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# Agent-based models of financial markets

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# Contents

# Executive summary

The fundamental objective of WP6 is to project and develop the financial core of EURACE, designing an artificial financial market where agents are engaged in sequential trading of assets. During the first year of the project a preliminary financial market model that makes significant steps towards the declared objectives has been theoretically elaborated. A prototype of the model has been implemented and tested on a grid of computers. The model has been conceived with an incremental approach (coherently with the whole project approach) and it has been therefore designed with a very general and flexible core, which is capable of hosting new features and different implementation. Moreover, considering that the EURACE project consists in the interaction of many different markets, the main effort has been to design a financial frame-work taking into account its potential interactions with the embedding economy. Thus, some economic processes that affect the financial market has been taken into consideration and modeled as exogenous processes. Once the different modules of EURACE will start to be integrated, the economic dynamics generated by other modules (i.e., the labor market, the goods market, the credit market), will substitute the current exogenous stochastic processes.

The main developing directions of the model, which are illustrated in detail in this deliverable, can be briefly summarized as:

- the design of an endogenous decision rule for saving/consumption that could be successfully integrated in the financial market
- the implementation of multi-asset model where a large number of stocks and bonds can be traded
- the design of a preferences structure that reflects the human psychological attitude rather then following optimization rules that are unfit for agent-based models
- the investigation of different price formation mechanisms
- the inclusion of a preliminary learning mechanism for households
- the incorporation of households aging, taking into account different profiles of consumption/saving according to the working or the retirement condition of households
- the study of a realistic policy decision making by firms, to be implemented when integrating the financial market with the goods and the credit markets

# Chapter 1

# The financial side of the EURACE Project

## Introduction

The interest devoted to agent-based design of financial markets in the last twenty years is testified by a large literature on the subject. Many of these studies share a number of common features. They generally concentrate on agents' behavior, comparing different trading strategies (Chiarella and Iori 2001; Hommes 2002; Cincotti et al. 2003; Raberto et al. 2003; LiCalzi and Pellizzari 2003), and implement some learning mechanisms covering different hypotheses such as zero-intelligence random traders (Raberto et al. 2001), genetic algorithms (Arifovic 1996), or neural networks (Arthur et al. 1997). The main goal of these studies is to reproduce the evolution of market prices and to capture some empirical stylized facts such as volatility clustering of price returns or fat tails in the distribution of returns (see LeBaron 2006; Samanidou et al. in press for recent surveys).

What is important in the EURACE project, and has been scarcely investigated in the field of agent-based modelling, is the interaction between the financial market and the embedding real economic system (the production side of the economy). Just recently the scientific community has begun confronting this issue (see Raberto et al. 2006 for a contribution). In this report, we address this issue in Chapter 6 where the objective is to provide a model for the financial policy decisions of firms, and so to provide the link between the economic fundamentals and the financial variables.

Modelling the connection between macroeconomic variables and the financial market has been a goal in financial economics for a long time (see Lettau and Ludvigson 2001) and this connection is supported by the evidence that returns of common stocks appear to vary along the business cycle. In recent years increasing attention has been focused on this topic and many empirical studies show that a range of financial and macroeconomic variables can be used to predict stock market returns (see Campbell 1987; Pesaran and Timmermann 1995; Chen 1991). Standard stock valuation models point out that stock prices are determined as the discounted value of expected future cash flows. It has been shown that there are strong correlations between real economic activities, the interest rate and stock market returns (Fama 1990). Most of the earlier studies consider the short-run relationship between financial and macroeconomic variables, which may remove important information contained in the permanent component of economic activity. Conversely, in the field of agent-based modelling of financial markets, there has been relatively little attention from the scientific community to the long-run relationships between stock market returns and real economic variables (Nasseh and Strauss 2000). Results support the existence of a long-term relationship between stock prices and interest rates, consumer prices, real domestic macroeconomic innovations and international activity. Thus, stock prices and their returns are related to the underlying economic activity in the medium and long run.

This document presents the models considered for the financial decision making of the economic agents in the EURACE Project. The economic agents considered consist of: households, firms (corporations), banks, asset management companies, the Central Bank and Government. The models considered are partial models (i.e., they are not fully integrated with the other market models of the overall EURACE framework), with some of the relevant economic aspects such as labor income or dividend cash flows being modelled as exogenous stochastic processes.

This is in line with our general modelling strategy in which we model each market as a separate submodule that can be interfaced with the other market modules, or can be switched on/off by effectively replacing the market mechanism by random processes that serve as inputs to the other markets.

Nevertheless, these exogenous processes have a key role in determining the agents' behavior, playing an essential part in their decision making, and thus are strongly integrated in the model dynamics. The endogenous modelling of the real economic variables (that are thus to be considered as purely exogenous stochastic processes in this document), is addressed by other workpackages of the EURACE Project. Future work will address the issue of further integration of the different market models.

## 1.1 Financial decision making by economic agents

### 1.1.1 Households

The system includes a large number of households. The households are simultaneously taking the roles of workers, consumers and market traders. Households receive, if employed, a labor income at a common wage, and have to determine how much to spend and how much to save. Special attention is devoted to the savings-consumption decision which is modeled in reference to the theory on buffer-stock saving behavior (see Deaton 1991, 1992; Carroll 1997, 2001). In this theory, a consumer has an amount of total resources (called cash-on-hand following Deaton 1991), that is the sum of current wealth and labor income. Given this starting position, the consumption rule is a rule-of-thumb which depends on the expectations of both future labor income and future financial wealth, thus considering both the real economy and financial markets. Once households have determined the amount of money to save, they can either invest their savings in the asset market, by trading stocks or bonds, or they can put it in a savings account that pays a fixed, risk-free interest rate. Firms' stocks yield dividends that are correlated with the economic trend, while bonds pay a fixed coupon. Special attention is devoted to the belief formation process that takes into account economic aspects when forming expectations about asset returns and may depend on many heterogeneous parameters that might be specific to each trader. It is worth noting that beliefs are derived from observations of both the financial side and the real side of the economy which establishes an endogenous integration between the two parts of the economy.

### 1.1.2 Corporations

Four types of corporations are considered:

- Investment goods producers (IGPs),
- Consumption goods producers (CGPs),
- Asset management companies (AMCs),
- Banks.

The financial structure of corporations is divided into equity and debt. The ownership of a corporation is separated from its control. Stock emission is the equity instrument available to each kind of corporation. As a debt instrument producers can resort to bank loans and issue corporate bonds, while banks can only issue corporate bonds. The market value of claims on a corporations future profits is not necessarily independent from its financial structure, i.e., the Modigliani-Miller theorem does not apply.

The equity of corporations is divided into shares that households may posses and trade. Corporations may posses and trade shares of other corporations under some restrictions as well. Shares can be exchanged in a centralized financial market or by bilateral trading.

In the next section two implementation schemes are proposed, a basic one and a more advanced one.

### 1.1.3 The Central Bank

The central bank implements monetary policy decisions using the available monetary policy instruments. The most important set of operations available to the Eurosystem consists of open market operations and, among open market operations, refinancing operations represent a key monetary policy instrument. Through refinancing operations the central bank lends funds to its counterparties. Following the current procedures of the European Central Bank, in EURACE the refinancing operations will be executed through variable rate tenders, where the central bank sets a minimum rate bid in order to signal the monetary policy position. Starting from current results in the literature a great effort will be dedicated to the conceptualization, implementation and analysis of monetary policy strategies using the minimum rate bid as the operational instrument.

### 1.1.4 The Government

The government runs a monthly financial budget. In the current model, government income is given by direct taxes, both on labor and capital income, while government expenditures are made by the funding of a public pension scheme and the interest rates payment on the government debt. The government may issue both short-term or long-term bonds in order to finance the budget deficit.

## 1.2 Financial schemes

### 1.2.1 The basic financial scheme

In the basic scheme (see Figure 1.1), three types of financial assets are considered:

- Stocks;
- Government bonds;
- Exchange Traded Funds (ETF) issued by AMCs.

In the basic scheme, asset trading is centralized in the financial market and short selling is not allowed. Households and AMCs trade in the financial market regularly and for speculative purposes. In general, financial assets can be owned and traded only by the households and AMCs.

Banks and firms may own and trade common stocks only for particular operating purposes; banks may agree with firms to transform debt into new equity, and can later sell the new firms' equity in the market. Firms trade only in their own stocks, raise new financial capital, or buy back shares.

### 1.2.2 The advanced financial scheme

In the advanced scheme (see Figure 1.2), the following assets are considered:

- Stocks

- Government bonds
- Corporate bonds
- Exchange Traded Funds (ETF) issued by AMCs
- Hedge funds long-short equity

Short selling is only allowed for the AMCs. In general, assets can be traded in a centralized financial market or by means of bilateral agreements. Households and AMCs may posses any kind of financial assets but trade for speculative purposes has to go through the centralized market. They are not allowed to trade through bilateral agreements. Apart from the operating reasons for banks described in the basic scheme, new operating reasons for banks and producers are considered, namely mergers and acquisitions. Banks, producers and AMCs may possess and trade stocks of other corporations of the same type. Banks and producers can trade stocks in the centralized financial market or by means of bilateral agreements.

## 1.3 The economic setting

### 1.3.1 Timing

We assume that the financial market opens on a daily basis, i.e., the minimum time step $t$ of the system corresponds to one day of trading activity. At each time step the asset prices are updated, while stock dividends, bond coupons and interests on the savings account are distributed every $k$ daily time steps, typically on a quarterly or semestrial basis.
On the other hand, the dynamics of exogenous economic variables, such as dividends, wages, and interest rates, are updated using a monthly frequency, denoted by $\tau$, which refers to the first trading day of each month. We assume each month consists of $D$ trading days.

### 1.3.2 The exogenous economic variables

Households receive a common labor wage $w_\tau$ that evolves according to an exogenous stochastic process, reflecting the general conditions of the economy. In particular, it is supposed that there is a positive correlation between labor income and stock dividends. Households further receive a capital income, composed of stock dividends and bond coupons (bonds pay a fixed coupon). Furthermore, households receive a risk-free interest rate $r_\tau$ on their savings account. This risk-free interest rate is a policy decision variable of the Central Bank.
Dividends evolve according to an exogenous stochastic process, i.e.,

$$\log d^a_\tau = \log d^a_{\tau-1} + g^a + \xi^a_\tau \sigma^a \tag{1.1}$$

where $g^a$ is the characteristic growth rate of the dividend of asset $a$, $\sigma^a$ is its characteristic standard deviation, and $\xi^a_\tau$ is a gaussian noise term with zero mean and unitary variance affecting the process at day $\tau$. All noise terms in the dividend generation process are correlated according to a given covariance matrix. It is worth noting that also the labor wage is correlated with dividends. This is due to the fact that we suppose that firms' profits and households' income come from the same economic environment and are therefore connected.

The stochastic process for wage updates is as follows:

$$\log w_\tau = \log w_{\tau-1} + g^w + \xi_\tau^w \sigma^w \,, \tag{1.2}$$

where $g^w$ is the wage growth rate, $\sigma^w$ is its characteristic standard deviation, and $\xi_\tau^w$ is a gaussian noise with zero mean and unitary variance, correlated with $\xi_\tau^a$.

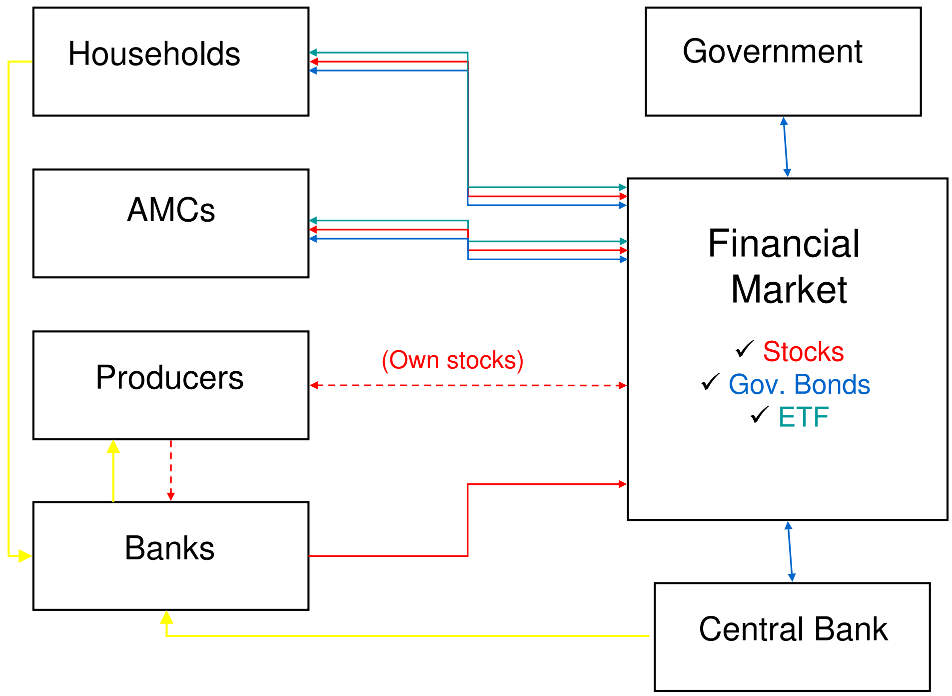


Figure 1.1: Basic Financial Scheme for EURACE.

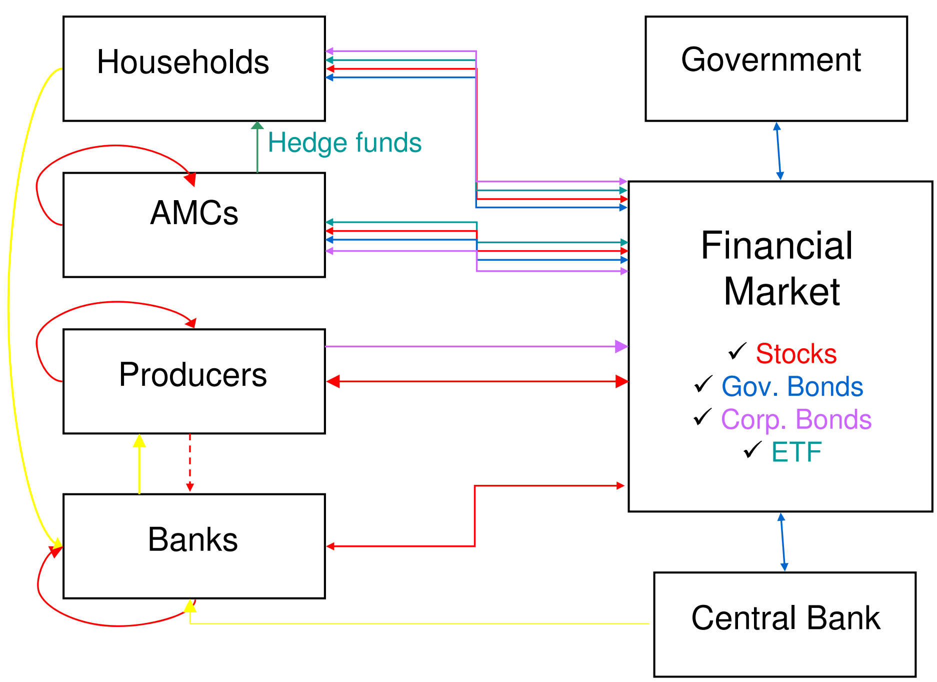


Figure 1.2: Advanced Financial Scheme for EURACE.

# Chapter 2

# A model of households' consumption and savings decisions

## Introduction

Households' savings/consumption decisions encompass three main motives that have been identified in recent years as quantitatively important for explaining individual saving behavior:

1. a precautionary savings motive, in order to protect consumption from unexpected shocks, e.g., unemployment, when subject to uninsurable labor income, or other kinds of risks, e.g., illness, for which insurance markets are often limited or do not exist;

2. a retirement or life-cycle savings motive, in order to integrate the pension income which is usually lower than mean working-life labor income;

3. a bequest motive.

See e.g. Gourinchas and Parker (2002); Cagetti (2003) for a detailed overview of the subject.

In the EURACE model, savings/consumption decisions are taken on a monthly basis, at the first trading day of each month after wages and capital incomes are paid. We will denote by $\tau$ the monthly time-index and by $\upsilon$ the yearly time-index.

## 2.1 Consumption during working age

### Precautionary saving

Precautionary saving has been modelled according to the buffer stock theory of saving (Deaton 1991; Carroll et al. 1992; Carroll 1997). The main idea is that households accumulate a buffer stock wealth, consisting of cash and liquid assets, to self insure against bad draws of labor income or other kinds of risks. Furthermore, household behavior depends on a target level of wealth to income ratio; if the buffer stock falls below this target households consume less than their expected income and holdings of liquid assets will increase, while if the holdings of liquid assets are in excess of their target households will spend freely and liquid asset holdings will diminish.

### Retirement saving

Retirement saving $I_\tau^i$ by each household $i$ is modelled by allowing households to invest a fraction of their net labor wage $\bar{w}_\tau$ in quotas of a pension fund, given by:

$$I_\tau^i = \nu_\tau^i \bar{w}_\tau \,, \tag{2.1}$$

where $\nu_\tau^i$ is a household decision variable, subject to learning and which is time dependent. $\bar{w}_\tau$ is net labor income of each household, given by:

$$\bar{w}_\tau = (1 - \alpha^w) w_\tau \,, \tag{2.2}$$

where $\alpha^w$ is the labor tax rate and is the gross labor wage evolving according to Eq. 1.2. Accordingly, the labor income tax $T_\tau^{(w)}$ payed monthly by each household is given by:

$$T_\tau^{(w)} = (1 - \alpha^w) w_\tau \,. \tag{2.3}$$

Tax deductible retirement saving is considered in order to give an incentive for saving for old age. Saving for retirement allows to detract from labor taxes an amount $T_\tau^{(d)i}$ given by:

$$T_\tau^{(d)i} = \alpha^w \min(\bar{I}, I_\tau^i) \,, \tag{2.4}$$

where $\bar{I}$ is the maximum amount of retirement saving which is allowed to be tax deductible. The pension fund is a not-for-profit company which invests the receipts in the financial market, buying different financial assets according to a given portfolio allocation strategy.

### The definition of cash-on-hand

Following Deaton (1991), we define the 'cash-on-hand' $X_\tau^i$ of household $i$, in a given month $\tau$, which is the maximum amount that can be spent by the household during that month, provided that the household does not borrow money to finance consumption. Cash-on-hand is then given by the sum at the beginning of the period of financial wealth and income, consisting of both labor and capital income, which are supposed to be payed on the first day of every month. The start-of-period wealth $\mathcal{W}_{\tau^*}$ is given by the sum of the market value of liquid assets and liquidity under the form of a savings account $S^i_{\tau^*}$, as follows:

$$\mathcal{W}_{\tau^*} = \sum_a q^{a,1}_{\tau^*} P^a_{\tau^*} + S^i_{\tau^*} \,, \tag{2.5}$$

where $q^{a,1}_{\tau^*}$ is the holding of asset $a$ by agent $i$ and $P^a_{\tau^*}$ is its market price. It is worth noting that all financial variables, i.e., wealth, assets holdings and prices, and the savings account, are evaluated at a daily frequency; in this respect the monthly index $\tau^*$ refers to the value of financial variables at the beginning of month $\tau$, following the transactions that occurred on the last trading day of month $\tau - 1$. Cash-on-hand $X^i_\tau$ is then given by:

$$X^i_\tau = \mathcal{W}^i_{\tau^*} + \varsigma^i_\tau \bar{w}_\tau + \bar{\pi}^i_\tau \,. \tag{2.6}$$

The terms $\varsigma^i_\tau \bar{w}_\tau$ and $\bar{\pi}^i_\tau$ refer to the net labor income and net capital income during month $\tau$, respectively. $\varsigma^i_\tau$ is a memory-less stochastic variable related to the household's unemployment state. $\varsigma^i_\tau$ can assume the value zero with probability $\mu$, meaning that the household is unemployed, and the value 1 with probability $1 - \mu$; $\mu$ is supposed to be an exogenously given unemployment probability.
The gross capital income $\pi^i_\tau$ is given by:

$$\pi^i_\tau = \sum_a q^{a,1}_{\tau^*} \Pi^a_\tau + r^S_{\tau^*} \,, \tag{2.7}$$

where $\Pi^a_\tau$ denotes the generic cash flow at month $\tau$ relative to the $a$-th asset, i.e., dividends for stocks and coupons for bonds, which are supposed to be payed, if present, on a monthly basis and during the first trading day of each month. $r^{(S)i}_{\tau^*}$ is the interest income received by each household on its savings account. Interests are payed on a quarterly basis at the beginning of the first month of the new quarter.
The net capital income is derived from the gross capital income as follows:

$$\bar{\pi}^i_\tau = (1 - \alpha^\pi) \pi^i_\tau \,, \tag{2.8}$$

where $\alpha^\pi$ is the capital income tax rate.

