



**Center for Empirical Macroeconomics**

Working Paper No. 98

**Progressive Taxation, Public Capital and  
Endogenous Growth**

by

Alfred Greiner

University of Bielefeld  
Department of Economics  
Center for Empirical Macroeconomics  
P.O. Box 100 131  
33501 Bielefeld, Germany

# Progressive Taxation, Public Capital and Endogenous Growth

Alfred Greiner\*

## Abstract

This paper presents and analyzes an endogenous growth model with public capital and progressive taxation. Two versions are considered: The first version assumes that the budget of the government is balanced at each point of time. The second allows for public debt but asserts that the ratio of the primary surplus to gross domestic income is a positive linear function of the debt income ratio which guarantees that public debt is sustainable. The paper then derives necessary conditions for the existence of a sustainable balanced growth path for the analytical model. Further, simulations are undertaken in order to gain insight into growth effects of varying the slope of the tax schedule and in order to find how the tax scheme affects the dynamics of the model.

JEL: E62, H54, H60

Keywords: Fiscal Policy, Progressive Taxation, Public Capital, Endogenous Growth

---

\*Department of Business Administration and Economics, Bielefeld University, P.O. Box 100131, 33501 Bielefeld, Germany

# 1 Introduction

In the book by Arrow and Kurz (1970) one type of growth models posits that public spending may be productive by raising a public capital stock which positively affects production possibilities in an economy. However, due to decreasing returns to scale of private and public capital, this model does not generate sustained per-capita growth unless exogenous technical progress is assumed. Futagami et al. (1993) analyzed this model assuming that the aggregate production function is linear homogeneous in private and public capital leading to ongoing growth. The study of the model presented by Futagami et al. (1993) found that the long-run growth rate is an inverted U-shaped function of the income tax rate, thus, confirming the result obtained by Barro (1990) in a simpler model where public spending as a flow variable affects aggregate production.

Empirical studies investigating the effect of public spending and public capital on the productivity of economies do not reach definite conclusions. Instead, the outcomes differ in part significantly. However, this is not too surprising because it is to be expected that the time period under consideration as well as the countries which are considered are important as to the results obtained. A survey of the empirical studies dealing with public spending, public capital and the economic performance of countries is given in the paper by Pfähler et al. (1996) and by the more recent contribution by Romp and de Haan (2005).

Most of the theoretical models incorporating public capital assume a constant marginal tax rate and a balanced government budget. One exception is the model presented by Turnovsky (1995, chap. 13), who took into consideration public debt in his analysis and showed that more public investment going along with higher public debt always raises the long-run growth rate (p. 418). In Greiner (2005) it was shown that the latter result does not necessarily hold when the primary surplus positively depends on public debt which guarantees sustainability of public debt. But the tax scheme in both contributions is characterized by constant marginal income tax rates.

Li and Sarte (2004) study an endogenous growth model with heterogeneous households and a balanced government budget but a progressive income tax scheme. They find that a more progressive tax scheme reduces the share of government expenditures, which equals the tax revenue, relative to output because the relative income of the rich household declines. If public spending is productive, a more progressive income tax scheme reduces economic growth through two channels: It reduces both the incentive to invest and the share of productive public spending to output at the same time, implying a smaller growth rate.

In this paper, we study the effects of progressive taxation in an endogenous growth model with and without public debt where we assume that the primary surplus is an increasing function of public debt. In contrast to Li and Sarte (2004) we analyze a symmetric equilibrium where households have the same capital stock and supply the same amount of labour. Our goal is to study growth effects of progressive tax schemes and to analyze effects of tax progression on the dynamics of the model.

The rest of the paper is organized as follows. In the next section we present our endogenous growth model with public capital and government debt. Section 3 studies the implications of the model, first assuming a balanced budget of the government and, then, allowing for public debt. Section 4, finally, concludes the paper.

## **2 The model**

Our economy consists of three sectors: A household sector which receives labour income and income from its saving, a productive sector and the government. First, we describe the household and the productive sector.

