^* - e^* - b\right) - \left(T^* + \gamma\left(1-\chi\right)\right)}{\left(1-\lambda\chi\right)\left(r+q(\theta)\right)}
\end{aligned}
$$

Noting that in equilibrium $B_1^b = B_1^f$ and $F_1^b = F_1^f$ and using (21) to substitute η^* with $\delta\left(\eta^*\right)$ the two equilibrium relations can be obtained by dividing one over the other. ■

Moreover, given these two equilibrium relations, the following proposition can been shown to hold concerning the existence of equilibria

Proposition 7 *Let the equilibrium relations be given by proposition (6). Then, there exists at most two equilibria. In addition, there exists a degree of product market competition, $\overline{c}$, such that*

$$
\left|\left\{\left(\phi^*, \theta^*\right)\right\}\right| = \begin{cases} 2 & \text{for } c > \overline{c} \\ 1 & \text{for } c = \overline{c} \\ 0 & \text{for } c < \overline{c}. \end{cases}
$$

where the couple $\left(\phi^*, \theta^*\right)$ describes a steady state in the $\phi - \theta$-quadrant.

P roof. Given proposition (3) and the concavity condition regarding y, $Y\left(\theta\right)$ will react negatively to changes in labor market liquidity, and hence the $\mathcal{FF}$ describes a downward sloping graph in the (ϕ, θ)-quadrant.

Regarding the $\mathcal{BB}$ schedule, the right-hand side unambiguously decreases with increasing labor market liquidity, θ; the overall sign therefore depends on its reaction to ϕ. Here, the right-hand side of the equation increases with ϕ while the left-hand side of the equation has an ambiguous reaction with respect to ϕ, leaving the overall sign ambiguous as well. However, as both the numerator and the denominator of the left-hand side increase monotonically with financial market liquidity, only one crossing points exists, yielding at most one maximum or minimum. Given that the numerator of the left-hand side unambiguously decreases with θ, the sign of the partial derivative of $\mathcal{BB}$ with respect to ϕ will be determined by the denominator of the left-hand side for low θ and by the numerator of the left-hand side for high θ; in total this yields a $\mathcal{BB}$-schedule that takes a minimum in the $\theta - \phi$-quadrant.

Finally, the $\mathcal{FF}$-schedule moves downward with increasing entry barriers, while $\mathcal{BB}$ rotates to the left. Nevertheless, given that $\mathcal{BB}$ has a minimum with respect to θ there exists an entry barrier value such that for $c = \overline{c}$, only one equilibrium exists. ■

Corollary 8 *There exist a degree of competition $\overline{c}$ for which the equilibrium A does no longer exist.*

Figure **??** illustrates the shape of the equilibrium relations as well as the possibility for multiple equilibria to arise. As the figure shows, with varying degrees of competitive pressure on product markets (represented by different $\mathcal{FF}$-schedules) these equilibria may disappear.