### The consumption function

Let us denote with $C^i_\tau$ the real consumption budget for month $\tau$ of household $i$. The monthly dynamics of cash-on-hand $X^i_\tau$ during working age is given by:

$$X^i_{\tau+1} = R^i_\tau \big( X^i_\tau - C^i_\tau - I^i_\tau + T^{(d)i}_\tau \big) + \varsigma^i_{\tau+1} \bar{w}_{\tau+1} \,, \tag{2.9}$$

where $R^i_\tau$ represents the gross total return of month $\tau$ savings, i.e., $X^i_\tau - C^i_\tau - I^i_\tau + T^{(d)i}_\tau$. $R^i_\tau$ includes both capital gains and after tax asset cash flows and interests on the savings account.
Let us now consider the ratio $x^i_t$ between cash-on-hand and permanent labor income, i.e.,

$$x^i_\tau = X^i_\tau / w_\tau \quad \forall \tau. \tag{2.10}$$

Defining the consumption-to-income ratio $c^i_\tau$ as $c^i_\tau = C^i_\tau / w_\tau \ \forall\tau$, and considering the beginning of the month conditional expectations $E^i_{\tau^*}$ by household $i$, Eq. 2.9 now becomes

$$E_{\tau^*}(x^i_{\tau+1} w_{\tau+1}) = E_{\tau^*} R^i_\tau \left((x^i_\tau - c^i_\tau) w_\tau - I^i_\tau + T^{(d)i}_\tau\right) + (1-\mu) E_{\tau^*} w_{\tau+1}\,, \tag{2.11}$$

where $E_{\tau^*} R^i_\tau$ is the conditional expectation at month $\tau$ of the monthly portfolio gross return[1] and $(1-\mu)$ is the conditional probability[2] to be employed. By imposing the steady-state solution, i.e., $x^i_\tau = \bar{x}^i$ and $c^i_\tau = \bar{c}^i \ \forall\tau$, we obtain a relation between the target levels of the consumption-to-income ratio and the cash-on-hand to income ratios as follows:

$$\bar{c}^i = \frac{(w_\tau\, E_\tau R^i_\tau - E_\tau w_{\tau+1})\bar{x}^i + (1-\mu) E_\tau w_{\tau+1} - E_\tau R^i_\tau (I^i_\tau - T^{(d)i}_\tau)}{w_\tau\ E_\tau R^i_\tau}\,. \tag{2.12}$$

Following Allen and Carroll (2001), we consider a first-order Taylor expansion of the consumption-to-income function $c(x^i_\tau)$ around the steady state solution $\bar{x}^i$, i.e.,

$$c(x^i_\tau) = \bar{c}^i + m(x^i_\tau - \bar{x}^i)\,, \tag{2.13}$$

where $m$ is the marginal propensity to consume, which is considered agent independent. Substituting Eq. 2.12 in Eq. 2.13, and multiplying by the permanent labor income $w_\tau$, one obtains the approximate consumption function $\widehat{C}^i_\tau$ given by:

$$\widehat{C}^i_\tau = \frac{(w_\tau\, E_\tau R^i_\tau - E_\tau w_{\tau+1})\bar{x}^i + (1-\mu) E_\tau w_{\tau+1} - E_\tau R^i_\tau\, (I^i_\tau - T^{(d)i}_\tau)}{E_\tau R^i_\tau} + m(X^i_\tau - \bar{x}^i w_\tau)\,. \tag{2.14}$$

$\widehat{C}^i_t$ has some intuitive features; namely, it is an increasing function of the permanent labor income $w_\tau$ and of the ratio between expected labor income and expected portfolio gross return. Furthermore, it increases with the difference between actual cash-on-hand $X^i_\tau$ and its target value $\bar{x}^i w_\tau$. The target cash-on-hand to income ratio is then a main driver of consumption; empirical research (Carroll 1997) shows that the media household at every age before age 50 typically holds total non-housing net assets worth somewhere between a few weeks worth and a few months worth of labor income. It is worth noting that the value of $\bar{x}^i$ can be explained according to a model of household preferences.
Considering the liquidity constraint $X^i_t - I^i_\tau + T^{(d)i}_\tau$, the consumption rule becomes

$$C^i_\tau = \min\left(\widehat{C}^i_\tau, X^i_t - I^i_\tau + T^{(d)i}_\tau\right). \tag{2.15}$$

Finally, it is worth noting that Eq. 2.14 is an approximation that will be close to the true consumption rule in the neighborhood of the target level of cash-on-hand; if actual wealth tends to stay relatively close to target cash-on-hand, the approximation can be considered as relatively good.

[1] It is worth noting while expectation formation and consumption decision are referred to the beginning of each month, i.e., during the first trading day of month $\tau$, the portfolio gross return refers to the whole incoming month, i.e., to all trading days which and so $R^i_\tau$ it is unknown by the household at the time the expectation is considered.

[2] It is worth noting that, being $\varsigma^i_{\tau+1}$ a memoryless stochastic variable, the conditional probability to be unemployed is equal to the unconditional one.

### 2.1.1 Learning optimal consumption

Following the work of Allen and Carroll (2001), we considered the possibility for a household to learn at every time step its optimal target values of both $x$ and $\nu$, given a set of expectations about the wage growth rate, returns of investment and life expectancy. As a measure of optimality, we consider a standard intertemporal utility function $U$ computed as the discounted sum of instantaneous monthly utility functions defined on consumption. In particular, the instantaneous monthly utility functions $u_\tau$ are CRRA utility functions that are defined as follows:

$$u_\tau = \frac{C_\tau^{(1-\sigma)}}{1-\sigma}\,, \tag{2.16}$$

where $\sigma$ is the so-called coefficient of relative risk aversion, usually with values in the interval between 1 and 5. Intertemporal utility $U_{\tau^*}$ at any given month $\tau^*$ is defined as:

$$U_{\tau^*} = u_{\tau^*} + \beta \sum_{\tau=\tau^*+1}^{\tau^L} \delta^{\tau-\tau^*} u_\tau\,, \tag{2.17}$$

where $\tau^L$ is the expected last month of life, $\delta$ is the usual discount factor, and $\beta$ is a time inconsistency factor which for $\beta < 1$ models present-biased preferences.[3] Optimal target values for the cash-on-hand to income ratio and for the fraction of salary saved for retirement, $\bar{x}^o$ and $\nu^o$, can be computed at any given month as follows:

$$\bar{x}^o, \nu^o = \max_{\bar{x},\nu} U\,. \tag{2.18}$$

## 2.2 Consumption after retirement

We assume that households retire at age $T^L$.[4] Retired people receive both a public and a private pension income, denoted respectively by $y$ and $\widehat{y}$. The public pension scheme is funded by the government by means of taxation and deficit spending; the public pension income $y$ is household independent and is set to a fixed fraction $\xi$ of the last salary before retirement, i.e.,

$$y_\tau = \xi\, w_{\tau^R-1} \quad \text{for all } \tau \geq \tau^R\,, \tag{2.19}$$

where $\tau^R$ is the month of retirement. $\xi$ is a policy parameter set by the government. Private pension income $\widehat{y}$ is household dependent and is computed according to the market

[3] Standard economic models based on the rational choice hypothesis assume that intertemporal preferences are time-consistent, i.e., that people discount exponentially the utility for future consumption; however, there is wide empirical evidence suggesting that individuals are characterized by present-biased preferences, i.e., people discount delays in gratification more severely in the short term than in the long term. Qualitatively, people discount more than exponentially in the short run and less than exponentially in the long-run. A natural consequence of present-biased (or time inconsistent) preferences, (for example explained by hyperbolic discounting), is that a taste for immediate gratification can lead a person to sacrifice too much future consumption for the sake of current consumption. See Laibson (1997); Angeletos et al. (1998); Harris and Laibson (2001); O'Donogue and Rabin (1999, 2001); Frederick et al. (2001) for further details.

[4] The age of retirement is a policy parameter set by the government, e.g., in most Euro area countries people are allowed to retire at an age between 60 and 65.

value of each household's pension fund quotas at the time of retirement, $Z^i_{\tau^R}$, as follows

$$\widehat{y}^i_\tau = \varphi Z^i_{\tau^R} \,, \tag{2.20}$$

where $\varphi$ is a transformation coefficient determined according to an actuarial computation depending on the age of retirement $T^R$, and the household's life-time expectancy $T^L$. The dynamics of $Z^i_\tau$ during working age is given by

$$Z^i_{\tau+1} = R^{(PF)}_\tau Z^i_\tau + I^i_\tau \,, \tag{2.21}$$

where $R^{(PF)}_\tau$ is the gross return of the pension fund at month $\tau$.
It is supposed that retired households do not save at all and consume all their net income, consisting of both pension income and capital income, plus a fraction of their financial wealth, as follows:

$$C^i_\tau = y_\tau + \widehat{y}^i_\tau - T^{(y)}_\tau + \frac{W^i_\tau - B^i}{12(T^L - t + 1)} \,, \tag{2.22}$$

where $t$ is the current household's age in years, $B^i$ is the household's planned bequest, and $T^{(y)}_\tau$ is the amount of taxes paid on both the public and private pension income. The denominator $12(T^L - t + 1)$ is the household's estimate of its remaining lifespan in months.

## 2.3 Computational evidence of the model

In the following, we present the dynamics of the main economic and financial variables which is expected by each household during its average lifespan according to our consumption/savings model. We want to investigate how these expectations critically depend on the value of both the given target ratio of cash-on-hand to income and on the ratio of consumption to income.

Figures 2.1–2.4 show trajectories for three different target values of the cash-on-hand to income ratio $\bar{x} \in \{1.5, 2.5, 3.5\}$, given the fraction of labor income saved for retirement $\nu$ set to zero. Initial conditions in terms of wealth are identical for the three cases considered. The expected values of the return on financial investments, the wage growth rate, the unemployment probability and the discount factor are all expected to be constant by the household during its entire lifespan. Results show that the three different values of $\bar{x}$ give significative differences in the lifetime trajectories for both wealth and cash-on-hand, considering that the labor and the pension incomes do not depend on them. However, only small differences are observable for what concerns the three consumption paths, except for the initial months after the decision is taken at age 35 and during after-retirement ages. The initial difference is due to the adjustment of consumption in order to let the cash-on-hand match the target ratio with the permanent labour income; the rate of monthly adjustment depends of course on the value of the marginal propensity to consume, $m$. The difference in the consumption paths during after-retirement ages depends much on the accumulated wealth during lifetime. Significant differences in $\bar{x}$ yield small differences in $\bar{c}$ during working ages, see Eq. 2.12 for the analytical details. The small differences in the consumption paths are reflected by small differences in the instantaneous utility paths. The intertemporal utility gives an overall measure of the likeness of different paths of consumption as the discounted sum of instantaneous future utilities. According to that measure, the best target value of the ratio of cash-on-hand to income among 1.5, 2.5, and 3.5, would be 2.5.

Figures 2.5–2.8 regard simulations where the fraction of salary saved for retirement $\nu$ varies assuming three possible values: $\nu \in \{0, 0.04, 0.08\}$, given the target value of $x$ fixed and set to 2.5. Different values of the fraction implies different consumption paths during the working ages and the retiremtn ages. The choice between less consumption now and more consumption during retirement or viceversa may be addressed by looking at the intertemporal utility. In the present simulations, $\nu = 0.04$ would be the value giving the higher intertemporal utility.

Figures 2.9–2.10 show the optimal target values of cash-on-hand for a household of 35 years old, maximizing its intertemporal utility, considering the target values constant during the entire household lifespan. Optimal values are reported with respect to four independent variables considered, namely the expected gross return of financial investment, the unemployment probability, the monthly wage growth rate, and the discount factor. Results show that both $\bar{x}^o$ and $\bar{c}^o$ exhibit a similar upward or downward trend dynamics within the considered range of the four independent variables. Furthermore, Figures 2.9 and 2.10 show sensible qualitative results corroborating the validity of the model. In particular, the optimal target values of the cash-on-hand and consumption to income ratio increase with the expected total return on investment and with the value of the discount factor. Indeed, if the household gives more weight to future consumption, then it is prone to save more during working ages and its $\bar{x}^o$ would be higher. Further, higher expected returns of investment foster consumption. On the other hand, higher expected wage growth rates reduce the necessity for precautionary savings and a higher unemployment probability affects consumption.
Finally, Figure 2.11 shows the optimal values of the fraction of the salary to be saved for retirement, $\nu^o$, with respect to four independent variables, namely, the expected gross return of financial investment, the unemployment probability, the monthly wage growth rate, and the discount factor. The optimal ratio of retirement saving to salary increases with the discount factor (the future is more important) and the expected gross return, while it decreases with the unemployment probability (there is not enough money for present consumption already) and the wage growth rate (future public pension income is expected to be high).

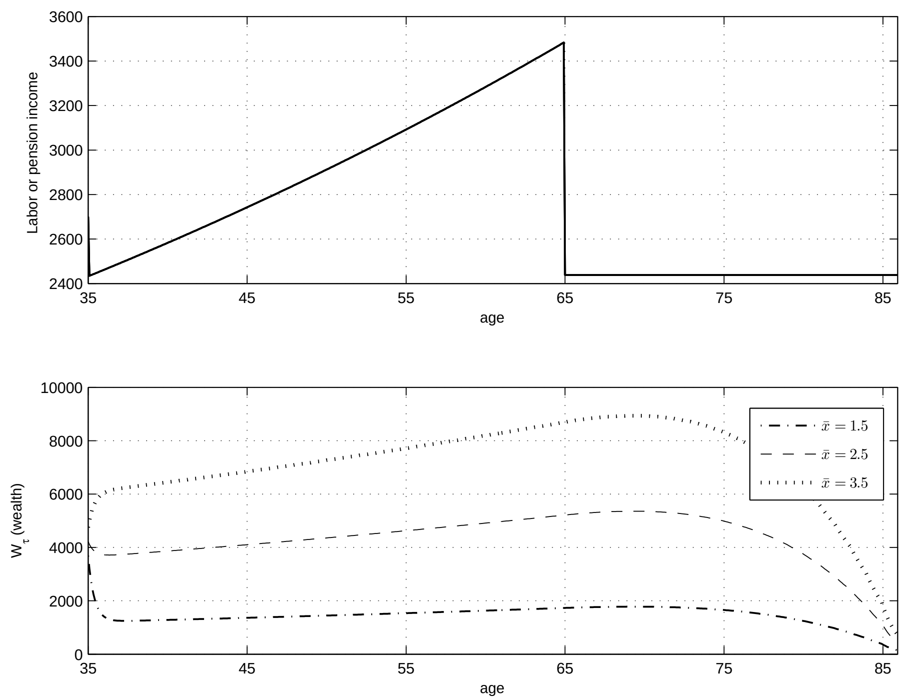


Figure 2.1: Expected dynamics of labor and pension income (top) and of financial wealth (bottom) during a household's average lifespan with respect to three different target values of the cash-on-hand to income ratio.

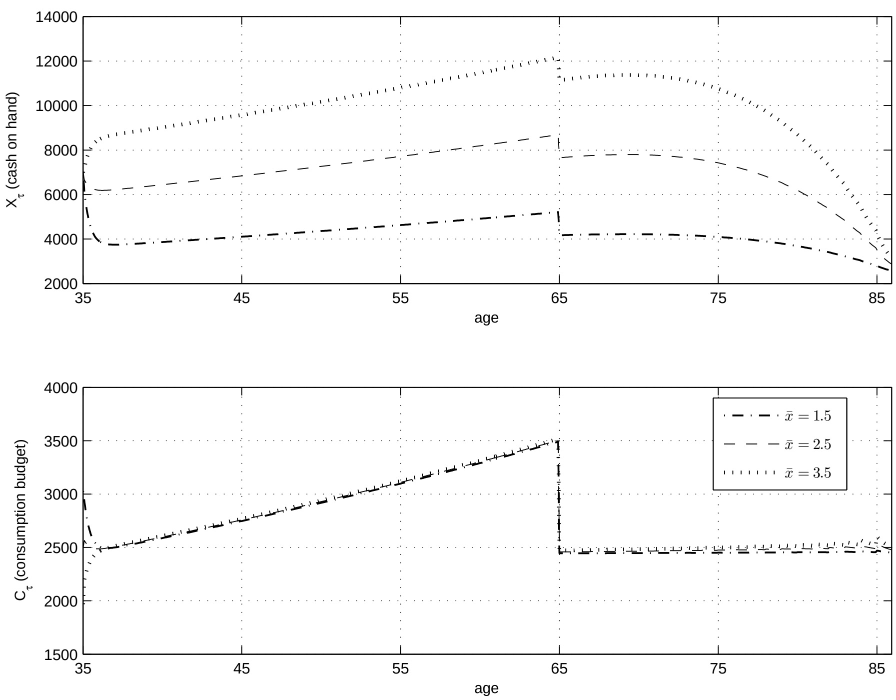


Figure 2.2: Expected dynamics of cash-on-hand (top) and consumption (bottom) during a household's average lifespan with respect to three different target values of the cash-on-hand to income ratio.

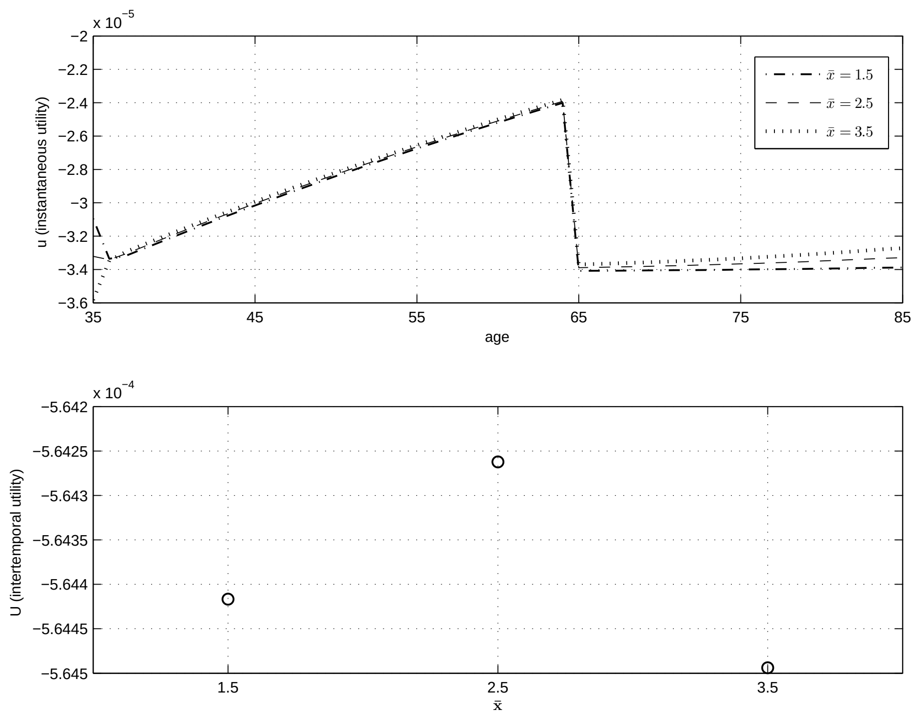


Figure 2.3: Expected dynamics of instantaneous utility (top) during a household's average lifespan and values of the intertemporal utility (bottom) for three different target values of the cash-on-hand to income ratio.

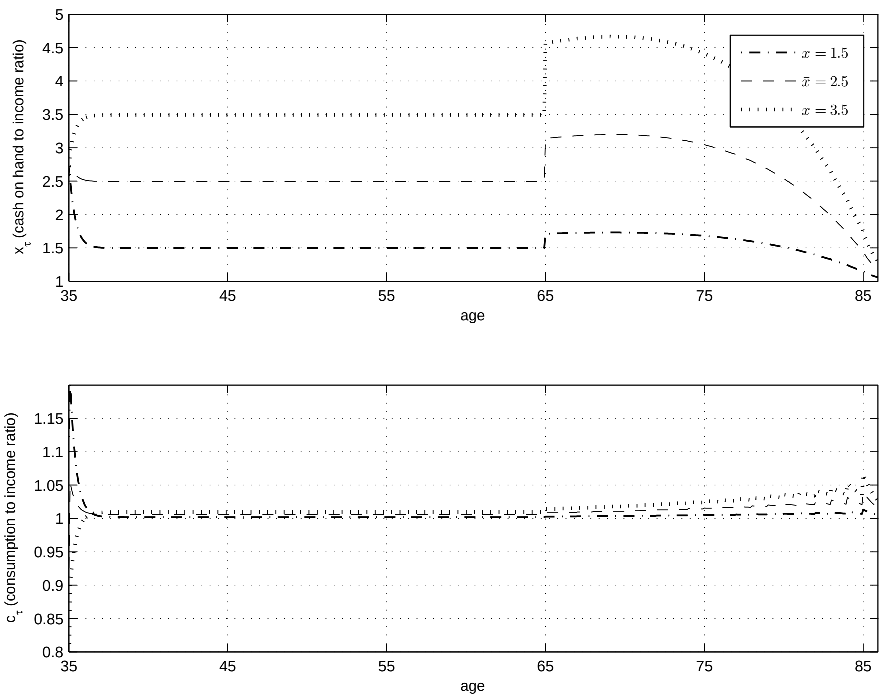


Figure 2.4: Expected dynamics of cash-on-hand to income ratio (top) and consumption to income ratio (bottom) during a household's average lifespan with respect to three different target values of the cash-on-hand to income ratio.