## 2.1 The household and the productive sector

The household sector consists of a unit measure of identical households maximizing the welfare functional

$$\max_{c(t)} \int_0^{\infty} e^{-\rho t} \ln c(t) dt, \quad (1)$$

where  $c$  is consumption<sup>1</sup> and the utility function is assumed to be logarithmic,  $U(c) = \ln c$ .  $\rho$  is the subjective discount rate and the household supplies labour  $l$  inelastically. The budget constraint is given by

$$(1 - \tau)(wl + ra) = \dot{a} + c. \quad (2)$$

$w$  is the wage rate and  $r$  is the interest rate.  $a \equiv b + k$  denotes assets which are equal to public debt held by the household,  $b$ , and private capital,  $k$ . All variables give per-capita quantities.  $\tau \in (0, 1)$  is the income tax rate. The dot gives the derivative with respect to time and we neglect depreciation of private capital.

As to the income tax rate we assume a progressive tax scheme where the tax rate  $\tau$  is given by (cf. Guo and Lansing, 1998, and Slobodyan, 2005)

$$\tau = 1 - \psi \left( \frac{\bar{y}}{y} \right)^{\phi}, \quad \psi \in (0, 1], \quad \phi \in [0, 1), \quad (3)$$

with  $\bar{y}$  a base level of income, which is taken as given by households, and  $y$  the income of the household. The parameters  $\psi$  and  $\phi$  give the level and the slope of the tax schedule. The marginal tax rate  $\tau_m$  is given by  $\tau_m = 1 - \psi(1 - \phi)(\bar{y}/y)^{\phi}$  and exceeds the average tax rate given by  $\tau y/y = 1 - \psi(\bar{y}/y)^{\phi}$ . For  $\phi = 0$  the marginal tax rate is constant and equals the average tax rate.

The necessary conditions for optimality are obtained as

$$\dot{c} = c \left( \psi(1 - \phi) \left( \frac{\bar{y}}{y} \right)^{\phi} r - \rho \right) \quad (4)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} a/c = 0. \quad (5)$$

---

<sup>1</sup>From now on we omit the time argument  $t$  if no ambiguity arises.

The productive sector is represented by one firm which behaves competitively and which maximizes static profits. The production function of the firm is given

$$Q = K^{1-\alpha}(LG)^\alpha, \quad (6)$$

with  $\alpha < 1$ .  $(1 - \alpha)$  is the private capital share and  $\alpha$  gives the labour share and we normalize aggregate labour,  $L$ , to be equal to one.  $G$  denotes public capital which is assumed to be a purely public good which is labour augmenting. Normalizing labour by setting  $L \equiv 1$ , profit maximization yields

$$w = \alpha K^{1-\alpha} G^\alpha \quad (7)$$

$$r = (1 - \alpha) K^{-\alpha} G^\alpha \quad (8)$$

### 2.1.1 The government

The government in our economy receives tax revenues from income taxation and has revenues from issuing government bonds it then uses for public investment. Further, the government sets the primary surplus such that it is a positive linear function of public debt which guarantees that public debt is sustainable as will be shown next.

The accounting identity describing the accumulation of public debt in continuous time is given by:

$$\dot{B} = r B - S = r B - T + I_p, \quad (9)$$

where  $B$  stands for public debt,  $r$  is the interest rate and  $S$  is the government surplus exclusive of interest payments.  $T$  gives the tax revenue of the government and  $I_p$  is public investment.

As concerns the primary surplus to gross national income ratio, we assume that this ratio is a positive linear function of the debt to gross domestic income ratio and of a constant. The primary surplus ratio, then, can be written as

$$\frac{T - I_p}{Y} = \varphi + \beta \frac{B}{Y}, \quad (10)$$

with  $\varphi, \beta \in \mathbb{R}$ .