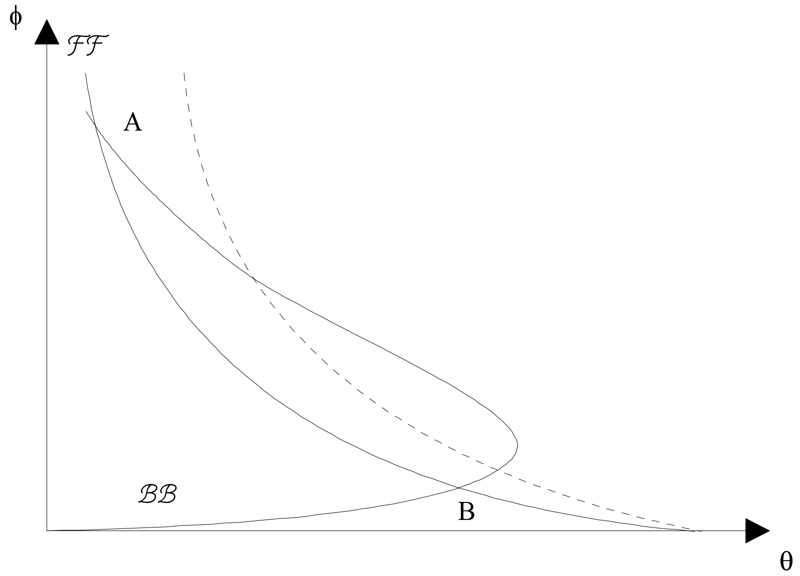

Figure 2: Market liquiditiy and multiple equilibria

For sufficiently high entry barriers, $c > \bar{c}$, the model identifies two quite distinct regimes on both the labor market and the financial market. In equilibrium A both financial market liquidity - as measured by the ratio $1/\phi = \mathcal{B}/\mathcal{F}$ - and labor market liquidity - $\theta = \mathcal{V}/\mathcal{U}$ - are relatively tight from the point of view of financial intermediaries and workers respectively: Financial investors are getting more picky with strong firm competition for funds. At the same time, low labor market liquidity pushes firms to adopt more specific technologies while at the same time they can reduce their spending for more sophisticated monitoring technologies[4]. The specific capital invested in a particular match is therefore particularly high in this equilibrium and can be protected through a relatively low liquidity on both financial and labor markets that reduces the value of the outside option for financial investors and workers.

On the other hand, in equilibrium B financial and labor markets are relatively liquid, allowing for a rapid turnover of firms and their workforce. Consequently, invested specific capital is low but the higher matching ratio on labor markets compensates for the loss in productivity in each single match. Without further specification of the production and matching process it is therefore impossible to Pareto-rank the two equilibrium that are qualitatively distinct.

While the $\mathcal{FF}$-schedule moves rightwards with decreasing entry barriers, the $\mathcal{BB}$-schedules is displaced to the left when financial intermediation cost, k, is rising (not presented in figure **??**). Rising funding costs reduce the available financial market liquidity and make firm entry

[4]Notice that in our set-up, much in line with earlier literature (see e.g. Bowles, 1985), expenditures for monitoring technologies are social waste as they do not contribute to the productive output.

more difficult while increasing ϕ in equilibrium. Together, this decreases the optimal labor market liquidity, θ^*.

Notice, finally, that the $\mathcal{BB}$-schedule allows a maximum labor market liquidity to be sustainable. For too high a degree of competition, i.e. $c < \overline{c}$, any equilibrium will disappear as firm turnover is too high to generate sufficient incentives for workers and firms to make any upfront investment to guarantee the firm's profitability. This in turn proves unprofitable for financial intermediaries to enter the market, generating low financial market liquidity. When competition is too strong, no financial investor will be ready to enter the market to provide funds for fear of too low firm profitability that could allow to recover the monitoring costs. Consequently, financial investors prefer to quit the market altogether at that point.

Multiplicity of equilibria arises in this context due to a particular strategic complementarity between the incentive structures shaping specific investment undertaken by the three actors in the model. This may be called institutional complementarities (Aoki, 1995; Amable, Ernst, Palombarini, 2005). Institutional complementarity refers to the fact that the incentive structures on different markets affect each other in providing a global incentive landscape in which the different agents locate their actions: In our case, the decisions to invest in particular technologies, T, to provide effort, e, and to monitor firms, η, are all interrelated in general equilibrium. Interestingly, only the monitoring of entering firms has non-trivial partial derivatives with respect to both θ and ϕ in partial equilibrium; nevertheless, the number of firms being endogenous in general equilibrium, both the technology choice as well as the effort decision will be affected by the monitoring effort and hence the financial market liquidity in general equilibrium.

5.2 Comparative statics

Despite the simplicity of the original Bellman equations, in order to derive some sensible comparative statics some rearrangement of equations has to be carried out first. Before deriving these results, however, we will look at how the liquidity on financial and labor markets will affect unemployment, labor productivity and GDP before looking in more detail into the relationship between structural parameters and equilibrium liquidity rates.