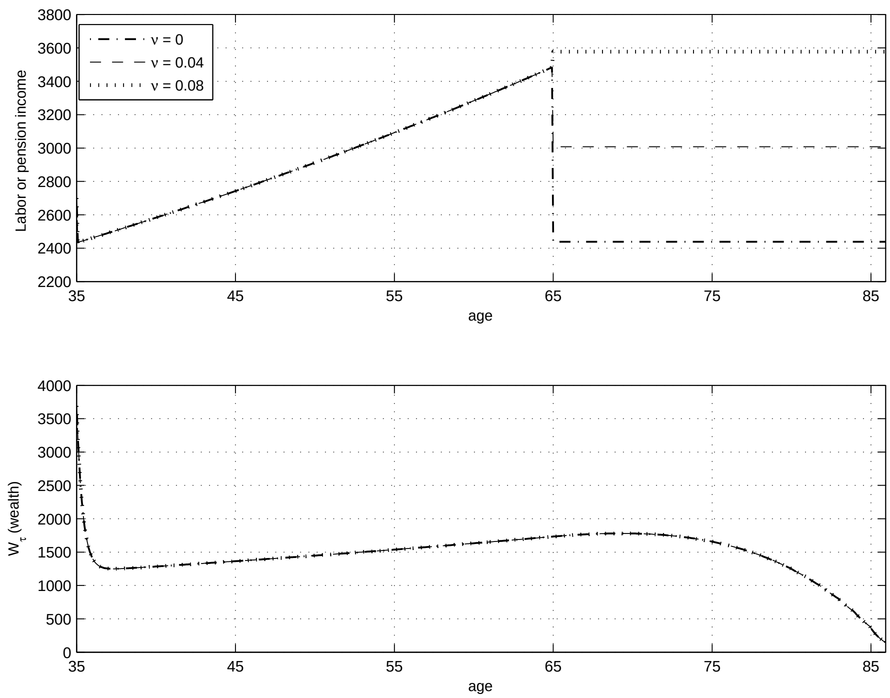


Figure 2.5: Expected dynamics of labor and pension income (top) and wealth (bottom) during a household's average lifespan with respect to three different fractions of labor income saved for retirement.

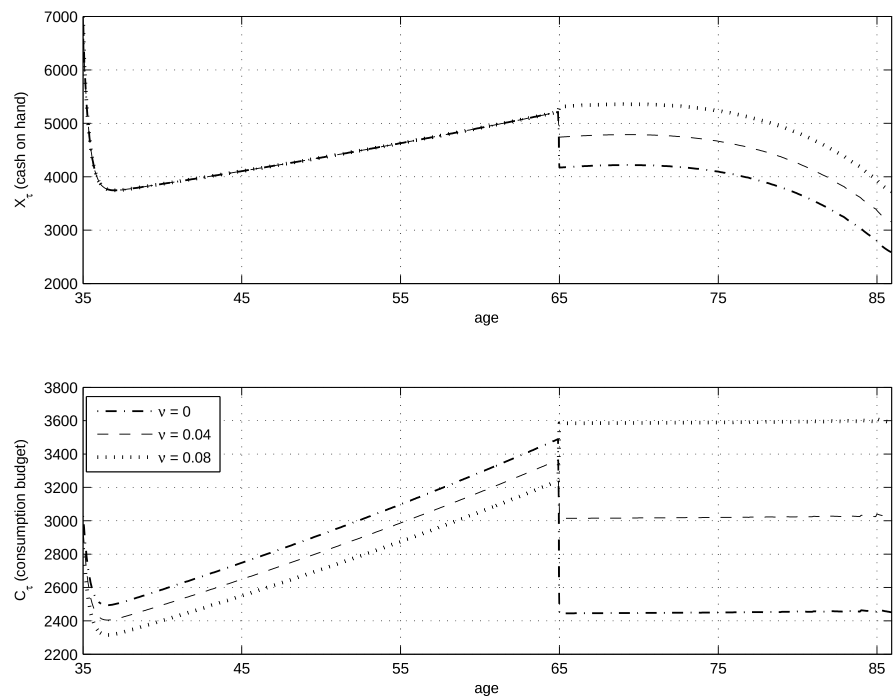


Figure 2.6: Expected dynamics of cash-on-hand (top) and consumption (bottom) during a household's average lifespan with respect to three different fractions of labor income saved for retirement.

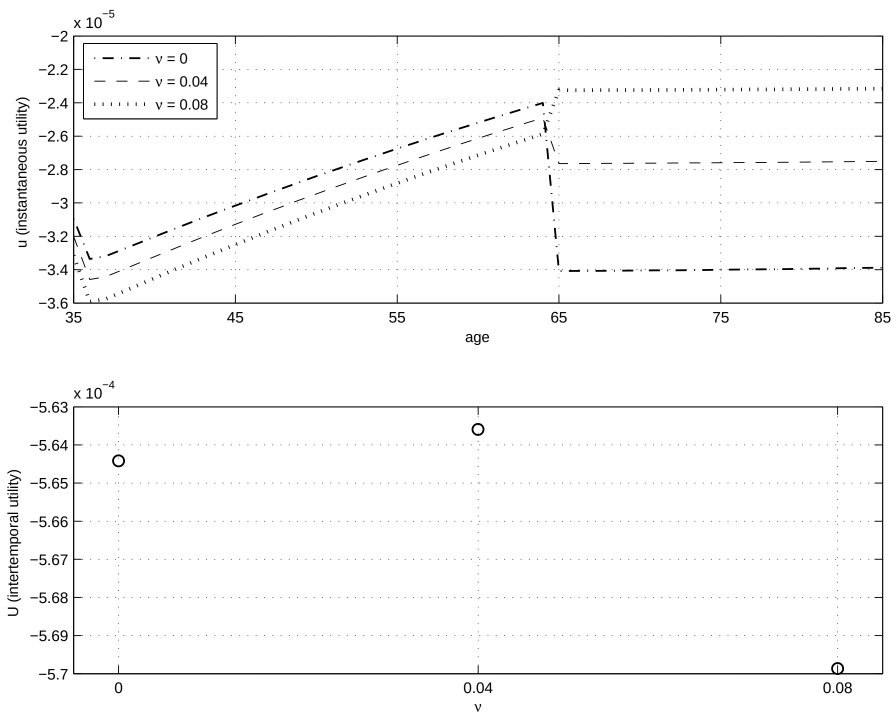


Figure 2.7: Expected dynamics of instantaneous utility (top) during a household's average lifespan and values of the intertemporal utility (bottom) for three different fractions of labor income saved for retirement.

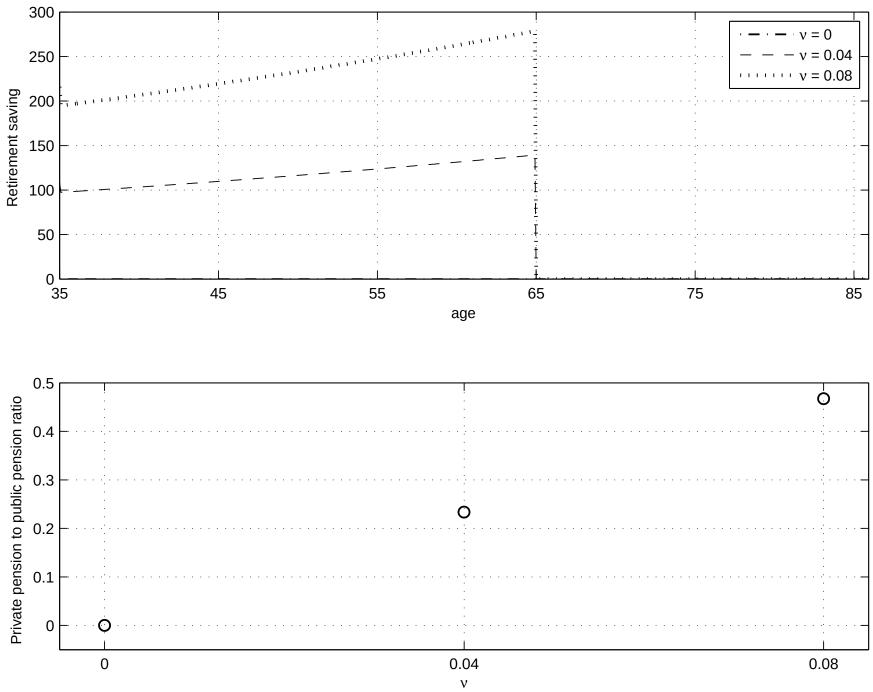


Figure 2.8: Expected dynamics of retirement saving (top) during a household average lifespan and values of the ratio of private to public pension income (bottom) for three different fractions of labor income saved for retirement.

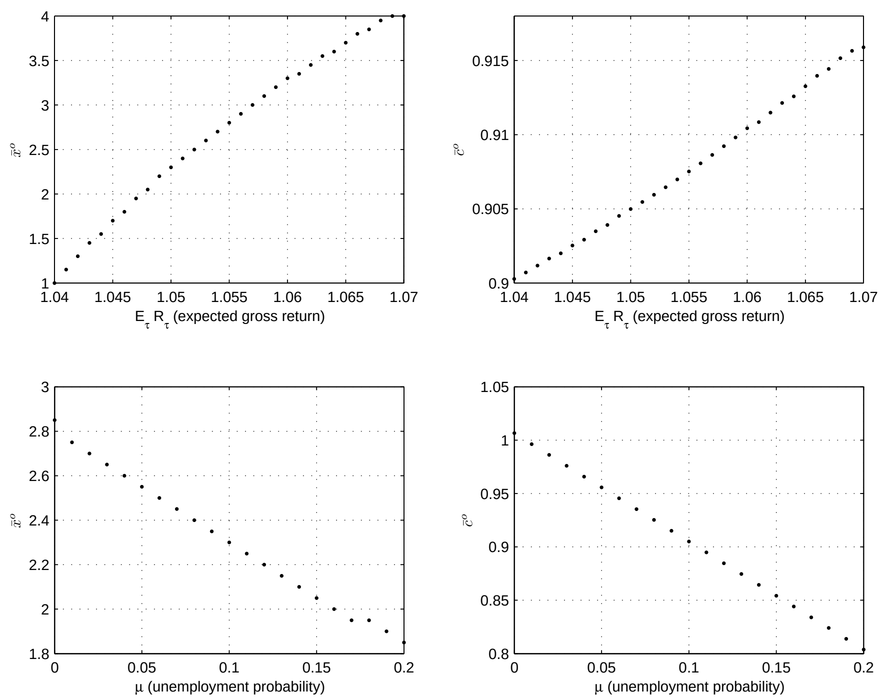


Figure 2.9: Optimal target values of the cash-on-hand to income ratio $\bar{x}^o$ and of the consumption to income ratio $\bar{c}^o$ with respect to the unemployment probability and to the expected gross return.

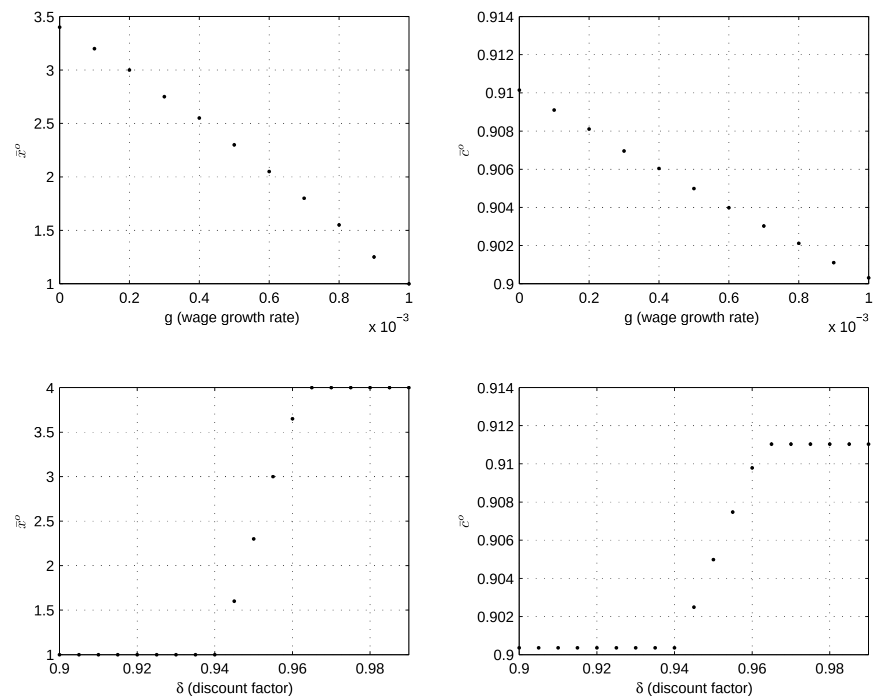


Figure 2.10: Optimal target values of the cash-on-hand to income ratio $\bar{x}^o$ and of the consumption to income ratio $\bar{c}^o$ with respect to the wage growth rate and to the discount factor.

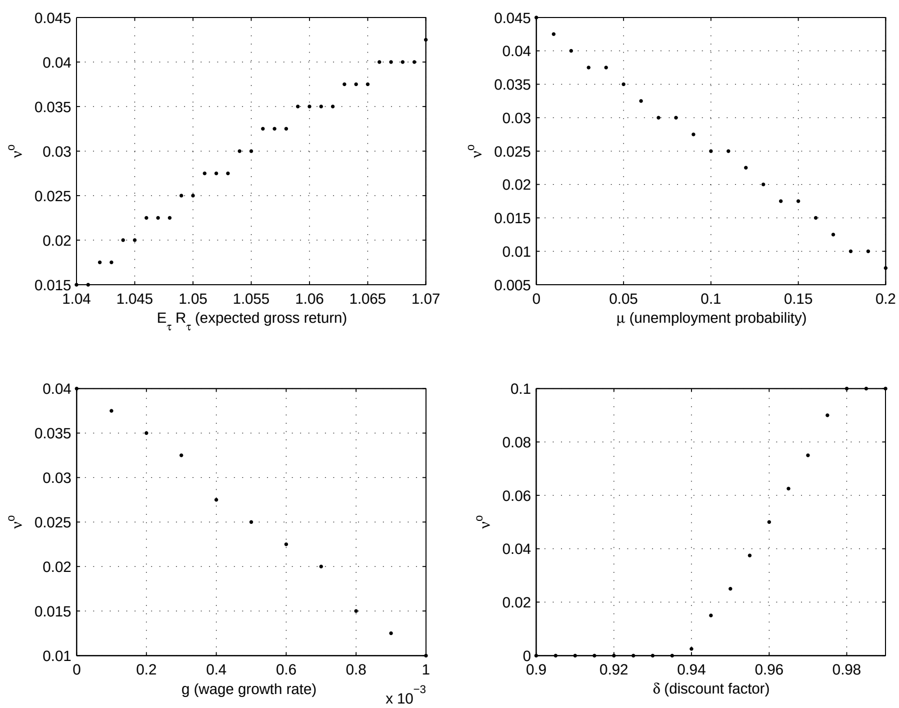


Figure 2.11: Optimal values of the fraction of salary to be saved for retirement with respect to four independent variables.

# Chapter 3

# A model of households' financial investment decisions

The asset market is populated by $N$ heterogeneous households indexed by $i$ and $M$ assets (i.e., stocks and bonds), indexed by $a$.
Denote by $\Pi_\tau^a$ the generic cash flow of asset $a$ on day $\tau$ (i.e., dividends for stocks and coupons for bonds), then total income $I_\tau^i$ at time $\tau$ for household $i$ is given by

$$I_\tau^i = \varsigma_\tau^i w_\tau + \sum_a \Pi_\tau^a + rS_{\tau-1}\,, \tag{3.1}$$

where $S_{\tau-1}$ is the savings account, and $\varsigma^i$ is a random variable representing the household's employment condition, that is modelled according to an exogenous memoryless stochastic process. $\varsigma_\tau^i$ may be either 1 with probability $(1-\mu)$ or 0 with probability $\mu$. If $\varsigma_\tau^i = 0$ the $i$-th household is unemployed at month $\tau$ and does not receive any labor income in this month.

## 3.1 Belief formation

Households have a given exogenous probability to update their beliefs about several financial rates of return at each daily time step $t$. In particular, they form expectations about three main classes of returns:

- Cash flow yields ($\bar{\rho}$);
- Price returns ($\hat{\rho}$);
- Total returns ($\rho$).

Each of these elements are expected rates on an annual base.
Each of the $N$ households is characterized by a memory corresponding to the dimension of a backward-looking time window $T^i$ through which the household looks at the past (i.e., $T^i$ is the number of previous daily time steps the $i$-th household considers as its historical time

series). Moreover, each household has a forward-looking time-horizon $h^i$ that indicates the household's forward time perspective (i.e, each household participates in the market with probability $1/h^i$, forming new beliefs and renewing its portfolio). The forward-looking horizon also fixes a forward-looking time window within which the household considers its future cash flows.

Given a cash flow $\Pi_t^a$, in order to calculate the return rate $\rho$ of an initial investment $K$, we use the concept of the internal rate of returns (IRR), defined as the annualized effective compounded rate of return that can be earned on the invested capital. I.e., $\rho$ is the return rate for which the following condition holds:

$$K = \sum_{t=1}^{t_{final}} \frac{\Pi_t^a}{(1+\rho)^t}. \tag{3.2}$$

Let us examine the case of cash flow yields expectations. Traders form beliefs about the future stream of the asset's cash flows. It is supposed that each trader knows the growth rate of each stock's dividend with a specific error. This error is represented by a coefficient that multiplies the real growth leading to an overestimation or an underestimation. Considering this perceived growth rate, traders build their future cash flow stream prediction $\Pi_t^{a,i}$, as a vector that incorporates each cash flow relative to the corresponding dividend distribution date, or coupon payment, within the trader's forward time horizon. Concerning bond coupons, traders do not have to form specific beliefs because the coupon payments are known. Given the cash flow vector $\Pi_t^a$, each trader is able to formulate its belief relative to the cash flow yield $\bar{\rho}_t^a$, according to

$$P_{t-1}^a = \sum_{\tau=0}^{h^i} \frac{\Pi_{t+\tau}^{a,i}}{(1+\bar{\rho}_t^{a,i})^{\frac{\tau}{12}}} + \frac{P_{t-1}^a}{(1+\bar{\rho}_t^{a,i})^{\frac{h^i}{12}}}\,, \tag{3.3}$$

where $P_{t-1}^a$ is the last price of the $a$-th asset. The first term of the right-hand side of Eq. 3.3 ($\Pi_{t+\tau}^{a,i}$) represents the cash flow (dividends and coupon payments), whereas the second term ($P_{t-1}^a$) represents the investment realization that corresponds to the asset's sale at the expected final price (considered in this phase as equal to the last price). It is worth noting that cash flows are zero when dividends or coupons not distributed.

The next step is to determine traders' expectations regarding price returns.

Traders have to estimate the assets' future price at the time step corresponding to their specific time horizon $h^i$. These expectations depend both on the observation of some real economic variables and of some purely financial variables. Traders' beliefs about price returns $\hat{\rho}_t^{i,a}$ are composed of different components: a fundamentalist component (that considers the excess return of dividends with respect to the risk-free interest rate), a chartist component (or trend following part, that considers the mean returns calculated over the backward-looking time window $T^i$), and a random component (that considers the past volatility of past asset returns), i.e.,

$$\hat{\rho}_t^{i,a} = \gamma_f^i\Big(\frac{1+\bar{\rho}_t^{i,a}}{1+r} - 1\Big) + \gamma_c^i\langle r^a\rangle^{T^i} + \ \gamma_r^i\sigma_t^{i,a}\varepsilon_t^i\,, \tag{3.4}$$

where $\gamma_f^i$, $\gamma_c^i$, $\gamma_r^i$ are the weights for each expectation component that sum to one, $\sigma_t^i$ represents the annualized stock returns volatility at time $t$ calculated over the time window $T^i$ of the $i$-th trader, and $\varepsilon_t^i$ is a gaussian noise term with zero mean and unitary variance. Finally, $\langle r^a \rangle^{T^i}$ is the mean value over $T^i$ of the past returns of the $a$-th asset.

It is worth noting that the fundamentalist term in Eq. 3.4 is derived by considering a no-arbitrage argument; equating the total pay-off of the asset's investment with a risk-free rate pay-off in a risk neutral environment, i.e.,

$$(1+\bar{\rho}_t^{i,a})P_{t-1}^a = (1+r)\tilde{P}. \tag{3.5}$$

Deducing $\tilde{P}$ from Eq. 3.5 and deriving returns allows one to write the fundamentalist term as in Eq. 3.4. Once traders have calculated their expected asset returns, they are able to predict the asset price at the end of their time horizon $h^i$. Such final price will be equal to the last market price of the stock multiplied by their gross asset returns forecast, relative to the whole time horizon period,

$$P_t^a(h^i) = P_{t-1}(1+\hat{\rho}_t^{i,a})^{\frac{h^i}{12}}. \tag{3.6}$$

As a general note, let us remark that bond returns have the same structure as stock returns with the only difference that the bond cash flow is deterministic and fixed in advance (depending on the stream of coupon payments) whereas for stocks the cash flow is stochastic.