This was suggested by Bohn (1995, 1998) because with this assumption any path of public debt is sustainable if  $\beta$  is strictly positive. Since Bohn (1995, 1998) takes time to be discrete and since he writes the government budget constraint in a slightly different manner, namely as  $B_{t+1} - B_t = r_{t+1}B_t - S_t(1 + r_{t+1})$ , we briefly show that  $\beta > 0$  is sufficient for public debt to be sustainable in our model, too. To see this we insert (10) in (9) which gives

$$\dot{B} = (r - \beta)B - \varphi Y. \quad (11)$$

Solving equation (11) and multiplying both sides of (11) by  $e^{-\int_0^t r(\tau)d\tau}$ , to get the present value of government debt at time  $t$ , yields

$$e^{-\int_0^t r(\tau)d\tau} B(t) = e^{-\beta t} B(0) - \varphi Y(0) \frac{\int_0^t e^{\beta\tau} e^{-\int_0^\tau (r(\mu) - \gamma_y(\mu))d\mu} d\tau}{e^{\beta t}}. \quad (12)$$

with  $B(0)$  and  $Y(0)$  public debt and gross domestic income at time  $t = 0$  and  $\gamma_y$  the growth rate of gross domestic income.

For  $r < \gamma_y$  the intertemporal budget constraint is irrelevant because in this case the economy is dynamically inefficient implying that the government can play a Ponzi game. Therefore, we only consider the case  $r > \gamma_y$ . (12) shows that  $\beta > 0$  is a necessary condition for  $\lim_{t \rightarrow \infty} e^{-\int_0^t r(\tau)d\tau} B(t) = 0$ , i.e. for the present value of public debt to converge to zero for  $t \rightarrow \infty$  which characterizes a sustainable debt policy.

If the numerator in the second expression in (12) remains finite, implying that  $\int_0^\tau (r(\mu) - \gamma_y(\mu))d\mu$  converges to infinity, the second term converges to zero for  $\beta > 0$ . If the numerator in the second expression in (12) becomes infinite, l'Hôpital gives the limit as  $e^{-\int_0^t (r(\mu) - \gamma_y(\mu))d\mu} / \beta$ . This shows that  $\beta > 0$  and  $\lim_{t \rightarrow \infty} \int_0^t (r(\mu) - \gamma_y(\mu))d\mu = \infty$  are sufficient for sustainability of public debt. Thus, the intertemporal budget constraint of the government is fulfilled if the ratio of the primary surplus to gross domestic income is a positive linear function of the debt ratio, which can also be observed for economies in the real world (see e.g. Bohn, 1998, and Greiner et al. 2004). Therefore, we posit that

the government sets the primary surplus according to (11) implying that public debt is sustainable.

Since the government sticks to rule (11) public investment can be obtained from (9) as

$$I_p = T - \beta B - \varphi Y = Y \left( \left( 1 - \psi \left( \frac{\bar{Y}}{Y} \right)^\phi \right) - \varphi \right) - \beta B, \quad (13)$$

where we used that  $y = Y$  and  $\bar{y} = \bar{Y}$  hold in a symmetric equilibrium.

Neglecting depreciation, the differential equation describing the evolution of public capital, then, is written as

$$\dot{G} = I_p = Y \left( \left( 1 - \psi \left( \frac{\bar{Y}}{Y} \right)^\phi \right) - \varphi \right) - \beta B. \quad (14)$$

It should be mentioned that the budgetary rule (10) imposes a constraint on the possibility of the government to control public investment. This holds because a rise in public debt, for whatever reasons, implies that public investment must decrease, for given values of the parameters  $\varphi$  and  $\beta$  and for a given tax revenue. The reason is that the government must raise the primary surplus such that a fiscal policy remains sustainable when public debt rises. This generates a crowding-out effect of public debt in the model.

## 2.2 Equilibrium conditions and the balanced growth path

An equilibrium allocation is defined as an allocation such that the firm maximizes profits implying that factor prices equal their marginal products (equations (7) and (8)), households solve (1) subject to (2) and the budget constraint of the government (11) is fulfilled. Further, we consider a symmetric equilibrium such that  $c = C$ ,  $b = B$ ,  $k = K$ ,  $l = L$ ,  $y = Y$ , and  $\bar{y} = \bar{Y}$  hold, where capital letters denote aggregate variables.  $\bar{Y}$  is set to the economy-wide average income as in Slobodyan (2005) so that  $Y = \bar{Y}$  holds.