5.2.1 Unemployment, productivity and GDP

In equilibrium inflows and outflows of unemployment are equal, hence steady state unemployment can be written as:

$$(1-u)\,\sigma = u \cdot \theta q(\theta) \Leftrightarrow u = \frac{\sigma}{\sigma + \theta q(\theta)}. \tag{23}$$

Moreover, given that a match involves only one entrepreneur and one worker, labor productivity equals firm production and can be defined as:

$$LP = \psi(e)\, y\left(T(\theta), \eta(\theta, \phi)\right). \tag{24}$$

Finally, GDP can be derived from the number of workers that are currently employed in a match. Hence, provided that there is no firm heterogeneity this writes as:

$$GDP = (1-u) \cdot LP = \frac{\theta q(\theta)}{\sigma + \theta q(\theta)} \psi(e)\, y\left(T(\theta), \eta(\theta, \phi)\right). \tag{25}$$

The following table gives an overview of the reaction of u, LP and GDP with respect to labor and financial market liquidity:

	u	LP	GDP
θ	$-$	$-$	$+/-$
ϕ	0	$+$	$+$

Interestingly, while labor market liquidity unambiguously decrease unemployment (as expected), it has a negative impact on labor productivity (due to the incentive effect) and hence an ambiguous effect on GDP (depending on the strength of the relative effects). Financial market liquidity (i.e. $1/\phi$), on the other hand, does not affect unemployment (at least not in a partial equilibrium sense) and decreases labor productivity and GDP.

5.2.2 The impact of structural reforms on market liquidity and macroeconomic performance

The impact of labor and financial market liquidity on macroeconomic variables constitutes, however, just one side of the effects of structural reforms on the macroeconomy. Hence, in order to establish the comparative statics results of the impact of the parameter space $\{c, k, \sigma, b, \chi, \lambda\}$, we fully differentiate the system $(\mathcal{FF}, \mathcal{BB})$ with respect to the different parameters, taking into account the differential behaviour of the $\mathcal{BB}$-schedule depending on the equilibrium the economy is. Here, the first two parameters (c, σ) describe product market policies, the second two (b, χ) labor market policies and the last two (k, λ) financial market policies. The following table gives and overview of the reaction of ϕ and θ with respect to these different policy variables[5]. The proposition summarises the effects that can be expected in the two equilibria when changing those parameters of the model that can be interpreted as referring to structural reforms, i.e. a reduction in c, k, b, χ, λ and an increase in σ.

[5]Note that the reported comparative statics results are based on the simplified IS-LM model and not on the underlying search framework.

Proposition 9 *Reducing entry barriers to product (c) and financial markets (k), reducing replacement ratios (b), the wage bargaining (χ) and banking bargaining power (λ) or increasing exit probabilities (σ) through more intense competition leads to distinctively different reactions of labor and financial market liquidity, depending on whether the economy is in equilibrium A or B. In particular the two market liquidities evolve as follows:*

Table 1: Comparative statics

		c	σ	b	χ	k	λ
A	θ	+	−	+	+	−	+
	ϕ	−	+	−	+	+	−
B	θ	+	−	+	+	+	+/−
	ϕ	+	−	+	−	−	+

Note: The table reports the effects of policy changes in the sense of structural reforms on the basis of the above IS-LM model; in particular the following policy changes have been accounted for: $c\downarrow$, $\sigma\uparrow$, $b\downarrow$, $\chi\downarrow$, $k\downarrow$, $\lambda\downarrow$.

P roof. See appendix. ■

As can be seen from the table, the two equilibria do not show the same behaviour with respect to all parameter changes. For some policies, the outcome on the labor market may be the same (such as in the case of a change in c and b) but differences in labor productivity and GDP will occur following the impact these structural reforms have on financial market liquidity. In general, however, the two equilibria will show a distinct behaviour following structural reforms given the differences in the importance of specific investment relative to market liquidity. Consequently, very different outcomes can be expected for these structural reforms.

In order to assess the impact these parameter changes have on macroeconomic performance in terms of unemployment, labor productivity and GDP, one has to go back to the original definitions spelled out in the last section (equations (23)-(25)). Given the sometime ambiguous nature of the effects that parameter changes have in the case of equilibrium B, however, macroeconomic effects cannot be determined for all structural policies without further specification of the concrete functional form[6]. The following table gives an overview of the effects that can be expected from structural policies in this model.