The last step for traders is to form beliefs about the total asset returns $\rho$, considering the price $P_t^a(h^i)$ from Eq. 3.6 as the realized price

$$P_{t-1}^a = \sum_{\tau=0}^{h^i} \frac{\Pi_{t+\tau}^{a,i}}{(1+\rho_t^a)^{\frac{\tau}{12}}} + \frac{P_t^a(h^i)}{(1+\rho_t^a)^{\frac{h^i}{12}}}. \tag{3.7}$$

Hence, the return value $\rho_t^a$ is the total expected return taking into account both cash flows and the asset price variations; $\rho_t^a$ will be a key variable for the traders when they are taking decisions about their portfolio allocations.

**Some remarks.** Modeling traders belief formations is a crucial part in developing the financial market. There are many examples of different possible choices in the literature (see the state of the art for references). Our proposal is to consider expectations about the real part of the economy when forming beliefs about assets prices, in the perspective of the integration with other EURACE modules. The beliefs formation mechanism is quite simple and there are no complex learning algorithms implemented, neither strategic interaction. However, these possibilities are taken into consideration in the next chapters.

## 3.2 Preferences

Once traders have formed beliefs about the economic and the financial environment, as discussed in the previous section, they have to form preferences in order to issue assets orders. Traders have $M$ assets at their disposal and can put money in a savings account at a risk-free interest rate $r$.

### 3.2.1 Preferences according to optimal portfolio selection theory

Agents can select portfolios according to the modern portfolio theory developed by Markowitz (1952) (henceforth referred to as MPT). It has been shown that traders form expectations about total asset returns, including dividend returns, and that they calculate the variance-covariance matrix ($\Omega^i$ for the $i$th trader) of total returns obtained in the past according to their specific backward-looking time window $T^i$. Considering every possible asset portfolio $P$ in the risk-return space they are therefore able to locate the efficient frontier and to chose the optimal portfolio maximizing the following mean-variance utility function:

$$U^i(\vec{\omega}^P) = \rho^{P,i} - \frac{1}{2}\nu^i \sigma^{2P,n} \tag{3.8}$$

where $U^i(\vec{\omega}^P)$ is the utility of portfolio P, identified by a vector of weights $\vec{\omega}^P$ for the $i$-th trader. $\rho^{P,i}$ is the $i$-th trader's return estimation of portfolio P, $\nu^i$ is trader $i$'s risk aversion, and $\sigma^{2P,i}$ is its expectation about the variance of portfolio returns, i.e.,

$$\sigma^{P,i} = \vec{\omega}^P \Omega^i (\vec{\omega}^P)^T. \tag{3.9}$$

Maximizing utility allows the trader to fix the fraction of the complete portfolio allocated to the risky portfolio (risky fraction), and to find the set of weights $\omega^P$ to assign to each asset that identifies the optimum portfolio.

### 3.2.2 Preferences according to prospect theory

An alternative method that is proposed for modeling household's preferences is based on Prospect Theory, developed by Kahneman and Tversky (1979); Tversky and Kahneman (1992). In order to address several experimental shortcomings of the vonNeumann-Morgenstern expected utility theory, Kahneman and Tversky proposed a far more general theory of decision making under risk, namely, the Prospect Theory, that departs from expected utility theory in the following ways,

- agents derive utility not from wealth, but from gains and losses defined with respect to some reference level (typically assumed to be the status quo). This is called the endowment effect.
- a loss hurts more than an equally large gain produces joy. This is called loss aversion.
- agents are risk averse over gains but are risk lovers over losses.
- agents weigh low probability states by too much, and high probability states by too little.

Some attempts to model the decision making of agents that operate in a financial market, applying the prospect theory's psychological assumptions, have already been made.

Benartzi and Thaler (1995) propose an answer to the equity premium puzzle that was raised by Mehra and Prescott ten years before (Mehra and Prescott 1985), using the behavioral concepts derived from prospect theory. Barberis et al. (2001) design a model that incorporates a value function that is sort of a dynamic adaptation of the one proposed by Kahneman and Tversky. This dynamic adaptation consists of keeping account of returns that households obtained in the past, i.e., prior gains and losses, when designing the value function; thus following the idea that prior outcomes may affect subsequent risk-taking behavior (see Tahler and Johnson 1990).

With respect to the existing theory, a major step that has to be made is to extend the framework, that at present permits to model the decision between a risk-free and a risky asset, to the specific EURACE framework where many assets are considered.

In the following, a method to incorporate concepts from prospect theory is proposed, and a way to translate them into household's preferences in a multi-asset context.

Specifically, in Kahneman and Tversky's prospect theory utility is defined over gains and losses (i.e., returns $\rho$) rather than levels of wealth. They propose a value function of the following form:

$$v(\rho) = \begin{cases} \rho^{\alpha} & \text{if} \quad \rho \geq 0\,, \\ -\lambda(-\rho^{\beta}) & \text{if} \quad \rho < 0\,, \end{cases} \tag{3.10}$$

where $\rho$ is a generic return and $\lambda$ is the coefficient of loss aversion. By means of behavioral experiments Kahneman and Tversky estimated $\alpha$ and $\beta$ to be equal to 0.88 and $\lambda$ to be equal to 2.25.

According to prospect theory, the value of each outcome is generally multiplied by a decision weight. The *prospective utility* of a gamble $G$ which pays off $\rho_G$ with probability $p_G$ is thereby given by $V(G) = \sum \pi(p_G)v(\rho_G)$ where $\pi(p_G)$ is the decision weight, generally a non-linear transformation of $p_G$. Barberis et al. (2001) argue that weighting the value of gains and losses by their probabilities themselves, instead of by a non-linear transformation $\pi(p_G)$, should not affect the model qualitatively. Accordingly, for the sake of simplicity, the weighting function $\pi(p_G)$ will not be considered at this design stage.

In Section 3.1, it has been explained how households form their beliefs about future stock returns. Equation 3.4 shows the value of the expected return of the $a$-th asset for the $i$-th household, where $\sigma_t^{i,a}$ is the annualized stocks returns volatility at time $t$, calculated on window $T^i$ of the $i$-th trader. Given its belief on returns $\hat{\rho}_t^{i,a}$ and its standard deviation $\sigma_t^{i,a}$, each household should have a perception of the future return of the $a$-th asset that follows a normal probability density function. Therefore, the utility of the prospect relative to the $a$-th asset for the $i$-th household is given by

$$U_t^{a,i} = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_t^{i,a}}\; e^{\frac{1}{2}\left(\frac{\rho-\hat{\rho}_t^{i,a}}{\sigma_t^{i,a}}\right)^2} v(\rho)\; d\rho. \tag{3.11}$$

It is worth noting that the utility described by Eq. 3.11 incorporates all the main concepts of prospect theory exposed at the beginning of the section. Moreover, by modifying the value function form (Eq. 3.10) it is possible to take into consideration other features of prospect theory (e.g., introducing concavity for gains and convexity for losses one takes into account that marginal values of returns decrease with their magnitude). Evaluating Eq. 3.11 for every value of $a$, the $i$-th household will obtain the prospective utility relative to each asset.

An alternative method to calculate the prospective utility is to build the histogram of past returns (i.e., the frequency distribution) without passing through a predefined probability density function. Indeed, each household remembers the past $T^i$ values of past returns and is therefore able to build a normalized histogram $H_t^{a,i}[\Upsilon_t^{a,i}]$ where past returns of asset $a$ are ordered and grouped in $N_b^i$ bins. $\Upsilon_t^{a,i}$ is the set of the past $T^i$ values of the $a$-th asset returns, with respect to time $t$. Given the histograms, households may calculate the asset's utilities as,

$$U_t^{a,i} = \sum_{m=1}^{N_b} H_t^{a,i}(\rho_m)\ v(\rho_m), \tag{3.12}$$

where $\rho_m$ are the central values of each bin and $H_t^{a,i}(\rho_m)$ is the height of the histogram evaluated in $\rho_m$. Notice that creating the histogram means, for each household, to formalize a belief structure that is much more coherent with the prospect theory perspective. Indeed, the histogram is a kind of simple mental representation of the perception by the household of the past performance of a specific asset. In this sense, the value of $N_b^i$ is highly relevant. Since the number of bins $N_b^i$ is related to the level of detail of the household's beliefs, using only a small number of bins $N_b^i$ means that the household had just a look at the past performance of the asset and keeps in mind the essential aspects of this performance. A larger number of bins means that the household is more careful in examining the asset's past performance and takes more elements into consideration (it uses a higher resolution to construct the histogram). If the number of bins equals the number of asset's past returns within the time window (i.e., $N_b^i = T^i$), then the household weighs one by one all the available asset returns.

According to these considerations $N_b^i$ could be an interesting source of heterogeneity among the households. At present, the value of $N_b^i$ will be kept low since agents are representing real households instead of more structured organizations like AMCs (Asset Management Companies), that are assumed to behave according to Markowitz's portfolio allocation theory.

Another element to underline is that, in this implementation of prospect theory, traders' beliefs are only based on the past values of stock returns. Each agent, as it will be shown, has its specific way to project past prices into the future due to the significant heterogeneity, but every agent is supposed to behave as a kind of chartist.

Prospect utilities of assets will be compared in order to determine the relative weight of each asset. According to these relative weights, each agent decides how much to allocate to its savings account, how much to invest in assets, and which specific assets to buy or to sell. In order to compare the risk-free rate with other asset returns we need to take into account the expectations about assets cash flow yields ($\bar{\rho}_t^{a,i}$ from Eq. 3.3). It is worth emphasizing that the historical price returns of assets are daily prices, implying that in

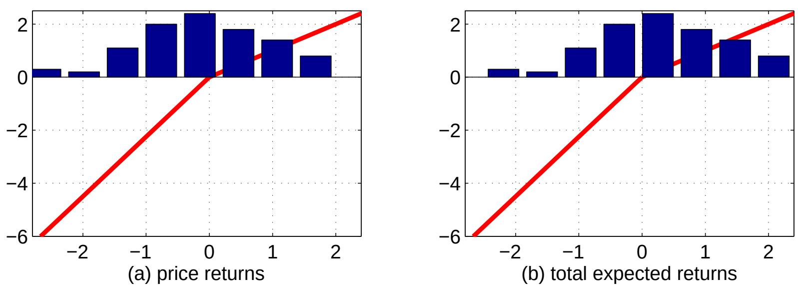


Figure 3.1: The solid line shows the prospect value function, while the histogram shows the perception of the agent about past asset return values. In figure (b) the histogram is shifted to the right to take into account the expected cash flow yields (dividend or coupon payments).

order to compare them we have to decompose the cash flow yields (dividends and coupons) and the risk-free interest rate into daily components, i.e, the function $D(\cdot)$ returns the daily component of a non-daily rate. Therefore, the quantity $D(\bar{\rho}_t^{a,i})$ will be added to each element of $\Upsilon_t^{a,i}$ with the result of shifting the histogram $H_t^{a,i}$ of $D(\bar{\rho}_t^{a,i})$ to the right. In this context, the prospect utility of the risk-free rate, that is always positive, will be exactly the daily risk-free rate. Analytically, it can be derived by evaluating Eq. 3.12 at $\rho_m = D(r)$, considering that in this case the histogram is composed of the unique value $D(r)$. The qualitative example of Figure 3.1 shows graphically how the prospect utility of an asset is obtained according to Eq. 3.12. The histogram represents the past values of asset returns, grouped into 8 bins ($N_b^i = 8$ for this particular trader), while the solid line is the prospect value function $v(\rho)$ of Eq. 3.10.

Once households have at their disposal the prospective utility $U_t^{a,i}$ for each asset, they can build a preference by ranking the assets according to these values. This ranking should give an order to be used for trading purposes, suggesting to buy assets with a high ranking and to sell assets with a low ranking. Therefore, we need a specific method to map the structure of utilities into an order issuing schedule. To be more clear, households' utilities can answer the question 'which assets would I prefer to buy or to sell?', but they are still unable to suggest how many of these different assets to buy, and how many to sell. This implies to define a rule that includes also the definition of a metric in order to translate preferences into orders. We use the same concept as the asset weights $\omega_{a,i}$ that sum to one, as introduced in Section 3.2.1 on the optimal portfolio theory. In this way we can identify the portfolio desired by the $i$-th household. The household will compare its actual portfolio to its desired portfolio, and will issue orders equal to the difference. The main problem will be how to map the prospect utilities into asset portfolio weights.

A very simple hypothesis that can be formulated is that each household decides to buy only the asset which is associated with the maximum utility and to sell only the asset which is associated with the minimum utility. In terms of weights this corresponds to setting the weight of the minimum utility asset to zero while leaving all the other assets' weights unchanged except for the maximum utility asset that will be set to one minus the sum of all other assets weights. In a framework with a low number of assets (at present we consider only four assets, namely two stocks and two bonds), and considering that agents only constitute households, this can be considered a first sensible operative hypothesis. Another simple solution that could be used even with larger numbers of assets, is to adopt a linear mapping that preserves proportions between utilities and weights. This means that, given a ratio between the utility of asset S1 and asset S2, the corresponding ratio between the weight of S1 and the weight of S2 will remain the same.

**How and when to choose a preference structure.** The main differences between the two preference structures presented above, i.e, prospect theory and optimal portfolio selection, can be comprehended by examining the well-known difference between prospect theory and expected utility theory (Kahneman and Tversky 1979). In the first case the attention is focussed on bounded rationality and human psychology through a behavioral approach, while in the second one uses an axiomatic analytical approach, and agents are generally considered to be more rational.

In our model preferences are heterogeneous. Each single agent can behave either according to the prospect theory preference scheme or according to the optimal portfolio selection scheme (MPT).

As a general rule, we consider that households have a lower degree of rationality and less computational capacities, and so they adopt a prospect theory preference scheme, while the AMCs can actually use optimal portfolio selection.

For each type of agent it is possible to set the percentage of individuals that adopt a particular scheme, e.g. 60 percent of households adopt preferences according to prospect theory and 40 percent adopt preferences according to optimal portfolio selection theory. For AMCs these percentages should be tilted towards using more the optimal portfolio theory, for instance 30 percent using prospect theory versus 70 percent using MPT.

However, these percentages may not be fixed but might instead change over time. This is where we can introduce learning in the strategy selection procedure. Agents can adopt either of the two strategies based on past performance of the rules, using a learning mechanism such as Experience Weighted Attraction (EWA) learning. See Section 5 for more details.

**Planning horizon and evaluation period.** Since 1926, the annual rate on stocks has been about 7 percent, while the real return on treasury bills has been less than 1 percent (see Benartzi and Thaler 1995). This huge discrepancy between the returns on stocks and on fixed income securities is difficult to explain with plausible levels of investor risk aversion (see Mehra and Prescott 1985). The equity premium puzzle refers to this empirical fact, that stocks have outperformed bonds over the last century by a surprisingly large margin. Benartzi and Thaler proposed a solution of the equity premium puzzle based on what they call *myopic loss aversion*. It consists of the combination of two types of investor behavior, *loss aversion* and *mental accounting*, that contribute to an investor being unwilling to bear the risks associated to holding equities. Mental accounting refers to the implicit methods that individuals use to encode and evaluate financial outcomes (Kahneman and Tversky 1984; Tahler and Johnson 1985). In particular, the evaluation period of investments has, according to Benartzi and Tahler, a key importance in explaining the equity premium puzzle. Let us report the clarifying example that the authors use in Benartzi and Thaler (1995).

Consider a problem first posed by Samuelson (1963). Samuelson asked a colleague whether he would be willing to accept the following bet: a 50 percent chance to win $200 and a 50 percent chance to lose $100. The colleague turned this bet down, but announced that he was happy to accept 100 such bets. This exchange provoked Samuelson into proving a theorem showing that his colleague was irrational. Of more interest here is what the colleague offered as his rationale for turning down the bet: "I won't bet because I would feel the $100 loss more than the $200 gain." This sentiment is the intuition behind the concept of loss aversion. One simple utility function that would capture this notion is the one of Eq. 3.10. The role of mental accounting is illustrated by noting that if Samuelson's colleague had this utility function he would turn down one bet but accept two or more as long as he did not have to watch the bet being played out. The distribution of outcomes created by the portfolio of two bets [$400, .25; $100, .50; -$200, .25] yields a positive expected utility with the hypothesized utility function, though of course simple repetitions of the single bet are unattractive if evaluated one at a time. As this example illustrates, when decision-makers are loss averse, they will be more willing to take risks if they evaluate their performance (or have their performance evaluated) infrequently.

In our model each agent is characterized by a different planning horizon, that determines its investment horizon and the frequency with which the household evaluates it.

We implemented a method in order to consider how the structure of a prospect changes when the evaluation period changes. Let us consider a generic prospect $\mathcal{P} = [\rho_i, p_i]$ where $\rho_i$ are the returns and $p_i$ are the associated probabilities. Following the concepts of myopic loss aversion, we introduce a new prospect $\mathcal{P}_n$ that represents the mental accounting of the agent when considering the risky investment,

$$\mathcal{P}_n = F(\mathcal{P}, n). \tag{3.13}$$

When the myopic loss aversion feature is switched on in the model, each agent that uses prospect theory preferences considers a prospect that is no longer given by the histogram $H$ of Eq. 3.12 with the past return values as $\rho_i$ and their occurrences as $p_i$, but it is given by the prospect iteration according to Eq. 3.13.

It is worth noting that, each agent having a different time horizon, different agents will face different prospects for the same assets also if they posses exactly the same data. Consequently, each agent will associate an asset with a different utility, implying there is heterogeneity of choices in the issuing of orders. This heterogeneity does not depend on an exogenous random variable that drives the system, but simply depends on the time horizon of the traders. Finally, this time horizon can be completely endogenized by supposing learning behavior on the part of the agents (see Chapter ).

**Heterogeneity factors in prospect theory.** It is well known that a fundamental characteristic of agent-based models is heterogeneity of agents, that can range from initial endowments, psychological attitudes, social dimensions, to behavioral rules, preferences and degree of rationality. For what concerns prospect theory, at the present stage the main parameters that characterize the behavior of the individual agents are the following:

- *Loss aversion:* according to Kahneman and Tversky a proper value for the loss aversion factor is approximately 2.25. However we propose a moderate variability in the perception of losses and therefore allow for agents to have heterogeneous loss aversion parameters in the interval [2,5].
- *Memory:* A household can be attracted to a stock that performed well in the last week, or it can evaluate a much longer period simply by looking at the plots of past returns (for example, using the internet). Agents are of course heterogeneous also in this respect.
- *Bins number:* The number of bins $N_b^i$ that composes a household's histogram represents the attention that is paid by the household to the historical data at its disposal. The greater $N_b^i$, the higher is the attention of the household to past returns of the stock.
- *Evaluation period:* The evaluation period is the length of time over which an agent aggregates and evaluates returns. If the evaluation period of an agent is longer, the utility associated with a risky investment will in general be higher.

### 3.2.3 Order issuing

After the asset weights have been determined, whether using optimal portfolio theory or prospect theory, each household evaluates the fraction of disposable wealth to allocate to risky assets and hence to issue buy or sell orders for each stock. The amount of resources available to household $i$ at the beginning of period $t$ is given by the wealth saved in liquid assets, i.e., the savings account and assets at the end of period $t-1$, plus current cash flows given by capital and labor income. Disposable resources $X_t^i$, often referred to as the cash-on-hand in the literature, are then determined by

$$X_t^i = W_{t-1}^i + \pi_t^i + \varsigma_t^i w_t \,, \tag{3.14}$$

where $W_{t-1}^i$ includes the savings account and the value of the portfolio invested in risky assets, evaluated at the $t-1$ market prices. Cash-on-hand is therefore partially saved and partially consumed, according the consumption rule $C_t^i(X_t^i)$ described in the next paragraph, and the saved fraction $(X_t^i - C_t^i)$ constitutes the amount of money to be distributed among different forms of investment, (i.e., stocks, bonds and savings account). Thus, $X_t^i - C_t^i$ is the amount of resources available to the $i$-th household for the financial investments. In order to issue orders, the $i$-th household evaluates a limit price $L_t^{i,a}$ for each asset $a$, given by

$$L_t^{a,i} = P_{t-1}^a (1 + \widehat{\rho}_t^{a,i})^{\frac{h^i}{12}} . \tag{3.15}$$

Next, each household computes the newly desired asset holdings $q_t^{a,i}$, given by the integer part of

$$q_t^{a,i} = \frac{\omega_t^{a,i}(X_t^i - C_t^i)}{L_t^{a,i}} \quad \forall a, \tag{3.16}$$

where $\omega_t^{a,i}$ are the new desired asset weights determined according to the preference scheme. Finally, the household decides to issue buy or sell orders so as to cancel the gap $\Delta q_t^{a,i} = q_t^{a,i} - q_{t-1}^{a,i}$ between its current portfolio and the desired one. Each order is a limit order characterized by a limit price $L_t^{a,i}$ and a quantity $\Delta q_t^{a,i}$. If $\Delta q_t^{a,i}$ is positive the $i$-th household issues a buying order for asset $a$, if $\Delta q_t^{a,i}$ is negative it is a selling order.