The economy-wide resource constraint is obtained from equations (2) and (11). Thus, the following autonomous system of differential equations completely describes our econ-



omy,

$$\frac{\dot{K}}{K} = \left(\frac{G}{K}\right)^\alpha - \frac{C}{K} + \beta \left(\frac{B}{K}\right) + \left(\frac{G}{K}\right)^\alpha \left(1 + (1 - \alpha)\frac{B}{K}\right) (\varphi - (1 - \psi)) \quad (15)$$

$$\frac{\dot{G}}{G} = -\beta \left(\frac{B}{G}\right) - \left(\frac{K}{G}\right)^{1-\alpha} \left(1 + (1 - \alpha)\frac{B}{K}\right) (\varphi - (1 - \psi)) \quad (16)$$

$$\frac{\dot{B}}{B} = (1 - \alpha) \left(\frac{G}{K}\right)^\alpha - \beta - \varphi \left(\frac{G}{K}\right)^\alpha \left((1 - \alpha) + \frac{K}{B}\right) \quad (17)$$

$$\frac{\dot{C}}{C} = -\rho + (1 - \alpha)\psi(1 - \phi) \left(\frac{G}{K}\right)^\alpha \quad (18)$$

A balanced growth path (BGP) is defined as a path on which all endogenous variables grow at the same rate, i.e.  $\dot{K}/K = \dot{G}/G = \dot{B}/B = \dot{C}/C$  holds, and the intertemporal budget constraint of the government must hold. Note that the BGP is dynamically efficient<sup>2</sup> and the transversality condition of the household is fulfilled. Since we have posited that the government sets the primary surplus according to (10) with  $\beta > 0$  any path which satisfies  $\dot{K}/K = \dot{G}/G = \dot{B}/B = \dot{C}/C$  is associated with a sustainable public debt.

To analyze our economy around a BGP we define the new variables  $x \equiv G/K$ ,  $z \equiv B/K$  and  $v \equiv C/K$ . Differentiating these variables with respect to time yields a three dimensional system of differential equations given by

$$\begin{aligned} \dot{x} = & x \left( -\beta z/x - x^{\alpha-1} (1 + (1 - \alpha)z) (\varphi - (1 - \psi)) - x^\alpha - \beta z + \right. \\ & \left. v - x^\alpha (1 + (1 - \alpha)z) (\varphi - (1 - \psi)) \right) \end{aligned} \quad (19)$$

$$\begin{aligned} \dot{z} = & z \left( (1 - \alpha)x^\alpha - \beta - \varphi x^\alpha ((1 + \alpha) + z) - x^\alpha (1 + (1 - \alpha)z) (\varphi - (1 - \psi)) + \right. \\ & \left. v - x^\alpha - \beta z \right) \end{aligned} \quad (20)$$

$$\dot{v} = v \left( v - \rho + (1 - \alpha)x^\alpha \psi (1 - \phi) - x^\alpha - \beta z - x^\alpha (1 + (1 - \alpha)z) (\varphi - (1 - \psi)) \right) \quad (21)$$

A solution of  $\dot{x} = \dot{z} = \dot{v} = 0$  with respect to  $x, z, v$  gives a BGP for our model and the

---

<sup>2</sup>The difference between the interest rate and the growth on the BGP is strictly positive and constant so that  $\lim_{t \rightarrow \infty} \int_0^t (r(\mu) - \gamma_y(\mu)) d\mu = \infty$  holds.

corresponding ratios  $x^*, z^*, v^*$  on the BGP.<sup>3</sup>

### 3 Implications of the model

In this section we first analyze our model for a balanced government budget and, then, allowing for public debt.