[6] For a quantitative analysis of the model in this paper see Ernst (2005).

Table 2: Structural policies and macroeconomic performance

		c	σ	b	χ	k	λ
	u	$-$	$+$	$-$	$-$	$+$	$-$
A	LP	$-$	$+$	$-$	0	$+$	$-$
	GDP	0	0	0	0	0	0
	u	$-$	$+$	$-$	$-$	$-$	0
B	LP	0	0	0	$-$	$-$	0
	GDP	0	0	0	0	0	0

Note: The table reports the effects of policy changes in the sense of structural reforms on unemployment, productivity and GDP levels; in particular the following policy changes have been accounted for: a reduction in firm entry barriers $c \downarrow$, a rise in competitive pressure $\sigma \uparrow$, a reduction in replacement ratios for unemployed $b \downarrow$, a decrease of the wage share $\chi \downarrow$, a reduction in banks' entry barriers $k \downarrow$, a reduction in banks' bargaining power $\lambda \downarrow$. A '0' in the above table indicates an ambiguous effect.

The impact of structural reforms on GDP is in general ambiguous not least due to the ambiguous role of θ in determining the GDP growth rate but mainly due to the fact that either labor productivity and unemployment are moving in the same direction (as in equilibrium A) or that their impact on labor productivity is itself ambiguous (as is the case in equilibrium B). The overall impact depends on the relative importance of specific investments in the production function: the lower the share of specific assets in total production, the more the impact on GDP will be determined by the outcome on the labor market alone. Moreover, when θ is high (i.e. unemployment is low), the overall impact of further rising employment is likely to be positive as in this case, the incentive effect is already low and not playing an important role (i.e. there are increasing returns to scale for either the liquidity or the incentive effect of labor market liquidity).

6 Conclusion

When markets are characterized by transactional imperfections, market interaction may arise where imperfections on one market spill over to another, mutually influencing the macroeconomic outcome. Introducing search and matching on both financial and labor markets we have shown in this paper, how a trade-off between market liquidity and match-specific investment incentives can arise, leading to multiple equilibria characterised by distinct levels of unemployment and productivity. Moreover, the two equilibria react differently to changes in the underlying parameters - identified in this paper as different reactions to structural policies. In particular, we have shown that markets may interact in a way such that the economy with higher friction react differently to structural policy changes as those expected from standard economic theory. Especially any attempt to increase product or financial market flexiblity may not yield the expected result. Hence both partial and general equilibrium effects of market frictions have to be considered simultaneously in order to determine the likely impact of any change in structural policies.

The outcome of such an analysis - where lowering 'imperfections' or the level of 'frictions' does not necessarily yield the expected results - is similar to those results found in the literature (Amable and Gatti, 2002), where higher competition on product markets may increase unemployment because of the presence of an effort incentive mechanism on the labor market. More generally, more 'liquidity' or 'flexibility' may act as a disincentive to specific investment, be it work effort, entrepreneurial screening or innovative outlays.

This analysis can be extended easily to account for the sectoral specialization a country may follow, laying the theoretical basis for "institutional comparative advantages". Indeed, different industries are identified by different technological characteristics (Kitschelt, 1991) that may determine the extent to which specific investment are necessary for its successful evolution. When only low levels of specific investments are required - or similarly when the marginal productivity of these kinds of investment is high - then lower market frictions may in fact lead to both higher employment and higher industrial growth. Conversely, where industries are characterized by high levels of specific investments, stronger frictions provide the necessary incentives for strong industrial performance. As in this situation, one size does not fit all , one might expect different industrial portfolios to be selected by countries characterized by different degrees of frictions on their credit and labor markets.