## 3.3 Model simulation

Computational experiments have been performed with the following parameter configuration. The model is populated by $N = 2,000$ traders who trade four assets (two stocks, two bonds). Stock dividends, bond coupons and interest on the savings account are distributed every six months. The wage has a growth rate of $g^w = 0.001$, and the standard deviation of noise is $\sigma^w = 0.004$. Dividend growth rates are $g^a$ and have been set equal to $g^w$, and $\sigma^a = 0.2$. Covariances among the dividend noise levels and among the wage noise level and the dividend noise level are summarized in the following variance-covariance matrix

$$\Omega = \begin{bmatrix} 1 & 0.4 & 0 \\ 0.4 & 1 & 0.25 \\ 0 & 0.25 & 1 \end{bmatrix} ,$$

where the third line and column represent the covariances of dividends with the wage. The two bonds pay semestrial coupons of 1% (Bond 1) and 0.6% (Bond 2) with respect to their face value. The annual risk-free rate on the savings account is 1%, i.e., $r = 0.01$. There is a 10% probability of being unemployed ($\mu = 0.1$). The adopted preference scheme for the simulations follows the optimal portfolio selection theory (MPT) described in Section 3.2. Backward-looking time windows $T^i$ through which a household looks at the past are uniformly distributed between 3 and 36 months, while the forward-looking time horizon varies from 1 to 12 months. For the traders' expectation component in Eq. 3.4, we only consider the following trader types: pure random traders ($\gamma_r^i = 1$, $\gamma_c^i = 0$, $\gamma_f^i = 0$) pure chartist traders $\gamma_r^i = 0$, $\gamma_c^i = 1$, $\gamma_f^i = 0$) or pure fundamentalist traders ($\gamma_r^i = 0$, $\gamma_c^i = 0$, $\gamma_f^i = 1$).

The presented simulations are performed with a percentage of 90% random traders, with the remaining 10% equally divided between chartists and fundamentalists. Note that the percentage of random traders is far greater than the percentage of chartists or fundamentalists. This can be motivated by the fact that the presence of a relatively high

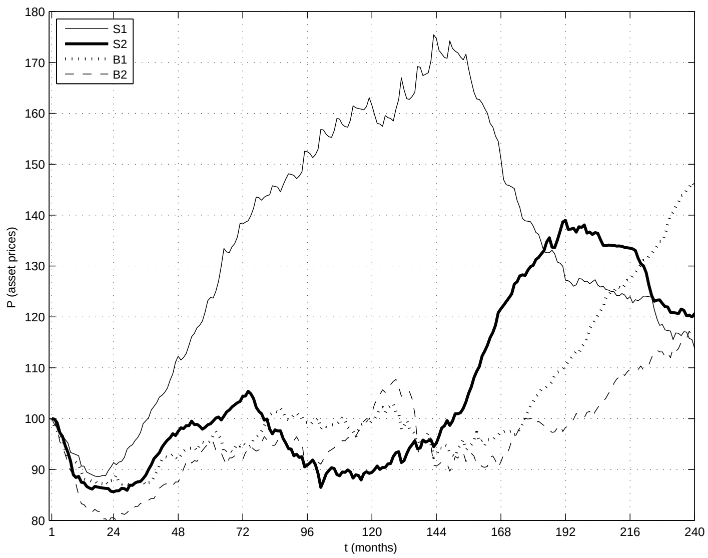


Figure 3.2: Time trajectories of asset prices.

percentage of chartists or fundamentalists may cause permanent bias effects on the asset price dynamics, due to the price returns belief formation process described by Eq. 3.4; see Raberto et al. (2003) for further evidence on this issue. Finally, the risk aversion $\nu^i$ is uniformly distributed among the agents, and taken to lie between the values of 1 and 5 ($\nu^i \in [1, 5]$). Concerning the savings/consumption dynamics, $\bar{x}^i$ is exogenously given and agent independent. The target level of cash-on-hand to labor income ratio is $\bar{x} = 2.8$, while the marginal propensity to consume is set to $m = 0.3$, following empirical studies (Deaton 1992; Carroll 2001). All traders are initially endowed with an equal amount of stocks, bonds and savings account. Moreover the initial cash-on-hand is set to a value such that the ratio of cash on hand to labor income $\bar{x}$ is higher than its target value. Figure 3.2 shows the price processes of a typical simulation relative to the four assets considered. The time span covered by each simulation corresponds to 240 months, i.e. 20 years. A graphical inspection of Figure 3.2 shows the existence of two regimes, a downward trend during the first two years, and then an increasing trend common to all assets. This is due to the initial values of $\bar{x}$, which is higher than the target value of $\bar{x}$, set at 2.8, as shown by the continuous line in Figure 3.3. In this case, according to buffer stock theory (see Eq. 2.13), agents consume on average more than the labor income, recurring to their asset's wealth. Thus, the consequent selling behavior induces a downward trend. Once the target buffer stock level is reached there is no more need to sell assets in order to finance consumption, and the wage growth rate has an effect on asset prices.

These considerations are corroborated by the experiment shown by the dashed lines in Figure 3.3, where from the 120th month agents exhibit a less prominent behavior than before, modeled by decreasing $\bar{x}$ from 2.8 to 2.4, thus requiring a lower level of buffer stock

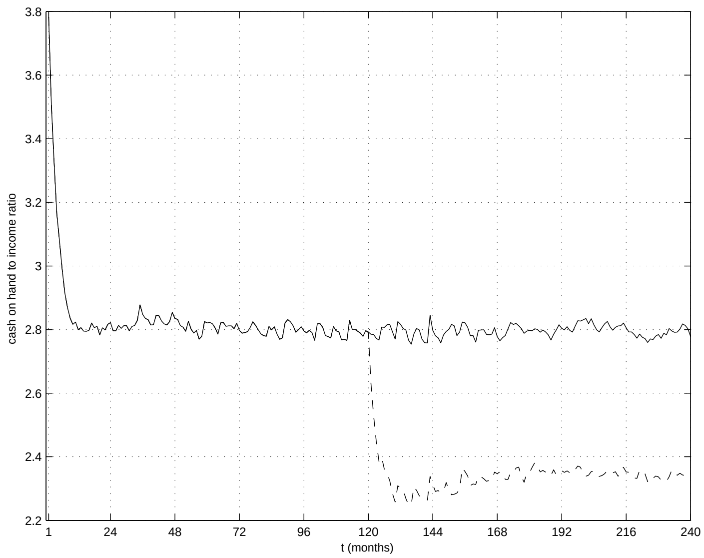


Figure 3.3: Time evolution of cash-on-hands to income ratio.

to deal with periods of unemployment. In order to match the new target agents consume more than their labor income until the lower desired buffer stock level is reached. Figure 3.4 shows the price processes relative to this experiment. Comparing Figure 3.2 and 3.4 shows that prices react to the change of $\bar{x}$ with a strong downward trend. This confirms the effect of consumption and saving behavior on the financial market.

Let us consider again Figure 3.2. In order to explain the relative different price trajectories, starting from the same initial conditions $P_0 = 100$ for each of the four assets, let us compare them with the respective cash flow dynamics. Figure 3.5 presents the cash flow yields of the assets, i.e, the ratio between asset cash flow $\Pi_t^a$ and asset price $P_t^a$ (dividend yields in case of stocks and coupon payments in case of bonds), computed at payment dates, i.e., every six months. It is worth reminding that the two bonds are characterized by constant coupons, while the cash flow dynamics for the two stocks is described by the stochastic process of equation 1.1. This explains why bond cash flow yields are far smoother than stock cash flow yields. Generally speaking, the no arbitrage argument which characterizes both the formation of beliefs and the structure of preferences of the agents, implies that prices move in order to reduce differencess in the cash flow yields, taking also into account a correction for risk bearing. Figure 3.2 shows that the bond that pays higher coupons (bond B1) is characterized by a higher price than bond B2, apart from the initial downward trend when assets are sold. This price behavior tends to reduce the coupon yields of the two bonds. A similar relative dynamics can be observed for the two stocks. Generally, the price of a stock is higher when its dividend yield is higher.

The simulated price processes exhibit a number of well-known stylized facts present in real financial time series, i.e., non stationarity, cointegration, serial correlations, and fat

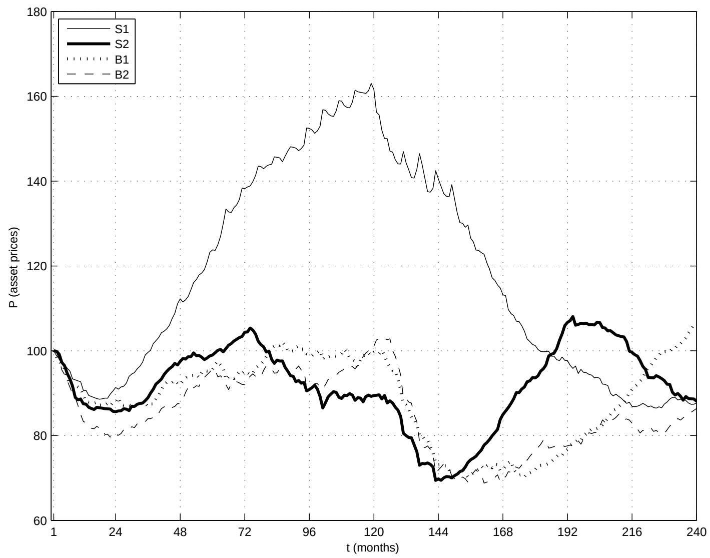


Figure 3.4: Asses prices. From t=120 $\bar{x}$ drops to 2.4.

tails. According to the augmented Dickey-Fuller test the null hypothesis of presence of a unit root can not be rejected for all time series. This result may be due to the unit root in the exogenous stochastic process given for labor income (see equation 1.2) which is one of the main drivers of the financial market. However, the Engle-Granger cointegration test rejects pair-wise cointegration between the wage process and each of the price processes. Pair wise cointegration is also rejected for each combination of price processes. Indeed, the Johansen test does not reject the presence of at most one cointegration relationship among the four price processes. Finally, for all four price processes, both the Jarque-Bera and the Kolmogorov-Smirnov tests reject the null hypothesis of normal distribution of returns, and the Ljung-Box test rejects the null hypothesis of absence of serial correlations both for the series of returns and the series of absolute values of returns. Indeed, empirical analysis by the authors has confirmed the presence of serial correlations even in the series of raw returns for the Dow Jones Euro Stoxx sectorial monthly indices. This result should be due to the usual presence of memory in the rates of returns of several economic variables, such as production, wages and profits, which are important in determining monthly financial returns. All test results have been considered at the significance level of 5%.

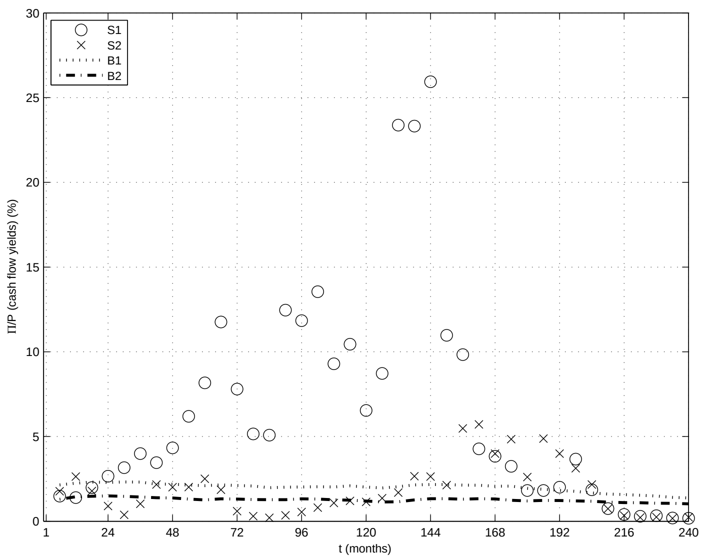


Figure 3.5: Cash flow yields.

# Chapter 4

# The asset market price formation mechanism

## 4.1 The clearing house

In this case, the price formation process is centralized and modeled according to a clearing house mechanism. Buying and selling orders are collected by the clearing house that builds cumulative demand and supply curves on a common price grid. The price $P_t^a$ that clears the market, at the crossing point between demand and supply, is chosen in order to maximize the transaction's amount.
All the traders whose limit prices are compatible with the clearing price ($L_t^{i,a} \geq P_t^a$ for buyers, $L_t^{i,a} \leq P_t^a$ for sellers) are selected for the transaction; however some of them will be rationed. A priority order is randomly generated and agents carry out their transactions following the order. When all the amount of stocks is traded, agents in the successive positions are rationed.

The price formation process is based on the equilibrium between supply and demand, and is set at the intersection of the two corresponding curves. The demand and supply curves are computed at every time step $h$. The procedure is the following. Suppose that at step $h$, traders have issued $U$ buy orders and $V$ sell orders. For each buy order let the pair $(a_u^b(h), b_u(h))$, $u = 1, \ldots, U$ indicate respectively the quantity of stocks to buy and the associated limit price. For each sell order in the same time step $h$, let the pair $(a_v^s(h), s_v(h))$, $v = 1, \ldots, V$ denote respectively the quantity of stocks to sell and the associate limit price. The demand curve and the supply curve are defined as follows:

$$f_h(p) = \sum_{u | b_u(h) \geq p} a_u^b(h) \qquad \text{(demand curve)}; \tag{4.1}$$

$$g_h(p) = \sum_{v | s_v(h) \leq p} a_v^s(h) \qquad \text{(supply curve)}. \tag{4.2}$$

The function $f_h(p)$ of the asset price $p$ represents the total amount of stocks that would be bought at price $p$ (demand curve). It can be viewed as a monotone non increasing step function of $p$. The bigger $p$, the fewer the buy orders that can be satisfied. If $p$ is lower than the minimum value of $b_u(h), u = 1, \dots, U$, then $f_h(p)$ is equal to total amount of stocks to buy. If $p$ is greater than the maximum value of $b_u(h), u = 1, \dots, U$, then $f_h(p)$ is equal to zero. Conversely, $g_h(p)$ represents the total amount of stocks that would be sold at price $p$ (supply curve) and is a non decreasing step function of $p$. Its properties are symmetric with respect to those of $f_h(p)$.

The clearing price computed by the system is the price $p^*$ at which the two functions graphically cross; in formulas, $p^*$ is the clearing price when for all $\epsilon_1, \epsilon_2 > 0$, if $f_h(p^* - \epsilon_1) \geq g_h(p^* - \epsilon_1)$ then $f_h(p^* + \epsilon_2) < g_h(p^* + \epsilon_2)$, or, alternatively, if $f_h(p^* - \epsilon_1) > g_h(p^* - \epsilon_1)$ then $f_h(p^* + \epsilon_2) \leq g_h(p^* + \epsilon_2)$.

The new market price $p(h)$ at time step $h$ is defined as: $p(h) = p^*$. The value $f_h(p^*)$ indicates the number of shares for which, at time step $h$, there is a demand at a limit price equal to $p^*$ or lower; on the other hand, $g_h(p^*)$ is the number of shares offered at a limit price equal to $p^*$ or higher at the same time step. Generally, the following relation holds: $f_h(p^*) \neq g_h(p^*)$.

In a closed system, all transactions must have a balance equal to zero. Thus, the number of shares sold have to be equal to the number of shares bought; otherwise the total number of shares in the system would vary. Hence, if $f_h(p^*) < g_h(p^*)\big(f_h(p^*) > g_h(p^*)\big)$, only $f_h(p^*)$ $\big(g_h(p^*)\big)$ shares are traded. If $f_h(p^*) < g_h(p^*)$, then $g_h(p^*) - f_h(p^*)$ stocks offered for sale at $p^*$ or more are randomly chosen and discarded from the corresponding sell orders. Conversely, if $f_h(p^*) > g_h(p^*)$, then $f_h(p^*) - g_h(p^*)$ stocks demanded to buy at $p^*$ or less are randomly chosen and discarded from the corresponding buy orders. After this procedure, the quantity of shares offered for sale at $p^*$ or more equals the quantity of shares offered for buy at $p^*$ or less. The corresponding buy and sell orders are then executed at the clearing price $p^*$. After transactions have taken place, traders' cash and portfolios are updated. Orders that do not match the clearing price are discarded.

It should be noted that two rare pathological cases may occur. In the first case, $f_h(p)$ and $g_h(p)$ do not cross at a single point but have a common horizontal segment whose abscissas are $p^{*1}$ and $p^{*2}$. In this case, the clearing price is assumed to be: $p^* := (p^{*1} + p^{*2})/2$ . In the second case, $f_h(p)$ and $g_(p)$ do not cross at all, i.e., $f_h(p_1) = 0$ and $g_h(p_2) = 0$ as $p_1 < p_2$. In this case, the time step is discarded and a new iteration begins.

For the sake of simplicity, let consider an example regarding the intersection of the demand and the supply curve. In the example, at time step $h$, $U = 5$ buy orders have been issued with the following sizes and linit prices:

1. $a^b_1(h) = 120, b_1(h) = \$99.4$;
2. $a^b_2(h) = 80, b_2(h) = \$99.6$;
3. $a^b_3(h) = 40, b_3(h) = \$100.0$;
4. $a^b_4(h) = 20, b_4(h) = \$100.4$;
5. $a^b_5(h) = 20, b_5(h) = \$100.6$.

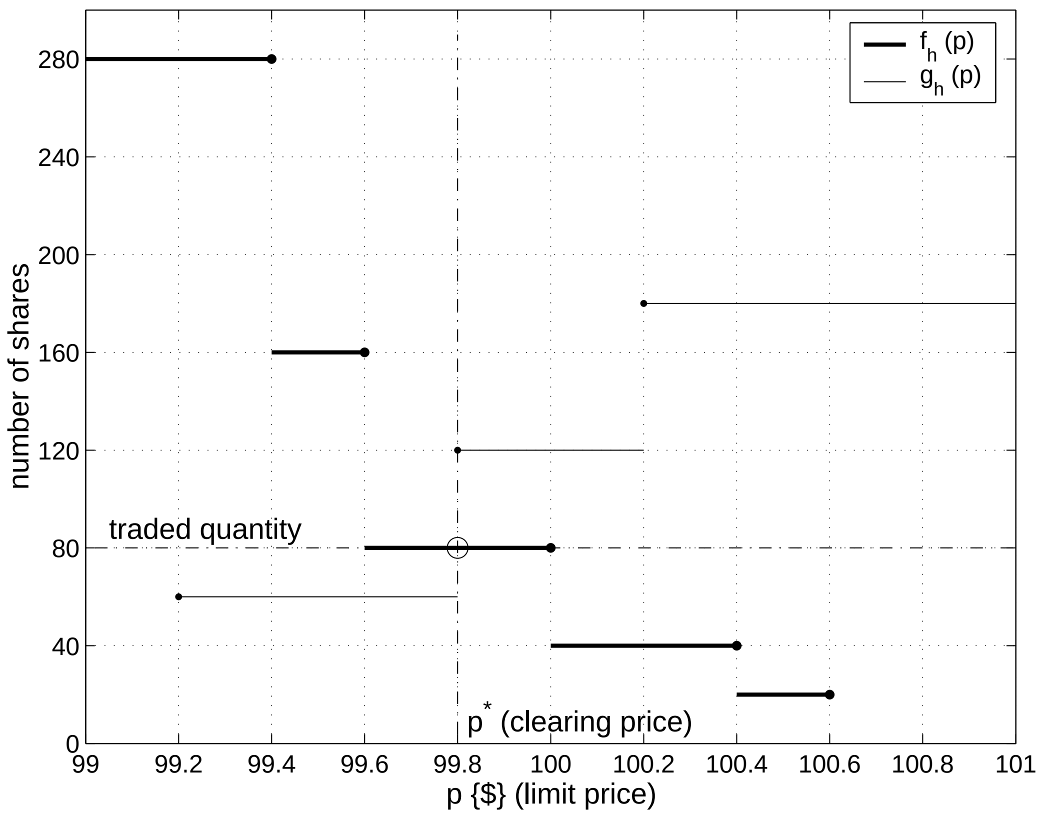


Figure 4.1: Matching of the demand $f_h(p)$ and of the supply $g_h(p)$ curve.

On the other hand, $V = 3$ sell orders have been issued at time step $h$. Sizes and limit prices are given by:

1. $a_1^s(h) = 60, s_1(h) = \$99.2$;
2. $a_2^s(h) = 60, s_2(h) = \$99.8$;
3. $a_3^s(h) = 60, s_3(h) = \$100.2$.

Following Eq. 4.1, the demand curve $f_p(h)$ is computed as follows:

- $f_h(p) = a_1^b(h) + a_2^b(h) + a_3^b(h) + a_4^b(h) + a_5^b(h) = 280$ for $p \leq \$99.4$;
- $f_h(p) = a_2^b(h) + a_3^b(h) + a_4^b(h) + a_5^b(h) = 160$ for $\$99.4 < p \leq \$99.6$;
- $f_h(p) = a_3^b(h) + a_4^b(h) + a_5^b(h) = 80$ for $\$99.6 < p \leq \$100.0$;
- $f_h(p) = a_4^b(h) + a_5^b(h) = 40$ for $\$100.0 < p \leq \$100.4$;
- $f_h(p) = a_5^b(h) = 20$ for $\$100.4 < p \leq \$100.6$;
- $f_h(p) = 0$ for $p > \$100.6$.

As regarding the supply curve $g_h(p)$ of Eq. 4.2, the values are:

- $g_h(p) = 0$ for $p < \$99.2$;
- $g_h(p) = a_1^s(h) = 60$ for $\$99.2 \leq p < \$99.8$;
- $g_h(p) = a_1^s(h) + a_2^s(h) = 120$ for $\$99.8 \leq p < \$100.2$;
- $g_h(p) = a_1^s(h) + a_2^s(h) + a_3^s(h) = 180$ for $p \geq \$100.2$.