#### 3.1 Balanced government budget

To analyze the model for a balanced government budget we set  $B = \beta = \varphi = 0$ . The system (19)-(21) reduces to a two-dimensional system in  $x$  and  $v$  given by

$$\dot{x} = x(x^{\alpha-1}(1-\psi) - x^\alpha + v + x^\alpha(1-\psi)) \quad (22)$$

$$\dot{v} = v(v - \rho + (1-\alpha)x^\alpha\psi(1-\phi) - \psi x^\alpha) \quad (23)$$

To get insight into our model we first solve (23)=0 with respect to  $v$  and insert that value in (22). Since we exclude the economically meaningless BGP with  $x^* = 0$  we can divide the resulting equation by  $x$  giving

$$f(x, \cdot) = \rho - (1-\alpha)x^\alpha\psi(1-\phi) + (1-\psi)x^{\alpha-1} \quad (24)$$

It is easily seen that  $\lim_{x \rightarrow 0} f(\cdot) = +\infty$ ,  $\lim_{x \rightarrow \infty} f(\cdot) = -\infty$  and  $\partial f(\cdot)/\partial x < 0$  holds, implying that there exists a unique positive  $x^*$  which solves  $f(\cdot) = 0$  and, thus, a unique BGP. It should be noted that this result is independent of the value for  $\phi$  determining the slope of the tax scheme as long as  $\phi < 1$ , i.e. as long as a regressive tax scheme is excluded. As concerns the local stability of the BGP it can easily be shown that it is a saddle point.<sup>4</sup>

---

<sup>3</sup>The \* denotes BGP values and we exclude the economically meaningless BGP  $x^* = z^* = v^* = 0$ .

<sup>4</sup>See the appendix.

In order to study growth effects of  $\phi$  we note that the balanced growth rate is given by equation (16). Differentiating (16) with respect to  $\phi$  gives

$$\frac{\partial(\dot{G}/G)}{\partial\phi} = (\alpha - 1)x^{\alpha-2}(1 - \psi)\frac{\partial x^*}{\partial\phi}. \quad (25)$$

The sign of  $\partial x^*/\partial\phi$  is obtained by implicit differentiation from (24) as

$$\partial x^*/\partial\phi = -(\partial f(\cdot)/\partial\phi)/(\partial f(\cdot)/\partial x) > 0.$$

This shows that a more progressive tax system leads to a smaller growth rate. It should be mentioned that, in contrast to Li and Sarte (2004), a more progressive tax scheme raises the ratio of public to private capital on the BGP. So, a more progressive tax system implies that the tax revenue rises and, as a consequence, the ratio of public to private capital increases. This effect raises the marginal product of private capital and tends to lead to a higher the growth rate. However, a more progressive tax system has a negative direct effect on the marginal product of private capital and it is this negative direct effect which dominates the positive indirect effect. Thus, a more progressive tax always reduces the long-run balanced growth rate. In particular, we cannot observe an inverted U-shaped relationship between the growth rate and the degree of progression as in the case of a flat rate income tax.

### 3.2 The model with public debt

With public debt, the system (19)-(21) completely describes the economy. Again, a BGP is a vector  $x^*, z^*, v^*$  such that  $\dot{x} = \dot{z} = \dot{v}$  holds. To derive a first result for our economy on the BGP we set  $\dot{v}/v = 0$  and solve that equation with respect to  $v$ . Inserting the resulting value in  $\dot{b}/b$  gives

$$h \equiv (\rho - \beta) + x^\alpha ((1 - \alpha)(1 - \psi(1 - \phi)) - \varphi(1 - \alpha + 1/b)) \quad (26)$$

Assuming that  $b$  is positive on the BGP, implying that the government is a debtor which is more realistic than the assumption that the government is a creditor, a first result

can be derived from (26). If the subjective discount rate of the household,  $\rho$ , is larger than the coefficient giving the increase in the primary surplus to a marginal increase in public debt,  $\beta$ , the parameter  $\varphi$  must be positive such that a BGP can exist. This means that the primary surplus must rise with a rising gross domestic income if the government does not raise the primary surplus sufficiently as public debt rises, i.e. if  $\beta$  is not sufficiently large. From an economic point of view, this seems plausible. If the government does not react to higher debt, it must raise the primary surplus as the gross domestic income increases in order to stabilize public debt. Otherwise, a BGP cannot exist. Of course, this condition does not hold if the government is a creditor, that is if  $b < 0$  holds.