Finally, the impact of other generic policies may be analysed in this framework and its consequences for the emergence of multiple equilibria further assessed. In particular, the existence of the second, non-standard equilibrium depends on the importance of match-specific assets in the production function. To the extent that these assets may be secured through generic policies (patent law, public investment in education and educational standards, prudential regulation of corporate governance mechanisms), they may become more and more general in nature, hence weakening the trade-off between liquidity and incentives and - consequently - lead to the disappereance of this second equilibrium.

7 Annex

7.1 Proof of proposition 9

In order to prove proposition 9, we will first transform the system of equations set up by $\mathcal{FF}$ and $\mathcal{BB}$ in the following way:

$$\begin{cases} \frac{c}{p(\phi)} = \frac{1-\lambda}{(r+\sigma)(1-\lambda\chi)} \frac{q(\theta)}{r+q(\theta)} Y(\theta,\phi) \\ \frac{k+\eta^*}{\phi \cdot p(\phi)} = \frac{\lambda}{(r+\sigma)(1-\lambda\chi)} \frac{q(\theta)}{r+q(\theta)} Y(\theta,\phi) \end{cases}$$

with $Y(\theta) = (1-\chi)\,(\psi(e^*)\,y(T^*,\eta^*) - m^* - e^* - b - \gamma(r+\sigma)) - (r+\sigma)\,\frac{T^*}{q(\theta)}$.

Taking logs on both sides, yields:

$$\begin{aligned}
\log(c) - \log(p(\phi)) &= \log\left[\frac{1-\lambda}{(r+\sigma)(1-\lambda\chi)}\right] + \log\left(\frac{q(\theta)}{r+q(\theta)}\right) + \log Y(\theta,\phi) \\
\log(k+\eta^*(\theta,\phi)) - \log(\phi p(\phi)) &= \log\left[\frac{\lambda}{(r+\sigma)(1-\lambda\chi)}\right] + \log\left(\frac{q(\theta)}{r+q(\theta)}\right) + \log Y(\theta,\phi)
\end{aligned}$$

As we are only interested in the sign of the derivative, this system can be simplified noting that

$$\begin{aligned}
sign\left(\frac{\partial \frac{q(\theta)}{r+q(\theta)}}{\partial\theta}\right) &= sign\left(\frac{\partial Y(\theta,\phi)}{\partial\theta}\right) = -1 \\
sign\left(\frac{\partial \phi p(\phi)}{\partial\phi}\right) &= sign\left(\frac{\partial Y(\theta,\phi)}{\partial\phi}\right) = +1
\end{aligned}$$

in order to be written as:

$$\left\{\begin{array}{l}\Lambda_1 = 0\\ \Lambda_2 = 0\end{array}\right. \equiv \left\{\begin{array}{l}\log(c) + \log(\phi) - \log\left[\frac{1-\lambda}{(r+\sigma)(1-\lambda\chi)}\right] - \log\widetilde{Y}(\theta,\phi) = 0\\ \log(k+\eta^*(\theta,\phi|\,\lambda,\sigma,\chi)) - \log\left[\frac{\lambda}{(r+\sigma)(1-\lambda\chi)}\right] - \log\widetilde{Y}(\theta,\phi) = 0\end{array}\right.$$

where $\widetilde{Y}(\theta,\phi) \equiv \phi p(\phi)\left(\frac{q(\theta)}{r+q(\theta)}\right)\cdot Y(\theta,\phi)$. Moreover, from (21) follows that $\frac{\partial\eta^*(\theta,\phi|\sigma,\chi)}{\partial\sigma} < 0$, $\frac{\partial\eta^*(\theta,\phi|\sigma,\chi)}{\partial\chi} > 0$. Totally differentiating both equations yields the slope of the two schedules at A and B:

$$\begin{aligned}
\left.\frac{d\phi}{d\theta}\right|_{FF} &= \frac{\frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}}{\frac{1}{\phi} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}} < 0 \\
\left.\frac{d\phi}{d\theta}\right|_{BB} &= \frac{\frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} - \frac{\eta_\theta}{k+\eta^*}}{\frac{\eta_\phi}{k+\eta^*} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}} = \left\{\begin{array}{l}<0 \text{ for A}\\ >0 \text{ for B}\end{array}\right.
\end{aligned}$$

hence we have

$$\frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} < 0,\ \frac{1}{\phi} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} > 0,\ \left(\frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} - \frac{\eta_\theta}{k+\eta^*}\right)\left(\frac{\eta_\phi}{k+\eta^*} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}\right)\left\{\begin{array}{l}<0 \text{ for A}\\ >0 \text{ for B}\end{array}\right.$$