Fig. 4.1 shows the two curves $f_h(p)$ and $g_h(p)$ in the example considered before. Following the intersection algorithm, the clearing price is set at $p^* = \$99.8$. At the clearing price, the value of the two curves are: $f_h(p^*) = 80$ and $g_h(p^*) = 120$, so only 80 shares are traded (all the shares offered to buy) while 40 shares offered to sell at $p^*$ are randomly discarded.

## 4.2 The limit order book

In the clearing house mechanism, buy and sell limit orders are accumulated over time and the market is cleared periodically at the intersection of the cumulative demand and supply curves. The rationale of the clearing house is a key tenet of economic theory: market prices are formed at the intersection of demand and supply (Varian 2003). However, the clearing house is an unrealistic description of the way stock exchanges operate around the world. Progress in information technology and the increasing deregulation of exchanges have led to a wide adoption of the limit order book for price formation. The limit order book is based on a double auction mechanism, where traders freely announce bids and asks and get matched with orders on the other side of the market. The double auction requires no auctioneer (but a matching algorithm is used instead) and both theoretical and empirical studies have found that it has remarkable power to promote price formation (Cason and Friedman 1996).

The limit order book is a snapshot at a given instant of the queues of all buy and sell limit orders, with their respective price and volume. Limit orders are organized in ascending order according to their limit prices. All buy limit orders are below the best buy limit order, i.e., the buy limit order with the highest limit price (the bid price). The best buy limit order is situated below the best sell limit order, i.e., the sell limit order with the lowest limit price (the ask price). All other sell limit orders are above the best sell limit order. Orders are stored in the book. A transaction occurs when a trader hits the quote (the bid or the ask price) on the opposite side of the market. If a trader issues a limit order, say a sell limit order, the order either adds to the book if its limit price is above the bid price, or generates a trade at the bid if it is below or equal to the bid price. In the latter case, the limit order becomes a marketable limit order or more simply a market order. Conversely, if the order is a buy limit order it becomes a market order and is executed if its limit price is above the ask price, else it is stored in the book. Limit orders with the same limit price are prioritized by time of submission, with the oldest order given the highest priority. Order's execution often involves partial fills before it is completed, but partial fills do not change the time priority.

### 4.2.1 Modeling orders arrival according to the theory of point processes

The point processes are good tools to model processes in which in any instant an event happened. So they are very good in modeling a limit order book. In the last years, studies about the statistical properties of stock order books appeared in literature. Biais et al. (1995) analyze the interaction between the order book and the order flow in the Paris stock exchange. They find that the order flow is concentrated near the best quotes, whereas the book deepness is in some cases larger near the evaluation point. Maslov and Mills (2001) find a power law distribution for the dimension of limit orders and market orders. Bouchaud et al. (2002) find that the prices of arrival limit orders follow a power law distributed around the current bid/ask. Lo and MacKinlay (1990) estimate an econometric model for the execution time of limit orders, that considers the effects of exploration variables like limit price, the distance between bid and ask and the market volatility. They find that the execution times are very sensible to the limit price. Furthermore, the execution time of a limit order is a very important issue for the traders. The submission of a limit order implies a trade-off between the advantage to get a fixed price and the disadvantage of a not known execution time.

An important empirical variable is the waiting time between two transactions and between two issued orders. In fact the limit-order book is an asynchronous mechanism, which means that a transaction only happens if a trader issues a market order. The order waiting time is a variable defined as the difference between the time in which two orders are issued: $\tau^o = t^o_{i+1} - t^o_i$, whereas the trading waiting time is defined as the difference between two transactions: $\tau^T = t^T_{i+1} - t^T_i$. The empirical analysis shows that the distribution of the trading waiting times follow a Poisson process, whereas the order waiting times distribution doesn't reject the null hypothesis of Weibull distribution. The formula of Weibull distribution is

$$p(\tau) = \frac{\beta}{\eta}(\frac{\tau}{\eta})^{\beta-1} e^{-(\frac{\tau}{\eta})^{\beta}}, \tag{4.3}$$

where $\eta$ is the scale parameter and $\beta$ is the shape parameter, also known as the slope, because the value of $\beta$ is equal to the slope of the regressed line in a probability plot. The expected value $< \tau >$ of a random variable following a Weibull PDF is given by:

$$< \tau >= \eta\Gamma(1/\beta + 1), \tag{4.4}$$

where $\Gamma$ is the Gamma function. The survival probability distribution $P_>(\tau) = \int_\tau^\infty p(\tau)$ is given by:

$$P_>(\tau) = e^{-(\frac{\tau}{\eta})^{\beta}}. \tag{4.5}$$

The exponential distribution is a particular case of the Weibull distribution for $\beta = 1$. In the case $\beta < 1$, the Weibull distribution assumes the form of the so-called stretched exponential and great values of $\tau$ occur with higher probability than in the case of $\beta = 1$. So empirical analysis shows that in the case of exponentially distributed order waiting times, also trade waiting times are exponentially distributed. Conversely, if order waiting times follow a Weibull, the same does not hold for trade waiting times.

# Chapter 5

# Modelling learning on the financial asset market

## 5.1 Introduction

When we consider learning on the asset market we want to let the agents search for the best trading rules, which implies a search through a possibly very large space. For the rule set we consider a finite set of portfolio strategies. These are generic rules for portfolio selection that may consist of a parameterized rule for determining the asset weights in the portfolio. For example, we can formulate a portfolio selection strategy that follows Markowitz's Portfolio Theory (MPT), or we might use Kahneman and Tversky (1979) Prospect Theory (PT). These are two different portfolio selection mechanisms and they define a subset of rules in the rule population.

Another rule can be a random trading rule subject to a budget constraint, such as Zero-Intelligence trading rules (Gode and Sunder 1993). Such rules can be used to test whether sophisticated trading rules can be outperformed by purely random behavior. The random rules can also be used to test the claim that noise traders will be driven out of the market by more rational, optimizing rules, a claim that has led to a large literature on models with boundedly rational agents in financial markets (see the survey by Hommes 2006a and references therein).

Another use of the ZI-rule is to test the efficiency of market mechanisms. By confronting a market mechanism with random behavior it can be tested which part of the overall market efficiency can be attributed to the behavior of the traders and which part is purely due to the structure of the market mechanism itself (see e.g. the recent contribution by LiCalzi and Pellizzari 2007).

The rule space that agents must search through to find better performing rules consists of the space of parameter settings for the different rule classes (MPT, PT, Random). The search space may be vast since it depends not only on the number of parameters in each class of rules but also on the discretization of the parameter space.

For this a GA is by far the best option, since it includes experimentation and exploitation, and recombines rules that have performed well in the past to form new rules that may perform even better in the future. However, due to the co-evolution of the rule population, rules that are well-adapted under certain circumstances may not be faring too well under others, so therefore we keep track of the rule performances in a classifier system.

Rule performance can be measured using different performance measures, possibly affecting the results of the learning dynamics. We have selected as a performance measure the capital gains that the rule has produced, averaged over the holding period of the portfolio that was suggested by the rule. This seems a natural measure for a rule's performance since it matches the objective of the financial traders, which is to obtain as large a capital gain as possible on their investments.

This chapter on learning in the Artificial Financial Market combines three mechanisms: a Learning Classifier System (LCS) to register the performance of a population of rules, a Genetic Algorithm (GA) to modify the population of rules, and Experience Weighted Attraction (EWA) learning to update the rule attractions and select among them using a probabilistic discrete choice framework. Both the LCS and the GA function at the system level. A Learning Classifier System is maintained and updated centrally and receives rule performance updates from all the agents. The LCS is therefore incorporated in a central Financial Advisor Agent. Learning is however defined at the individual agent level since every agent is also maintaining an internal LCS that registers the agent-specific attractions. The central mechanism of the Financial Advisor Agent makes it possible to speak about social learning on the financial market as well since the traders receive information about the performances of other rules which they themselves have not used.

## 5.2 Summary of the main features

- The generic strategy classes are pre-defined and remain fixed throughout.
- From the portfolio strategies, a finite population of rules is enumerated. The *condition* of the rule consists of a portfolio strategy and a parameter setting for this strategy. The *action* of the rule consists of the prescribed asset portfolio to be selected.
- The size of the rule population per strategy class is fixed (e.g., there might be 100 rules for each class, depending on the number of parameters and the discretization of the parameter space).
- An initial population of rules is selected at random, based on equal probabilities.
- An external Learning Classifier System is embedded in a Financial Advisor agent, who keeps track of the performance measures of the current population of rules in a centralized database that can be accessed by the agents.
- An internal Learning Classifier System in each agent registers the agent-specific attractions for each rule.
- New attractions are computed using the EWA learning rule.

- Selection of the rule to activate is based on discrete choice probabilities, following a probabilistic multi-logit rule.
- The current population of rules can be modified at the system level by a GA that changes the rule's parameter settings. This rule modification occurs at a slower speed than the agent-level learning.

## 5.3 EWA learning mechanism

We include a learning algorithm on the asset market by using the EWA learning mechanism (see Camerer and Ho 1999; Pouget 2007). EWA learning incorporates two learning effects, namely the *law of actual effect* and the *law of simulated effect*. The law of actual effect refers to the fact that agents learn from information about the own choices in the past. Selected strategies that were successful in the past will have a higher probability to be selected in the future. This law is also at the core of reinforcement learning mechanisms such as Roth and Erev (1995) (see also Marks 2006; van der Hoog and Deissenberg 2007, pp. 29–40).

The law of simulated effect refers to the notion that agents learn from information about others' choices in the past. The agent observes (i.e. simulates) the payoffs from non-selected strategies and reinforces the successful ones. This law is at the core of belief-based learning (see e.g, Fudenberg and Levine 1998).

EWA learning therefore is a hybrid form of learning, combining the law of actual effect and the law of simulated effect. This hybrid model consists of two variables: the attractions $A_i^j(t)$ and an experience weighing parameter $N(t)$, which are both updated after every period of experience. The model adds a key feature to reinforcement and belief-based learning models which is the weight given by players to forgone payoffs from unchosen strategies, denoted by the parameter $\delta$. See Table 5.1 for a reference to the EWA learning parameters.

Table 5.1: Variables and parameters in the EWA learning model.

| Variable | Description |
|---|---|
| $N(t)$ | experience/number of previous observations |
| $A^j(t)$ | attraction to select strategy $j$ |
| $\pi^j(t)$ | reward from using strategy $j$ at time $t$ |
| **Param.** | **Description** |
| $\rho$ | 'memory' parameter: depreciation rate of previous observations $N(t-1)$ |
| $\phi$ | 'change' parameter: depreciation rate of previous attraction $A^j(t-1)$ |
| $\delta$ | 'imagination' parameter: law of simulated effect, weight on forgone payoff $\pi^j(t-1)$ |
| $\kappa$ | 'lock-in' parameter, $\rho = (1-\kappa)\phi$: $k = 0 \Leftrightarrow \rho = \phi$ |

The experience weight starts at an initial value $N(0)$ and is updated according to

$$N(t) = \rho N(t-1) + 1, \tag{5.1}$$

where $\rho$ is a depreciation rate that measures the fractional impact of previous experiences, compared to one new observation in the current period. If $\rho = 0$ all previous experiences are fully discounted and has no impact on the current strategy selection: there is no memory effect and $N(t) = 1 \forall t$. If $\rho = 1$ then there is a strong memory effect and past observations/experiences are fully taken into account. The experience weight in this case reduces to a counter of the number of observations: $N(t) = N(t-1) + 1$.

Attractions are initiated at $A_i^j(0)$ and are updated according to:

$$A_i^j(t) = \frac{\phi N(t-1) A_i^j(t-1) + [\delta + (1-\delta) I(s_i^j, s_i(t))] \pi_i^j(t)}{N(t)}, \tag{5.2}$$

where the factor $\phi$ is a discount factor which depreciates the previous attraction, $N(t-1)$ weighs the observations, and $\delta$ is the weight given to the forgone payoffs from the unchosen strategies.

The value of $\delta \in [0,1]$ can take two extreme values. When $\delta = 0$ only the law of actual effect is used, while when $\delta = 1$ this implies that both the law of actual effect and the law of simulated effect are used. Any value in between discounts the forgone payoffs from the unchosen strategies with $\delta$. When $\delta = 0$, the attractions simply represent the cumulated past payoffs from the used strategies only. When $\delta = 0, \rho = 0$, the past experiences do not matter (note that $N(t) = 1$ in every period), so there is no memory effect and EWA learning reduces to reinforcement learning.

When $\delta = 1$ and $\rho = \phi$, the attractions of EWA learning in (5.2) reduce to those given for weighted fictitious play.[1]

In a later version of the EWA learning framework, the parameter $\rho$ has been replaced by $(1-\kappa)\phi$, see Eqn. 2 in Camerer et al. 2002, p. 6. This implies that when $\kappa = 0$, the same case as $\rho = \phi$ is obtained. A cube with all possible parameter configurations $(\delta, \phi, \kappa)$ appears in Camerer et al. 2002, p. 42, Fig. 2. Special cases then appear as edges or corners of this EWA learning cube. In Table 5.2 we provide a summary of the properties of EWA learning for these eight extreme cases.

The formula in (5.2) can now be split up into two parts, relating to the law of actual effect and the law of simulated effect respectively:

$$\begin{cases} A_i^j(t) = \frac{\phi N(t-1) A_i^j(t-1) + \pi_i^j(t)}{N(t)}, & s_i^j = s_i(t), \text{(for the used strategy)}, \\ A_i^j(t) = \frac{\phi N(t-1) A_i^j(t-1) + \delta \pi_i^j(t)}{N(t)}, & s_i^j \neq s_i(t), \text{(for the non-used strategies)}. \end{cases} \tag{5.3}$$

In the first line, the law of actual effect is measured by the term $\pi_i^j(t)$ for the actually chosen strategy. The performance from the chosen strategy $j$ is fully taken into account in the attraction for $j$.

In the second line, the law of simulated effect is measured by the term $\delta \pi_i^j(t)$, since it weighs the foregone payoff of strategy $j$, $\pi_i^j$, even though strategy $j$ was not actually

[1] In human subject experiments, estimates of $\delta$ are generally around .50, $\phi$ around .8 to 1, and $\rho$ varies from 0 to $\phi$ (see Camerer and Ho 1999).

used by agent $i$ at time $t$. Nonetheless, it is given some weight $\delta$ in the updating of the attraction $A_i^j(t)$ for strategy $j$.

### 5.3.1 Learning algorithm

A general form of the learning algorithm runs as follows:

0. Initialize the choice propensities/attractions to initial values: $A^j(0)$, for $j = 1, ..., J$.
1. Generate the choice probabilities for all actions using the current attractions.
2. Choose an action according to the current choice probability distribution.
3. Update the attractions for all actions.
4. Repeat from step 1.

**Step 0. Setting default parameter values and initial conditions** The default parameter values are set to:

$$\rho \in \{0,1\}, \delta \in \{0,1\}, \phi \in \{0,1\}, \beta = 1, N(0) = 1, A_i^j(0) = 1. \tag{5.4}$$

Here $\{0,1\}$ denotes the discrete set with elements 0 and 1. See Table 5.2 for all eight cases.

The depreciation rate of previous observations can be set to $\rho \in \{0,1\}$. If it is set to $\rho = 0$, this implies that all previous experience is fully discounted and does not influence the current strategy selection (there is no memory effect): $N(t) = 1\ \forall t$. If $\rho = 1$ then there is a strong memory effect, and past observations are fully taken into account. The experience weight in this case reduces to a counter of the number of observations: $N(t) = N(t-1)+1$.

The choice of $\delta \in \{0,1\}$ determines two treatments: $\delta = 0$ implies that *only* the law of actual effect is used, while $\delta = 1$ implies that *both* the law of actual effect and the law of simulated effect are used.

The depreciation rate of previous attractions can be set to $\phi \in \{0,1\}$. If $\phi = 0$ this means that previous attractions are completely discounted, and the attraction equals the performance measure: $A_i^j = \pi^j(t)$. When $\phi = 1$ this means that previous attractions are not completely discounted but taken into account in the computation of new attractions, which now are a weighted sum of experience (the performance) and attraction (hence the term Experience-Weighted Attraction).

When $\delta = 1$ and $\rho = \phi$, the attractions in EWA learning reduce to those of weighted fictitious play.

The intensity of choice parameter lies in the interval $\beta \in [0, +\infty)$. By default, it is set to $\beta = 1$, which implies a low learning speed. This means the learning process converges with high probability.

The initial condition for the experience weight is set to $N(0) = 0$ and for the attractions to $A^j(0) = 1$ respectively. This implies that, initially, all rules have equal probability of being selected.

**Step 1. Generating choice probabilities.** In order to define an agent's probability to use a certain strategy, the attractions are mapped into predicted *choice probabilities* using some statistical rule. For example, a simple rule would be to use the *linear choice* rule:

$$p_j(t) = \frac{A^j}{\sum_{k=1}^{J} A^k(t)}. \tag{5.5}$$

However, it is currently more common in the literature on social interactions to let the choice probabilities follow from the Boltzmann distribution:

$$p_j(t) = \frac{exp[\beta A^j(t)]}{\sum_{k=1}^{J} exp[\beta A^k(t)]}. \tag{5.6}$$

This formulation is called the *discrete choice*- or *multinomial logit* model and can be derived from a random expected utility framework, see McFadden (1973), Diks and van der Weide (2003, p. 4) and Hommes (2006b, p. 1149). The parameter $\beta$ is often referred to as the *intensity of choice* and is related to the randomness in the strategy selection process. The larger the value of $\beta$, the smaller the noise level in the random expected utility, and the larger the probability that an agent chooses the strategy with the highest attraction. For $\beta = \infty$ the random utility term vanishes and the strategy with highest attraction is chosen. For $\beta = 0$ the random term dominates and all strategies are selected with equal probability. The value of $1/\beta$ can then be interpreted as the propensity of agents to err, if their intention is in fact to select the strategy with the highest attraction.[2]

The discrete choice mechanism represents a general probabilistic framework for strategy selection motivated by results from interacting particle systems in physics, see e.g. Blume (1993) and Föllmer (1974). For use of this framework in models of herding and social interactions, see Brock and Durlauf (2001a,b), and Durlauf and Young (2001). For surveys on the use of this framework in models of financial interactions, see the survey by Hommes (2006a).

**Step 2. Choosing an action according to the choice distribution.** The choice probabilities generated in step 1 are used to build a cumulative distribution function $F_i(.)$. To determine a trader's actual choice, a random variable $u$, uniformly drawn between zero and one, is compared to this cumulative distribution function. Action $j$, $1 \leq j \leq J$, is chosen if $F_i(j-1) \leq u \leq F_i(j)$. Action 1 is chosen if $0 \leq u < F_i(1)$, and action $J$ is chosen if $F_i(J-1) < u \leq 1$.

[2] According to the Wikipedia: "The Boltzmann distribution is often expressed in terms of $\beta = 1/kT$ where $\beta$ is referred to as thermodynamic beta. The term $exp(-\beta E_i)$ or $exp(-E_i/kT)$, which gives the (unnormalised) relative probability of a state, is called the Boltzmann factor and appears often in the study of physics and chemistry." (Source: `http://en.wikipedia.org/wiki/Boltzmann_distribution`). Here $E_i$ stands for the energy of a particle in state $i$. Since energy is a potential for change, it corresponds to a negative attraction. Hence the attraction terms $A^j$ are negative energy terms, and the corresponding factors in our case are given without the minus sign: $exp(\beta A^j)$.

**Step 3. Update the attractions for all actions.** The attractions are updated according to the EWA learning rule:

$$\begin{cases} A_i^j(t) = \frac{\phi N(t-1)A_i^j(t-1)+\pi_i^j(t)}{N(t)}, & s_i^j = s_i(t), \text{(for the used strategy)}, \\ A_i^j(t) = \frac{\phi N(t-1)A_i^j(t-1)+\delta\pi_i^j(t)}{N(t)}, & s_i^j \neq s_i(t), \text{(for the non-used strategies)}. \end{cases} \tag{5.7}$$

## 5.4 Classifier Systems

A standard Classifier System (CS) is a set of classifier rules. A classifier rule consists of a Condition, an Action, and other parameters, such as the Performance (or Strength or Accuracy) of the rules. We can list all the rules in a table with the corresponding strength of each rule. The *'bid'* of a rule is then usually defined as the rule-strength plus a small noise term:

$$b(j) = s(j) + \varepsilon, \qquad \varepsilon \sim U[0,1].$$

The bid is needed to provide some randomness for the rule selection mechanism, such that it is not just the best performing rule that gets selected and to ensure some variability in the rule usage. Of course, this slight adjustment is only needed if the rule selection mechanism does not already include such randomness. In our approach, we will use a discrete choice- or multinomial logit model to select the rules (see e.g., van der Hoog and Deissenberg 2007, p. 31). This framework can be derived from a random expected utility framework (McFadden 1973), and hence already includes randomness in the rule selection process. In fact, this framework allows us to tune the degree of randomness using a single parameter $\beta$.

In the next subsection we define the rule population and the modification of the rule set by the evolutionary operators. We refer to Fig. 5.5 to illustrate the interaction between the different parts of the learning mechanism.