In order to gain further insight into our model we resort to numerical examples. As a benchmark for our simulations we set the elasticity of production with respect to public capital to 25 percent, i.e.  $\alpha = 0.25$ .<sup>5</sup> The rate of time preference is set to 30 percent,  $\rho = 0.3$ . Interpreting one time period as 3 (5, 10) years then gives an annual rate of time preference of 10 (6, 3) percent.  $\psi$  is set to 0.9 implying an average tax rate of 10 percent.

Figure 1 shows the average tax rate (10 percent) and the marginal tax rate depending on the value of  $\phi$ . While the average tax rate is constant and only depends on  $\psi$ , which is set to  $\psi = 0.9$ , the marginal tax rate rises with the value of  $\phi$ . For example, for  $\phi = 0.1$  the marginal tax rate is about 20 percent and for  $\phi = 0.4$  the marginal tax rate is roughly 45 percent.

---

<sup>5</sup>For a survey of empirical studies giving estimates for that parameter see e.g. Pfähler (1996).

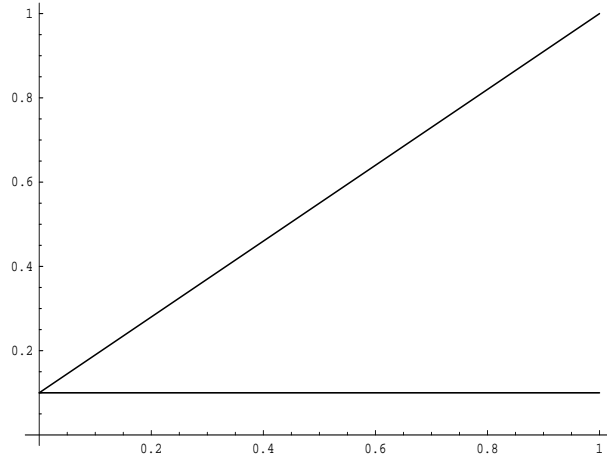


Figure 1: Average tax rate (constant) and marginal tax rate (rising) as a function of  $\phi$ .

In table 1 we next report results of our simulations for  $\beta = 0.15$  and  $\beta = 0.5$  and for different values of  $\phi$  with  $\varphi = 0.015$ , where  $\gamma$  denotes the balanced growth rate. Recall that  $\varphi > 0$  means that a higher gross domestic income raises the primary surplus.

Table 1

	$\beta = 0.15$				$\beta = 0.5$			
$\phi$	$b^*$	$x^*$	$\gamma$	eigenvalues	$b^*$	$x^*$	$\gamma$	eigenvalues
$\phi = 0$	0.06	0.29	0.195	(+,+,-)	-0.08	0.43	0.246	(+,-,-)
$\phi = 0.1$	0.05	0.36	0.17	(+,+,-)	-0.16	0.61	0.232	(+,-,-)
$\phi = 0.2$	0.04	0.46	0.144	(+,+,-)	no BGP			

Table 1 shows that higher values of  $\phi$  imply a smaller balanced growth rate although the ratio of public capital to private capital,  $x^*$ , rises. Thus, the result obtained for the model with a balanced government debt also seems to hold for the model with public debt. On the one hand, a more progressive tax system leads to a higher ratio of public to private capital as table 1 shows. On the other hand, however, a more progressive tax scheme by itself has a distortionary effect and makes the household reduce investment.

As in the model without public debt it is the negative direct effect which dominates and, thus, reduces the long-run balanced growth rate. Further, one also realizes that with  $\beta = 0.5$  no BGP with sustained growth exists for  $\phi \geq 0.2$ .