Moreover, regarding equilibria A we have:

$$\begin{aligned}
\left|\left.\frac{d\phi}{d\theta}\right|_{FF,\theta=\theta_A}\right| &> \left|\left.\frac{d\phi}{d\theta}\right|_{BB,\theta=\theta_A}\right| \Leftrightarrow \left|\frac{\frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}}{\frac{1}{\phi} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}}\right| > \left|\frac{\frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} - \frac{\eta_\theta}{k+\eta^*}}{\frac{\eta_\phi}{k+\eta^*} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}}\right| \\
\Leftrightarrow \left|\frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}\left(\frac{\eta_\phi}{k+\eta^*} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}\right)\right| &> \left|\left(\frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} - \frac{\eta_\theta}{k+\eta^*}\right)\left(\frac{1}{\phi} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}\right)\right|
\end{aligned}$$

In order to derive the comparative statics results, we will set up the Cramer matrix equation:

$$\begin{pmatrix} \frac{1}{\phi} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} & -\frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} \\ \frac{\eta_\phi}{k+\eta^*} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} & \frac{\eta_\theta}{k+\eta^*} - \frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} \end{pmatrix}\begin{pmatrix}\frac{d\phi}{d\Xi}\\ \frac{d\theta}{d\Xi}\end{pmatrix} = -\begin{pmatrix}\frac{\partial\Lambda_1}{\partial\Xi}\\ \frac{\partial\Lambda_2}{\partial\Xi}\end{pmatrix}$$

where $\Xi \in \{c, k, \sigma, b, \chi, \lambda\}$, leading to the following equations for the two derivatives:

$$\begin{aligned}\frac{d\phi}{d\Xi} &= \frac{\frac{\partial \Lambda_1}{\partial \Xi} \cdot \left(\frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} - \frac{\eta_\theta}{k+\eta^*}\right) - \frac{\partial \Lambda_2}{\partial \Xi} \cdot \frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}}{\Delta} = \frac{\frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}\left(\frac{\partial \Lambda_1}{\partial \Xi} - \frac{\partial \Lambda_2}{\partial \Xi}\right) - \frac{\partial \Lambda_1}{\partial \Xi}\frac{\eta_\theta}{k+\eta^*}}{\Delta} \\ \frac{d\theta}{d\Xi} &= \frac{\frac{\partial \Lambda_1}{\partial \Xi} \cdot \left(\frac{\eta_\phi}{k+\eta^*} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}\right) - \frac{\partial \Lambda_2}{\partial \Xi} \cdot \left(\frac{1}{\phi} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}\right)}{\Delta} = \frac{-\left(\frac{\partial \Lambda_1}{\partial \Xi} - \frac{\partial \Lambda_2}{\partial \Xi}\right)\frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} + \frac{\partial \Lambda_1}{\partial \Xi}\frac{\eta_\phi}{k+\eta^*} - \frac{\partial \Lambda_2}{\partial \Xi} \cdot \frac{1}{\phi}}{\Delta}\end{aligned}$$

where

$$\begin{aligned}\Delta &\equiv Det\left[\begin{pmatrix} \frac{1}{\phi} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} & -\frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} \\ \frac{\eta_\phi}{k+\eta^*} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} & \frac{\eta_\theta}{k+\eta^*} - \frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} \end{pmatrix}\right] \\ &= -\left(\frac{1}{\phi} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}\right)\left(\frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} - \frac{\eta_\theta}{k+\eta^*}\right) + \frac{\widetilde{Y}_\theta(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}\left(\frac{\eta_\phi}{k+\eta^*} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}\right).\end{aligned}$$