### 5.4.1 Learning Classifier Systems

In comparison to regular Classifier Systems, a Learning Classifier System (LCS) has the additional property that new rules can be created and old rules can go extinct (but the population size remains fixed). This is done through the evolutionary operators of crossover and mutation. We assume that the evolutionary operators can only operate on rules of the same strategy class:[3]

- Reproduction and extinction: a subset of rules is selected for reproduction. This can occur in two ways:
  - Roulette-wheel reproduction: from the entire population, rules are randomly selected for reproduction according to fitness proportional probabilities. This

[3]The strategy classes can be likened to animal species, in the sense that two members of the same species can produce offspring but members of two different species can not produce offspring. It does not make sense to have crossover between parameter settings (rules) of two different strategy classes since the number of parameters may be different and also it does not have any meaning to take a convex combination of two investment strategies.

implies that also lesser performing rules have a (small) probability of reproducing, thereby ensuring variability in the population of rules.

  – Rank-based reproduction: the 5% lowest performing rules get replaced by the offspring from the 5% highest performing rules. The original parents (the 5% highest) remain in the population. This ensures that the population size remains fixed.

- The cross-over operation only happens between rules within the same portfolio strategy class, but with different parameter settings. Rules of different classes cannot undergo cross-over.

- Each agent keeps account of its own internal attractions for the population of rules, based on its own experiences with each rule and the general performances of each rule. This is because the attractions are agent-specific, but rule performance is registered on a system level in the external Classifier System that is held centrally in the rule database.

Figure 5.5 gives a schematic overview of the learning mechanism. There is an external Learning Classifier System to register the performance of each rule in a centralized rule database. The updating of this general rule set occurs at the system level, through the application of a Genetic Algorithm that modifies the rules' parameter settings. In addition, each agent has an internal Learning Classifier System to record the agent's attraction for each rule. The EWA learning occurs at the agent level and concerns the computation of attractions on the basis of the performance registered in the external Learning Classifier System and the internal attraction in the internal Learning Classifier System. The Learning Classifier System of each agent transforms the system-level performances into agent-level attractions, which implicitly includes a finite memory of past performances due to the discounting of the previous period attraction in the EWA learning rule (5.2).

For the selection of which rule to activate agents use a probabilistic approach based on the multi-logit rule. Given the attractions in the internal Learning Classifier System, we construct a probability distribution based on the discrete choice probabilities given in (5.6).

## 5.5 The GA mechanism

An efficient sampling method of the space of parameter settings can be obtained using a Genetic Algorithm (GA). However, we have to define how the crossover and mutation operators modify the parameter settings in the rule population. This means we have to encode the parameter settings into bitstrings that enter into the GA mechanism.

**GA encoding:**

- *Bitstrings* represent Parameter Settings (PS points).

- The alleles (positions on the bitstring) are encoded with parameter values. Using real-encoded bitstrings (chromosomes), one bit represents one parameter value.
- The size of a bitstring equals the dimension of a PS point.
- *Search space:* The parameter space is the space of all Parameter Settings (PS points). Suppose we have a model with $n$ parameters, $\theta \in \mathbb{R}^n$, and each parameter has a range that has been discretized into 10 distinct values. Then we obtain a search space consisting of $10^n$ PS-points.
- Restricted search space: if the search space is too large, the complete set of bitstrings can be restricted to only consist of *deviations* from a baseline parameter setting (see below).

**Algorithm** Each generation of bitstrings undergoes several stages before the next generation appears:

Stage 1 Testing: the performance of the bitstring is compared to the performance of other bitstrings in the current population. This yields the relative fitness of each bitstring.

Stage 2 Reproduction: Sample $2N_{rep}$ parent strings from the population using fitness proportionate probabilities (roulette-wheel selection). Random draws are with replacement.

Stage 3 Cross-over: The selected sample of parent strings are randomly matched $N_{rep}$ times (with replacement). Cross-over occurs with a probability $p_{cross}$. This produces $2N_{rep}$ offspring that are potential new members in the population.

Stage 4 Mutation: random perturbation of the newly generated bitstrings.

Stage 5 Election: the new bitstrings are tested for higher fitness than their parents. New offspring is accepted only if it is at least as fit as the parents. If not, identical copies of the original parents are placed in the next generation.

**Reproduction of real-valued bitstrings:** The current generation of bitsrings is used to select $2N_{rep}$ copies. These copies are drawn at random with probabilities depending on the relative fitness (drawing is with replacement). This is called fitness-based reproduction. Other forms of reproduction such as rank-based or tournament-based reproduction are not considered here.

Details:

- Set probabilities depending on the relative fitness.
- Sample $2N_{rep}$ copies from the current generation of bitstrings.
- Randomly match the $2N_{rep}$ copies into pairs, using $2N_{rep}$ random draws with replacement based on equal probabilities of matching.

**Crossover between two real-valued bitstrings:** The cross-over operation is carried out with a certain probability $p_{cross}$. The type of cross-over operation is single-point cross-over, combining the genetic material from one parent that is on the left-hand side of the cross-over point with the material from the other parent that is on the right-hand side of the cross-over point.

Details:

- Select a random crossing point between $[1, L-1]$.
- With probability $p_{cross}$ a cross-over produces two offspring that are potential new members in the population.
- With probability $1 - p_{cross}$ no cross-over occurs and the offspring consists of two identical copies of the parents (the parents effectively remain in the population).

Suppose that for $n = 3$ parameters we consider a baseline parameter setting given by $\theta_1 = (\alpha_1, \beta_1, \gamma_1)$, and an alternative setting $\theta_2 = (\alpha_2, \beta_2, \gamma_2)$. The Parameter Space is encoded as a population of bitstrings as in Fig. 5.1, which shows a crossover operation between the two bitstrings at position 1. After the crossover the population size has remained the same, but two new parameter settings have been generated.

$$\begin{array}{c} \boxed{\alpha_1 \mid \beta_1 \mid \gamma_1} \\ + \\ \boxed{\alpha_2 \mid \beta_2 \mid \gamma_2} \end{array} \quad \rightarrow \quad \begin{array}{c} \boxed{\alpha_2 \mid \beta_1 \mid \gamma_1} \\ + \\ \boxed{\alpha_1 \mid \beta_2 \mid \gamma_2} \end{array}$$

Figure 5.1: Illustration of the crossover operation on position 1 between two bitstrings (parameter settings).

**Random perturbation of real-valued bitstrings:** Perturbations from a baseline parameter setting are encoded as deviations in the range $-10\%$ to $+10\%$, in 5% increments. The space of GA bitstrings therefore consists of the following parameter perturbations:

$$d\theta \in \{-10\%, -5\%, 0\%, +5\%, +10\%\}^n. \tag{5.8}$$

The search space now consists of $5^n$ elements, since each parameter value can be independently perturbed. A perturbation value of 0% means that the parameter has attained its baseline value. Note however that if one parameter has a 0% perturbation the other parameters can still have nonzero perturbations from the baseline setting.

**Creation of the initial set of perturbations:**

1. From the set of $5^n$ perturbations, we draw a random perturbation vector according to a uniform distribution.

$$\begin{array}{|c|c|c|}\hline \alpha_1 & \beta_1 & \gamma_1 \\ \hline \end{array} \qquad \begin{array}{|c|c|c|}\hline \alpha_1 + d\alpha_1 & \beta_1 + d\beta_1 & \gamma_1 + d\gamma_1 \\ \hline \end{array}$$

$$\vdots \qquad \rightarrow \qquad \vdots$$

$$\begin{array}{|c|c|c|}\hline \alpha_n & \beta_n & \gamma_n \\ \hline \end{array} \qquad \begin{array}{|c|c|c|}\hline \alpha_n + d\alpha_n & \beta_n + d\beta_n & \gamma_n + d\gamma_n \\ \hline \end{array}$$

Figure 5.2: Illustration of the mutation operation on a population of bitstrings (parameter settings).

2. For each iteration of the GA a new population of bitstrings is created by cross-over and mutation. The mutation part consists of the randomly drawn perturbation vector, that can be added either to the baseline parameter setting or to the current parameter setting.

3. Each iteration of the GA requires a run of the model (perhaps multiple runs if different random seeds are required).

## 5.6 Application to the financial asset market

### 5.6.1 The rule set

For the rule set we consider a finite set of portfolio strategies. These are generic rules for determining the asset weights in the portfolio and may consist of a parameterized rule. Modifications of the parameter settings yields a different rule set for each strategy class and the total population of rules defines a Classifier System with a fixed population size.

For example, we can formulate a portfolio selection strategy that follows Markowitz's Portfolio Theory (MPT), or we might use Prospect Theory (PT). These are two different portfolio selection mechanisms and they both define a subset of rules in the rule population.

For our purpose, we will make a distinction between the *performance* of a rule and the *attraction* of the rule for a particular agent. The rule performances are registered in an external Classifier System (see Table 5.3), while the attractions are registered in an internal Classifier System that is internal to each agent (see Table 5.4).

This separation seems useful since learning takes place at the agent level, while social learning (learning from the experiences of others) occurs at the system level and requires the performance of a rule to be defined as the average performance across the agent population. This implies that we also need to introduce a *user counter* for each rule that accounts for how many agents are using a particular rule, and then compute the average performance of this rule, see column 3 in Table 5.3.

### 5.6.2 GA encoding of the rules

In the asset allocation mechanism there are four main parameters:

1. The trader's investment horizon (this equals the household's expected holding period of the portfolio), encoded in the parameter `household.InvestmentHorizon`.

2. The trader's memory (the history used for econometric computations such as chartism), encoded in the parameter `household.memory`.

3. The trader's risk aversion to be used in combination with the Markowitz Portfolio Theory, encoded in the parameter `household.risk_aversion`.

4. The trader's degree of extrapolation to be used in combination with a chartist rule, encoded in the parameter `household.chartist_return_weight`.

Suppose we encode these parameters as $\theta = (\alpha, \beta, \gamma, \delta) \in \mathbb{R}^4$. To obtain a space of parameter settings we need to specify for each parameter its range, its increment and a benchmark value. Thus, a full specification requires the following minimal information:

$$\begin{aligned} \alpha : &\quad (\alpha_0, \alpha_{min}, \alpha_{max}, \Delta\alpha) \\ \beta : &\quad (\beta_0, \beta_{min}, \beta_{max}, \Delta\beta) \\ \gamma : &\quad (\gamma_0, \gamma_{min}, \gamma_{max}, \Delta\gamma) \\ \delta : &\quad (\delta_0, \delta_{min}, \delta_{max}, \Delta\delta). \end{aligned} \tag{5.9}$$

For each parameter we can now construct the space of parameter settings. The cardinality of these sets gives use a count of the number of elements:

$$\begin{aligned} |A| = &\quad (\alpha_{max} - \alpha_{min}/\Delta\alpha) \\ |B| = &\quad (\beta_{max} - \beta_{min}/\Delta\beta) \\ |C| = &\quad (\gamma_{max} - \gamma_{min}/\Delta\gamma) \\ |D| = &\quad (\delta_{max} - \delta_{min}/\Delta\delta). \end{aligned} \tag{5.10}$$

The total parameter space $\Theta$ is constructed by taking the Cartesian product of the individual parameter spaces, $\Theta = A \times B \times C \times D$, with $\Theta$ being the space on which the GA operates by modifying existing rules and producing new ones. This rule space needs to be specified beforehand in order to initialize the universe of rules from which the GA can randomly draw or generate new rules. An individual rule is then given by a parameter setting $\theta \in \Theta$.

For the mutation operator in the GA we need to be able to produce random perturbations, using deviations from the parameter benchmark value in discrete multiples of $(\Delta\alpha, \Delta\beta, \Delta\gamma, \Delta\delta)$:

$$\theta + d\theta = (\alpha_0 + \epsilon_\alpha \Delta\alpha, \beta_0 + \epsilon_\beta \Delta\beta, \gamma_0 + \epsilon_\gamma \Delta\gamma, \delta_0 + \epsilon_\delta \Delta\delta). \tag{5.11}$$

where $\epsilon_i \in \{-10, -5, 0, +5, +10\}$. This means that each parameter setting $\theta \in \Theta$ has $5^N$ neighbouring parameter settings. With $N = 4$ as in the example this implies that each rule has 625 neigbouring rules to which it can be perturbed by a single perturbation $d\theta$.

However, the universe of rules on which the GA operates is much larger than that. It consists of $|\Theta| = |A| \times |B| \times |C| \times |D|$ rules, the size of which depends on the discretization we choose for the individual parameters.

For the Learning Classifier System we initialize the rule population by randomly drawing a subset of rules from this universe, which then undergoes selection, cross-over, mutation and election as described in the previous sections.

### 5.6.3 Preliminary simulation results

We have implemented the learning mechanism described above and tested it for some special cases that were considered to be interesting benchmarks. The current implementation consists of the learning classifier system that is coupled to the EWA learning mechanism. The GA has not yet been implemented due to the fact that we so far only considered a small rule population that fits entirely in the table of the classifier system. Once the rule population grows in size we will need the GA to select among the rules.

The initialization values for the time parameters and EWA parameters can be found in Table 5.5. The population of agents consisted of 200 households trading in stocks of 10 firms.

We considered two market mechanisms: the clearing house and the limit-order book. See Table 5.6 for reference parameter values for the simulation runs. So far, we have used only the MPT rule and the random rule (RND) for testing purposes (the prospect theory PT rule and Chartist rules are left for future research). Further, the EWA parameters were fixed at: $\rho = 1, \delta = 1, \phi = 0$. This corresponds to the case that includes memory, agents are using the law of actual and simulated effect (foregone payoffs from non-selected rules), and attraction equals performance ($\phi = 0$).

Fig. 5.6 shows results for the simulation with the clearing house mechanism. It shows time series for the price and returns of a single asset, after an initialization period of 240 time steps. Fig. 5.7 shows the corresponding plot for rule usage and rule performances. Since there are multiple rules for both MPT and RND, traders can switch between any of the 40 rules in use, depending on the rules' performance. From the rule performance plot (Fig. 5.7, right) it is clear that some rules outperform others, and are consequently selected more often. This is also reflected in the plot of the rule user counts (left) that gives information about how how many users are using each rule.

Fig. 5.8-5.9 show the results for a simulation with the limit order book mechanism. In this run there were only two rules available: a single MPT rule and a single RND rule (both with an investment horizon of 1 time step, memory equal to 3 time steps, and the MPT rule uses a risk-aversion parameter set to 3). With only two rules available, the plot of the rule user count in Fig. 5.9 (left) shows the evolution of the rule usage more clearly. Looking at the performances of the rules we observe that one rule is clearly outperforming the other rule every once in a while and hence there is rule switching. However, we also see that over time the rule usage counters are converging, and also the spikes in the performance measure are decreasing in magnitude over time, so it seems that there is some balancing-out effect at work. This means that with only two rules available, we see that the performance of each rule stabilizes due to the presence of the other rule, and this would imply there is a co-existence of the rules without any one rule being driven out by the other. Whether or not this is a robust phenomenon will require more explorative simulation runs in the future.

Finally, Fig. 5.10-5.11 show a simulation with multiple MPT rules versus multiple RND rules using the limit-order book mechanism. Again, there are multiple rules that have a relatively large user base, without there being a single rule consistently outperforming all others and consequently driving the other rules out of the market. Instead, there is a co-existence of rules that alternate in their number of users and in their relative performance.

| 'memory' | 'imagination' | 'change' | 'lock-in' | description |
|---|---|---|---|---|
| $\rho$ | $\delta$ | $\phi$ | $\kappa = 1 - \rho/\phi$ | |
| 0 | 0 | 0 | 0 | no memory, law of actual effect only, attraction equals performance . |
| 0 | 1 | 0 | 0 | no memory, law of actual and simulated effect, attraction equals performance. |
| 0 | 0 | 1 | 1 | no memory, law of actual effect only, attraction discounted with performance. |
| 0 | 1 | 1 | 1 | no memory, law of actual and simulated effect, attraction discounted with performance. |
| 1 | 0 | 0 | – | memory, law of actual effect only, attraction equals performance. |
| 1 | 1 | 0 | – | memory, law of actual and simulated effect, attraction equals performance. |
| 1 | 0 | 1 | 0 | memory, law of actual effect only, attraction discounted with performance. |
| 1 | 1 | 1 | 0 | memory, law of actual and simulated effect, attraction discounted with performance. |

Table 5.2: EWA parameter settings. Eight possible configurations of EWA learning parameters. Note that $\rho \equiv (1-\kappa)\phi$, hence $\kappa := 1 - \rho/\phi$. A cube with all possible parameter configurations $(\delta, \phi, \kappa)$ appears in Camerer et al. 2002, p. 42, Fig. 2, with special cases denoted as edges or corners. The eight cases in the table correspond to the cases in this EWA learning cube. Extreme cases are: $\rho = 0$: no memory, $\rho = 1$: memory. $\delta = 0$: reinforcement learning, $\delta = 1$: belief-based learning. $\phi = 0$: attraction equals performance, $\phi = 1$: attraction weighs performance. Rule 1: reinforcement learning, rule 2: fictitious play, rule 8: Cournot best-response; weighted fictitious play.

**External Classifier System: The population of classifier rules.**

| rule id. | condition-action | total performance | user count | avg. performance |
|---|---|---|---|---|
| 1 | ⟨PortfolioRules.A[1], Prescribed portfolio A[1]⟩ | $\pi(1)$ | $N(1)$ | $\langle\pi(1)\rangle$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $na$ | ⟨PortfolioRules.A[na], Prescribed portfolio A[na]⟩ | $\pi(na)$ | $N(na)$ | $\langle\pi(na)\rangle$ |
| $na+1$ | ⟨PortfolioRules.B[1], Prescribed portfolio B[1]⟩ | $\pi(na+1)$ | $N(na+1)$ | $\langle\pi(na+1)\rangle$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $na+nb$ | ⟨PortfolioRules.B[nb], Prescribed portfolio B[nb]⟩ | $\pi(na+nb)$ | $N(na+nb)$ | $\langle\pi(na+nb)\rangle$ |
| $na+nb+1$ | ⟨PortfolioRules.C[1], Prescribed portfolio C[1]⟩ | $\pi(na+nb+1)$ | $N(na+nb+1)$ | $\langle\pi(na+nb+1)\rangle$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $na+nb+nc$ | ⟨PortfolioRules.C[nc], Prescribed portfolio C[nc]⟩ | $\pi(na+nb+nc)$ | $N(na+nb+nc)$ | $\langle\pi(na+nb+nc)\rangle$ |

Table 5.3: The parameter settings are discretized into a finite set of rules for each strategy class A,B,C. The number of rules in each class ($na$, $nb$ and $nc$ resp.) may differ depending on the number of parameters in the strategy and the cardinality of the discretized parameter range.

**Internal Classifier System for each agent.**

| rule id. | avg. performance | attraction | choice prob. |
|---|---|---|---|
| 1 | $\langle\pi(1)\rangle$ | $A(1)$ | $p^i(1)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $na$ | $\langle\pi(na)\rangle$ | $A(na)$ | $p^i(na)$ |
| $na+1$ | $\langle\pi(na+1)\rangle$ | $A(na+1)$ | $p^i(na+1)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $na+nb$ | $\langle\pi(na+nb)\rangle$ | $A(na+nb)$ | $p^i(na+nb)$ |
| $na+nb+1$ | $\langle\pi(na+nb+1)\rangle$ | $A(na+nb+1)$ | $p^i(na+nb+1)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $na+nb+nc$ | $\langle\pi(na+nb+nc)\rangle$ | $A(na+nb+nc)$ | $p^i(na+nb+nc)$ |

Table 5.4: The attractions for each rule in the population are registered in the agents' internal classifier system, based on the average performance registered in the external, system level Learning Classifier System. The internal LCS of the agent transforms these performances into attractions, which implicitly includes a finite memory of past performances of the rule (due to the EWA learning rule that discounts previous period attractions).

```
Algorithm 5.6.1 LEARNING ALGORITHM

function INITIALIZATION
    for Learning Classifier System do                                   ▷ LCS
        create a random population of N_pop bitstrings
        set all performances to zero
        set all user counters to zero
    end for
    for EWA learning do
        set experience to N(0) = 1
        set all attractions to A(0) = 0
    end for
end function

function MAIN
    use current rule in the market                       ▷ Market environment
    for every T_LCS periods do
        report performance to LCS
        run LCS routine
        run EWA learning
    end for
end function

function LCS                                      ▷ Learning Classifier System
    update all rule performances
    every T_GA periods run GA routine
end function

function EWA LEARNING                                          ▷ EWA learning
    read all updated rule performances from LCS
    compute attractions for all current rules in LCS
    for Multi-logit rule do                                  ▷ Multi-logit rule
        read attractions
        compute choice probabilities
        select rule at random according to choice prob.
    end for
end function
```

Figure 5.3: Pseudocode for a learning algorithm.