The column ‘eigenvalues’ in table 1 gives the sign of the eigenvalues of the Jacobian matrix evaluated at the BGP. If there is one negative real eigenvalue, (+,+,-), there exists a one dimensional stable manifold. If one takes  $x(0)$  and  $z(0)$  as given, this implies that the set of initial conditions  $\{x(0), z(0), v(0)\}$  lying on the stable manifold has Lebesgue measure zero. In this case the economy can converge to the BGP in the long-run only if the government levies a lump-sum tax at  $t = 0$  which is used to control  $B(0)$  implying that  $B(0)$ , and thus  $z(0)$ , are not fixed at  $t = 0$ .  $B(0)$  and  $C(0)$ , then, must be chosen such that  $z(0)$  and  $v(0)$  lie on the stable manifold and these values are uniquely determined. If there are two negative eigenvalues and  $B(0)$  is fixed the equilibrium is again determinate in the sense that there exists a unique  $C(0)$ , which can be chosen freely, so that the economy converges to the BGP. If both  $B(0)$  and  $C(0)$  can be chosen at  $t = 0$  there exists a continuum of initial values implying that the equilibrium is indeterminate.

For sake of completeness, we should mention that for  $\beta = 0.4$  no BGP exists for  $\phi \geq 0.05$ . In the next table we make simulations where we set  $\varphi = -0.015$ , implying that the primary surplus negatively depends on gross domestic income.

Table 2

	$\beta = 0.15$				$\beta = 0.5$			
$\phi$	$b^*$	$x^*$	$\gamma$	eigenvalues	$b^*$	$x^*$	$\gamma$	eigenvalues
$\phi = 0$	-0.05	0.4	0.238	(+,+,-)	0.08	0.26	0.184	(+,Re-,Re-)
$\phi = 0.1$	-0.04	0.48	0.206	(+,+,-)	0.11	0.24	0.126	(+,Re-,Re-)
$\phi = 0.2$	-0.04	0.6	0.174	(+,+,-)	0.15	0.15	0.035	(+,Re-,Re-)
$\phi = 0.3$	-0.03	0.77	0.143	(+,+,-)	0.13	0.06	-0.064	(+,Re+,Re+)

From a qualitative point of view, the results are the same as those in table 1. So, a more

progressive tax scheme reduces the balanced growth rate and destabilizes the economy.<sup>6</sup> Further, a higher value of  $\beta$  stabilizes the economy. So, the differential equation system is stable for  $\beta = 0.5$  (except for  $\phi = 0.3$ ) while it is unstable for  $\beta = 0.15$  if one takes  $B(0)$  as fixed.

## 4 Conclusion

This paper has presented an endogenous growth model with public capital and progressive income taxation. The analysis of the model has shown that a more progressive tax schedule leads to a higher ratio of public to private capital but to a smaller balanced growth rate because the direct negative growth effect of a more progressive tax system outweighs the indirect positive growth effect of a relatively higher public capital stock. So, there does not exist an inverted U-shaped relation between the growth rate and the parameter determining the slope of the tax schedule as in the model with a flat rate income tax. For the model with a balanced government budget, this could be shown analytically while for the model with public debt we derived this result using numerical examples.

As concerns stability of the system, the model without public debt is characterized by a unique balanced growth path which is saddle point stable, independent of tax parameters. In the version where we allowed for public debt, numerical examples showed that sustained per-capita growth may not be feasible when the degree of progression exceeds a certain threshold. Further, it also turned out that the system may become unstable with more progressive tax schedules, unstable in the sense that number of negative eigenvalues or eigenvalues with negative real parts decreases.

---

<sup>6</sup>Re- (Re+) means that the eigenvalue is complex with negative (positive) real part. Note that the system does not undergo a Hopf bifurcation as  $\phi$  is varied.