From the fact that $\mathcal{FF}$ is downward-sloping we know that the term $\frac{1}{\phi} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)}$ has to be positive. Moreover, from the proof of proposition (6) we know that $\frac{\eta_\phi}{k+\eta^*} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} > 0$ for equilibrium A and $\frac{\eta_\phi}{k+\eta^*} - \frac{\widetilde{Y}_\phi(\theta,\phi)}{\widetilde{Y}(\theta,\phi)} < 0$ for equilibrium B, i.e. $Det_A < 0$ and $Det_B > 0$. Moreover, for the RHS we have:

$$\begin{aligned}\frac{\partial \Lambda_1}{\partial k} &= \frac{\partial \Lambda_2}{\partial c} = \frac{\partial \Lambda_2}{\partial b} = 0 \\ \frac{\partial \Lambda_1}{\partial c} &> 0, \frac{\partial \Lambda_1}{\partial b} > 0, \frac{\partial \Lambda_1}{\partial \sigma} > 0, \frac{\partial \Lambda_2}{\partial \sigma} > 0, \frac{\partial \Lambda_2}{\partial k} > 0, \frac{\partial \Lambda_1}{\partial \lambda} > 0, \frac{\partial \Lambda_1}{\partial \chi} > 0, \frac{\partial \Lambda_2}{\partial \chi} > 0 \\ \frac{\partial \Lambda_2}{\partial \lambda} &< 0\end{aligned}$$

and

$$\frac{\partial \Lambda_2}{\partial \chi} > \frac{\partial \Lambda_1}{\partial \chi}, \frac{\partial \Lambda_2}{\partial \sigma} < \frac{\partial \Lambda_1}{\partial \sigma}.$$

- $\frac{\partial \theta}{\partial \Xi}$: For k, c, b, the sign of $\frac{\partial \theta}{\partial \Xi}$ will be determined by the sign of the determinant as the numerator is either unambiguously positive or unambiguously negative.

$$\begin{aligned}&\frac{\partial \theta}{\partial c} < 0 \\ &\left.\frac{\partial \theta}{\partial k}\right|_A > 0, \left.\frac{\partial \theta}{\partial k}\right|_B < 0 \\ &\left.\frac{\partial \theta}{\partial \sigma}\right|_A < 0, \left.\frac{\partial \theta}{\partial \sigma}\right|_B < 0 \\ &\frac{\partial \theta}{\partial b} < 0 \\ &\left.\frac{\partial \theta}{\partial \chi}\right|_A < 0, \left.\frac{\partial \theta}{\partial \chi}\right|_B < 0 \\ &\left.\frac{\partial \theta}{\partial \lambda}\right|_A < 0, \left.\frac{\partial \theta}{\partial \lambda}\right|_B \gtreqless 0\end{aligned}$$

- $\frac{\partial \phi}{\partial \Xi}$: The sign of $\frac{\partial \phi}{\partial \Xi}$ depends on the relative size of η_θ with respect to η_ϕ.

$$\left.\frac{\partial\phi}{\partial c}\right|_A > 0,\ \left.\frac{\partial\phi}{\partial c}\right|_B < 0$$
$$\left.\frac{\partial\phi}{\partial k}\right|_A < 0,\ \left.\frac{\partial\phi}{\partial k}\right|_B > 0$$
$$\left.\frac{\partial\phi}{\partial \sigma}\right|_A > 0,\ \left.\frac{\partial\phi}{\partial \sigma}\right|_B < 0$$
$$\left.\frac{\partial\phi}{\partial b}\right|_A > 0,\ \left.\frac{\partial\phi}{\partial b}\right|_B < 0$$
$$\left.\frac{\partial\phi}{\partial \chi}\right|_A < 0,\ \left.\frac{\partial\phi}{\partial \chi}\right|_B > 0$$
$$\left.\frac{\partial\phi}{\partial \lambda}\right|_A > 0,\ \left.\frac{\partial\phi}{\partial \lambda}\right|_B < 0$$

This completes the proof. ■

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