## References

- Arrow, K.J. and M. Kurz (1970) *Public Investment, the Rate of Return, and Optimal Fiscal Policy*, Baltimore: The John Hopkins Press.
- Aschauer, D.A. (1989) "Is Public Expenditure Productive?" *Journal of Monetary Economics*, Vol. 23, pp. 177-200.
- Barro, R.J. (1990) "Government Spending in a Simple Model of Endogenous Growth", *Journal of Political Economy*, Vol. 98, pp. S103-25.
- Bohn, H. (1995) "The sustainability of budget deficits in a stochastic economy", *Journal of Money, Credit and Banking*, Vol. 27, pp. 257-271.
- Bohn, H. (1998) "The behaviour of U.S. public Debt and deficits", *Quarterly Journal of Economics*, Vol. 113, pp. 949-963.
- Futagami, K., Y. Morita and A. Shibata (1993) "Dynamic Analysis of an Endogenous Growth Model with Public Capital", *Scandinavian Journal of Economics*, Vol. 95, pp. 607-25.
- Greiner, A. (2005) "An endogenous growth model with public capital and sustainable debt", *Japanese Economic Review*, forthcoming.
- Greiner, A., U. Koeller and W. Semmler (2004) "Debt sustainability in the European Monetary Union: Theory and empirical evidence for selected countries", *CEM Working Paper*, no. 71 (<http://www.wiwi.uni-bielefeld.de/~cem/>).
- Guo, J-T. and K.J. Lansing (1998) "Indeterminacy and stabilization policy", *Journal of Economic Theory*, Vol. 82, pp. 481-90.
- Li, W., and P.-D. Sarte (2004) "Progressive taxation and long-run growth", *American Economic Review*, Vol. 94, pp. 1705-16.



Pfähler, W., U. Hofmann and W. Bönnte (1996) "Does Extra Public Capital Matter? An Appraisal of Empirical Literature", *Finanzarchiv N.F.*, Vol. 53, pp. 68-112.

Romp, W. and J. de Haan (2005) "Public capital and economic growth: a critical survey." *EIB Papers*, Vol. 10, no. 1.

Slobodyan, S. (2005) "Indeterminacy, sunspots, and development traps." *Journal of Economic Dynamics and Control*, Vol. 29, pp. 159-85.

Turnovsky, S.J. (1995) *Methods of Macroeconomic Dynamics*, Cambridge, Mass.: The MIT Press.

## A Stability of the model with a balanced government budget

The Jacobian matrix to (22)-(23) is given by

$$J = \begin{bmatrix} x \cdot \frac{\partial(\dot{x}/x)}{\partial x} & x \cdot \frac{\partial(\dot{x}/x)}{\partial v} \\ v \cdot \frac{\partial(\dot{v}/v)}{\partial x} & v \cdot \frac{\partial(\dot{v}/v)}{\partial v} \end{bmatrix}$$

where we have used that  $\dot{v}/v = \dot{x}/x = 0$  holds on the BGP. The elements of the Jacobian matrix can be computed as

$$\begin{aligned} x \cdot \frac{\partial(\dot{x}/x)}{\partial x} &= v \cdot \frac{\partial(\dot{G}/G)}{\partial x} - v \cdot \frac{\partial(\dot{K}/K)}{\partial x} = -\gamma - \alpha v \\ x \cdot \frac{\partial(\dot{x}/x)}{\partial v} &= v \cdot \frac{\partial(\dot{G}/G)}{\partial v} - v \cdot \frac{\partial(\dot{K}/K)}{\partial v} = x \\ v \cdot \frac{\partial(\dot{v}/v)}{\partial x} &= v \cdot \frac{\partial(\dot{C}/C)}{\partial x} - v \cdot \frac{\partial(\dot{K}/K)}{\partial x} = \alpha v(\rho - v)/x \\ v \cdot \frac{\partial(\dot{v}/v)}{\partial v} &= v \cdot \frac{\partial(\dot{C}/C)}{\partial v} - v \cdot \frac{\partial(\dot{K}/K)}{\partial v} = v \end{aligned}$$

where we have used that  $\gamma = \dot{C}/C = \dot{G}/G = \dot{K}/K$  holds at the BGP. For  $\gamma > 0$  it can be easily seen that the determinant of the Jacobian is given by  $\det J = -v(\gamma + \alpha\rho) < 0$ .

Since a negative determinant is necessary and sufficient for saddle point stability, the model with a balanced government budget is a saddle point for positive balanced growth rates  $\gamma$ .