**Algorithm 5.6.2** LEARNING ALGORITHM

```
function GA                                                    ▷ Genetic Algorithm
    read current generation of bitstrings from LCS
    read current fitness of bitstrings from LCS
    Selection/Reproduction
    Cross-over
    Mutation
    Election
end function
function SELECTION/REPRODUCTION                                ▷ Reproduction
    create fitness based probabilities
    draw 2N_rep random copies from the LCS using fitness based probabilities
    (2N_rep can be a fixed percentage of the pop. size N_pop)
    create N_rep parent pairs by random matching
    (drawing is with replacement using uniform probabilities)
end function
function CROSS-OVER                                            ▷ Cross-over
    draw random cross-over point between [1, L − 1]
    for Parent pair 1 : N_rep do
        if rand > p_cross then
            perform single-point cross-over between parent pair
            add 2 offspings to potential new generation
        else
            2 offspring bitstrings are identical copies of parents
        end if
    end for
end function
function MUTATION                                              ▷ Mutation
    for Potential new bitstrings 1 : 2N_rep do
        if rand > p_mut then
            draw random mutation
            apply mutation to the bitstring
        end if
    end for
end function
function ELECTION                                              ▷ Election
    for Potential new bitstrings 1 : 2N_rep do
        test for higher fitness between 2 offspring and 2 parents
        add 2 out of 4 best bitstrings to new generation
    end for
end function
```

Figure 5.4: Pseudocode for a learning algorithm (cont.).

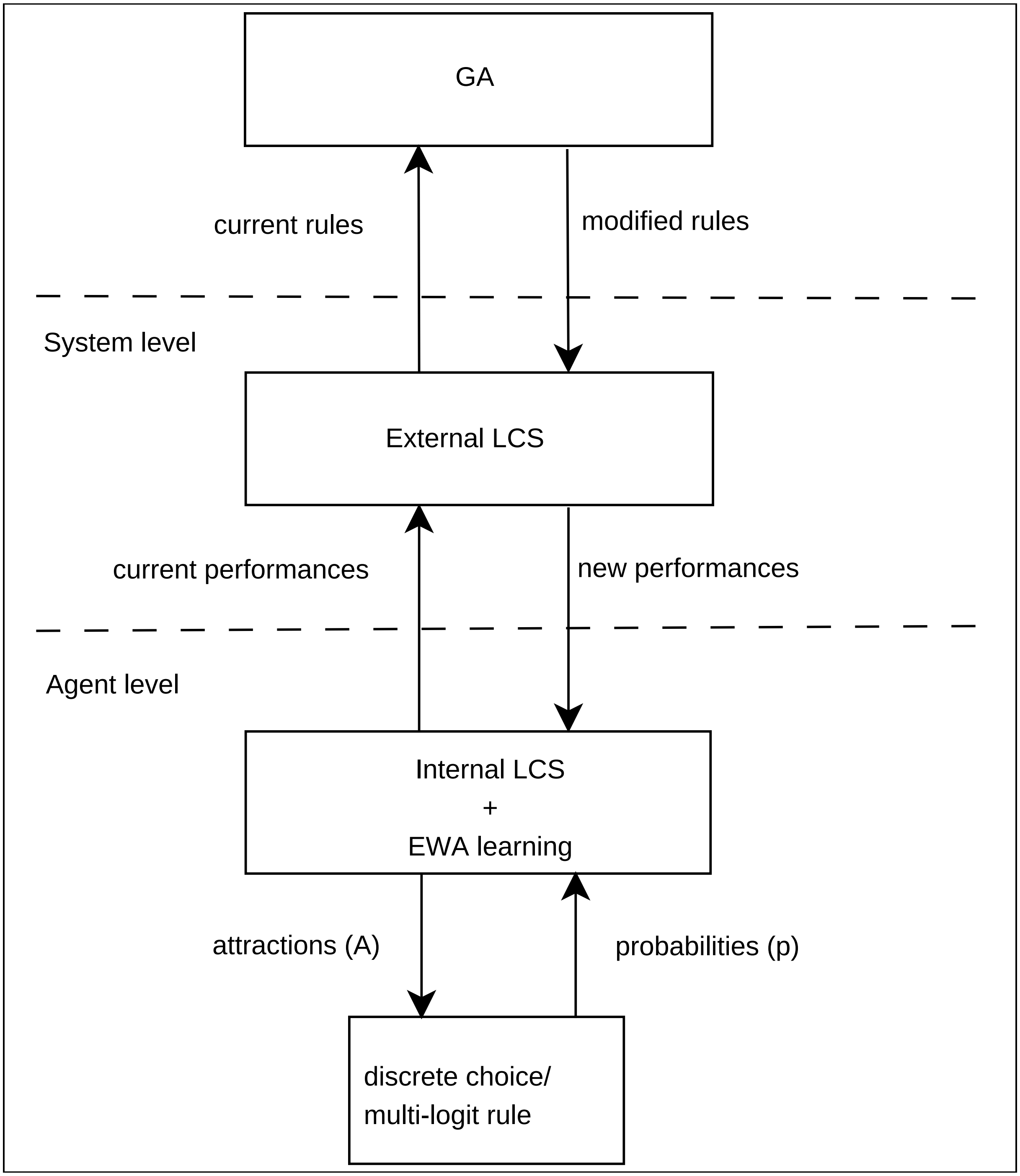


Figure 5.5: Diagram of the learning mechanism. The updating of the rule set in the Learning Classifier System occurs at the system level, through the application of a Genetic Algorithm that modifies the rules' parameter settings. The EWA learning is occurring at the agent level and concerns the selection of rules on the basis of the performances registered in the external, system level Learning Classifier System. The internal Learning Classifier System of the agent transforms system-level performances into agent-level attractions, which implicitly includes a finite memory of past performances.

| Initialization | |
|---|---|
| NrHouseholds | 200 |
| NrFirms | 10 |
| NrMonths | 200 |
| NrDaysInMonth | 10 |
| NrMonthsInitialization | 24 |
| TotalDays | 2240 |
| EWA variables | |
| $\rho$ | 1 |
| $\delta$ | 1 |
| $\phi$ | 0 |

Table 5.5: Parameter settings for simulation runs.

| Simulation 1: | Clearing house | |
|---|---|---|
| rule classes | multiple MPT | multiple RND |
| horizon | 1:10 | 1:10 |
| memory | 10 | 10 |
| risk aversion | 3:5 | - |
| total rules | 30 | 10 |
| Simulation 2: | Limit-order book | |
| rule classes | single MPT | single RND |
| horizon | 1 | 1 |
| memory | 10 | 10 |
| risk aversion | 3 | - |
| total rules | 1 | 1 |
| Simulation 3: | Limit-order book | |
| rule classes | multiple MPT | multiple RND |
| horizon | 1:10 | 1:10 |
| memory | 10 | 10 |
| risk aversion | 3:5 | - |
| total rules | 30 | 10 |

Table 5.6: Simulation runs to test the learning mechanism.

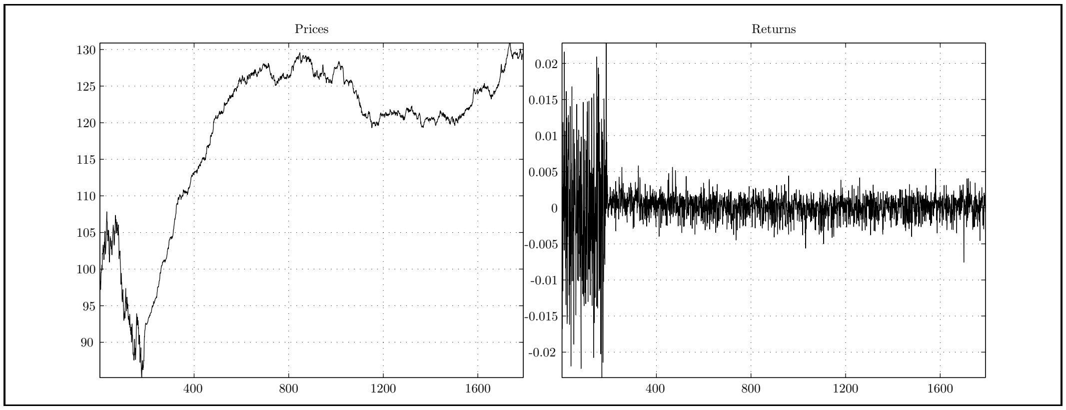


Figure 5.6: Simulation 1. Graph of asset prices and asset returns for a single asset.

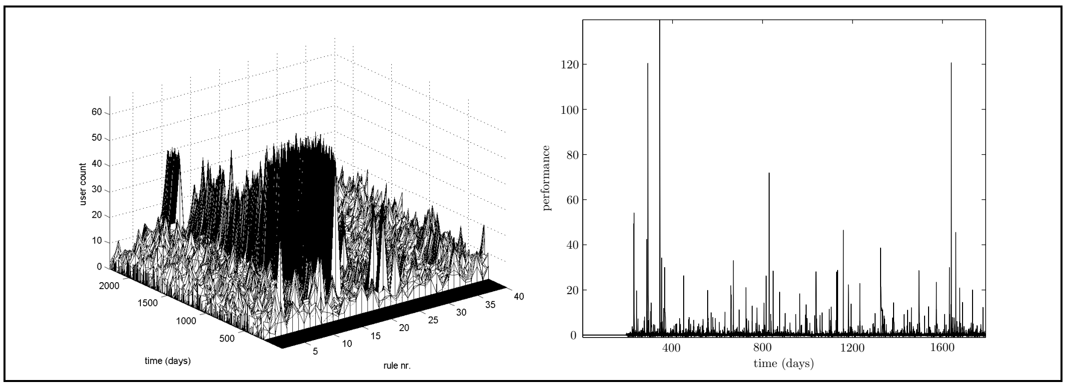


Figure 5.7: Simulation 1. Graph of rule usage (left) and rule performance (right). Plotted is the average performance of the rule (per time step) at each timestep.

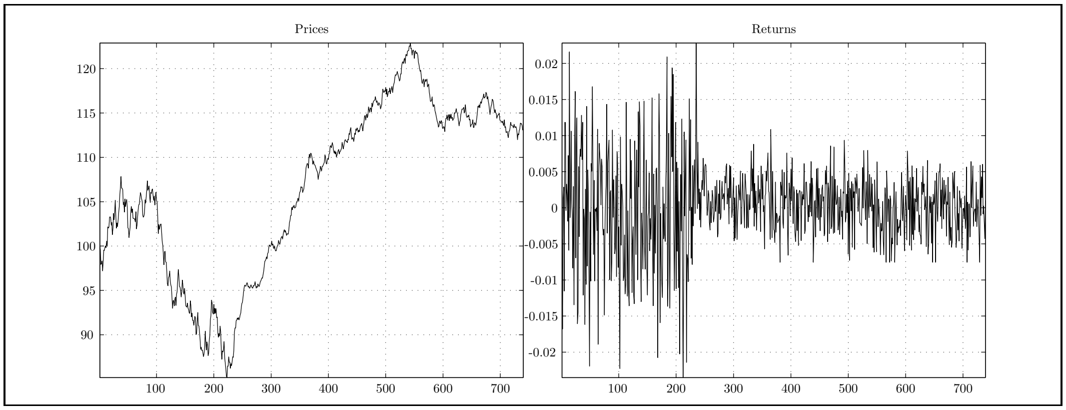


Figure 5.8: Simulation 2. Graph of asset prices and asset returns for a single asset.

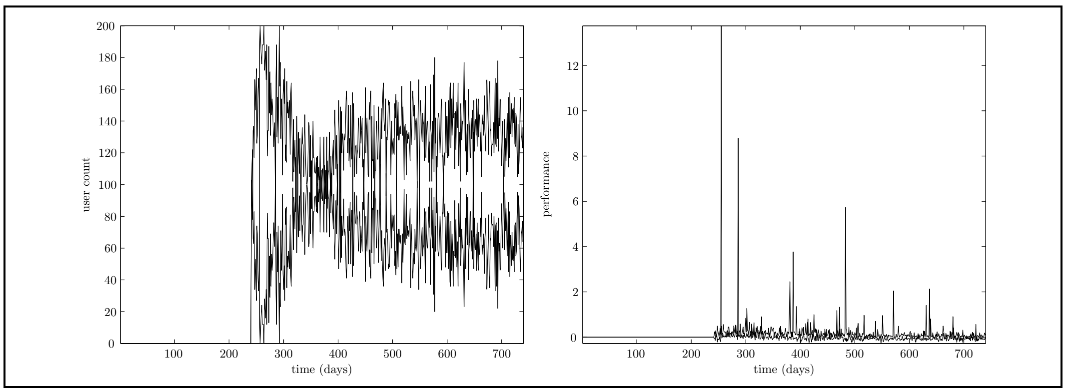


Figure 5.9: Simulation 2. Graph of rule usage (left) and rule performance (right). Plotted is the average performance of the rule (per time step) at each timestep.

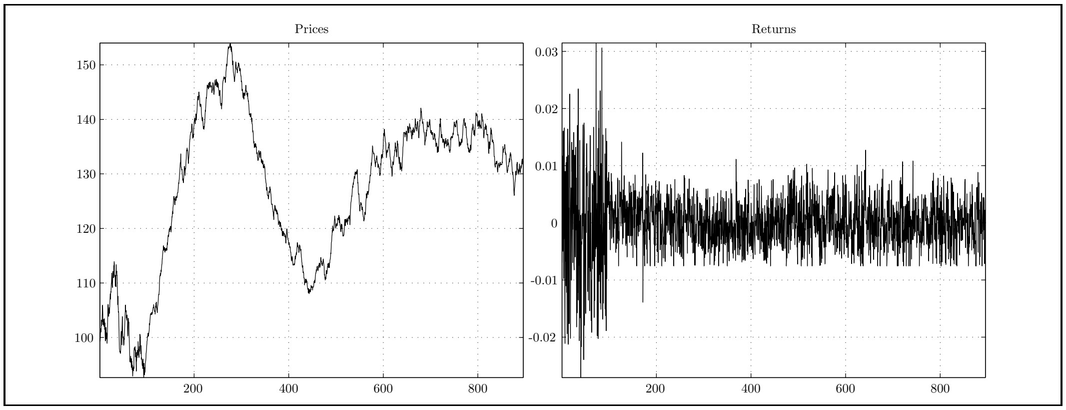


Figure 5.10: Simulation 3. Graph of asset prices and asset returns for a single asset.

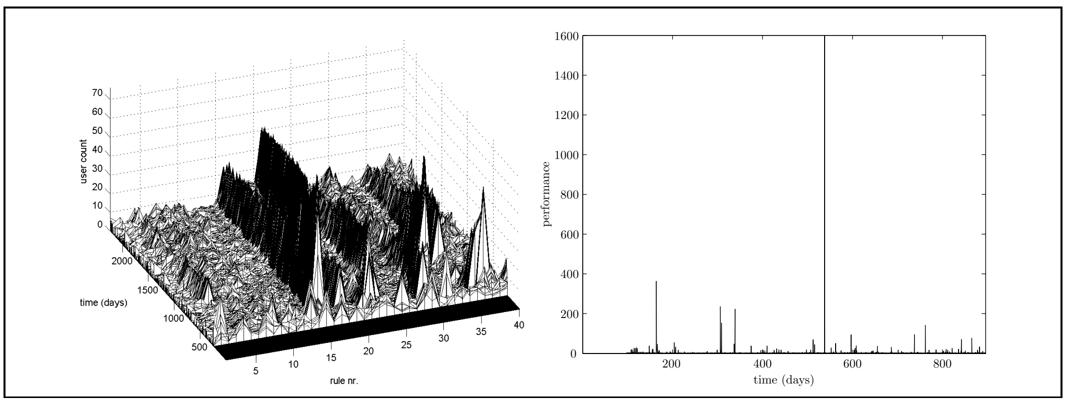


Figure 5.11: Simulation 3. Graph of rule usage (left) and rule performance (right). Plotted is the average performance of the rule (per time step) at each timestep.

# Chapter 6

# A model of firms' financial policy decisions

## 6.1 Introduction

An important part of agent-based macroeconomic modelling is to link the real economy to the financial economy. This is due to the fact that macroeconomic variables and the financial market are invariably connected, and the evolution of macroeconomic variables and financial variables are correlated along the business cycle. This is also the reason why in finance one speaks about 'economic fundamentals' to explain long-term fluctuations of financial variables, and about 'stochastic shocks' to explain short-run fluctuations away from the fundamentals.

As was stated in the introduction to Chapter 1, this document presents the models considered for the financial decision making of the economic agents in the EURACE Project. In Chapters 2-5 we discussed the financial decision making of households which dealt with modelling the households' belief formation, portfolio selection algorithms (optimization versus prospect theory), and learning.

In this chapter the objective is to provide a model for the financial policy decisions of the firms, and so to provide the link between the economic fundamentals and the financial variables. We try to connect neatly to the Bielefeld document that describes the modelling of the firms from the production side (i.e., the real economy). Section 3.1 of that document ends with the following sentence (Dawid et al. 2007, p. 14):

> 'The last aspect of considering the supply side of [the] consumption goods market is the matter of financing the needed investments. In general, all investments might be financed by own profits, savings or, if available by bank credits. In special cases financing might be possible by government subsidies (e.g. for training or R&D).'

This is exactly the point at which we continue to model the firm's behavior. The firms in EURACE are all corporations. Therefore we will rely on corporate finance and capital management theory for modelling the financial decisions. Firm types that are not considered include partnerships, sole proprietorships, and not-for-profit organizations.

There are several sources on behavioral theories of the firm, starting with Cyert and March (1963) and Myers (1984), dealing respectively with the behavioral theory of the firm and Pecking Order Theory, a behavioral theory of corporate finance. We also draw from Tirole (2006, Ch. 2) for information about firms' financing policy and payout policy. An overview of payout policies is given by Allen and Michaely (2004). We also have used the recent survey by Brav et al. (2004) that gives insights into the behavior of financial executives. They report the results from surveys with 384 financial executives and 23 in-depth interviews on the determinants of dividend and share repurchase decisions.

We are using the general-to-specific approach. First we describe the general decisions to be taken by the firm, and only then do we continue to describe them in more detail. The behavioral rules are taken as much as possible from what is used in practice. This means that we rely on practice to give us a description of what steps are actually taken by firms in order to finance their production plan.

We start by listing the general assumptions that are used in this chapter, followed by the sequence of decisions to be taken by the firm before the production cycle starts. This is followed by an overview of the empirical literature, listing some empirical findings and stylized facts concerning the financial policy decisions. Finally, we discuss a behavioral model of the firm's payout policy (dividend payout, share repurchase decisions, debt repayment decisions) and it's production financing policy (retained earnings, applications for bank loans).

## 6.2 General assumptions

- The production plan and the financial policy are *not* taken simultaneously: the production plan is made before the financial policy is decided on.
- This implies that new shareholders (in case of newly issued equity) do not have control over the production decision for the current period. Hence, preferred stocks and common stocks are indistinguishable and the price of equity on the primary and secondary equity markets are the same.
- We abstract from dividend payments made in common stock (which dilutes the stock value). All dividend payments are assumed to be made in cash.
- Stock buybacks are included.
- We abstract from corporations that buy equity in other corporations (for the moment).
- The equity and bond markets clear.
- We do not assume that corporate financial policy is indeterminate and irrelevant as suggested by the Modigliani-Miller Theorem.

The last point may need some qualification. A summary of the Modigliani-Miller Theorem (1958) is the following:

> '...with well-functioning markets (and neutral taxes) and rational investors, who can 'undo' the corporate financial structure by holding positive or negative amounts of debt, the market value of the firm – debt plus equity – depends only on the income stream generated by its assets. It follows, in particular, that the value of a firm should not be affected by the share of debt in its financial structure or by what will be done with the returns paid out as dividends or reinvested (profitably).' (Modigliani 1980, p. xiii)

Another statement of the same theorem is:

> 'In the absence of taxes, bankruptcy costs, and asymmetric information, and in an efficient market, the value of a firm is unaffected by how that firm is financed. It does not matter if the firm's capital is raised by issuing stock or selling debt. It does not matter what the firm's dividend policy is. [...]
>
> The MM-theorems do not pretend to describe how managers make decisions: they generate testable hypotheses about capital market outcomes that follow from firms' policies, not about whatever arcane means firms use to make or to implement their policies.' (Bailey 2004, p. 1)

However, since we are interested in how managers make decisions, and how firms make their financial policies, we will want to include in our models a description of actual firm behavior. This should include a description of how firms behave in the presence of imperfect capital markets and when there are transaction costs, and hence a trade-off exists in how a firm finances its production.

## 6.3 Financial policy decisions by producers

Figure 6.1 shows the sequence of decisions to be taken by the producers before it starts the production cycle. The figure refers to Consumption Good Producers (CGPs) since it includes the investments in new technologies and on-the-job training for production workers. Considering the Investment Goods Producers, their decision sequence is the same (apart from these two steps that should be ignored).

Below we describe the decisions how to finance the production plan, and the decisions that are taken right after the production cycle finishes.

**Decisions to be taken before the production cycle starts** Before the firm can enter the production cycle it has to make certain irreversible decisions:

- The target production level $Q$.
- Investment in R&D employees.
- Investment in new technology.

- Investment in training to increase the technology-specific skill levels of its production workers.

Once the firm has invested in R&D, it has hired R&D employees with a high general skill level, it has invested in a new technology (new machines) to produce, it has acquired additional labor, and it has invested in training of production workers to increase their technology-specific skill levels, the firm can now start production. At this moment the inputs for production (energy, labor) have already been contracted, but the financial policy for the production cycle is not yet in place.

**Decisions to be taken before actual production starts**

- How much debt to obtain from banks (business credits).
- How much equity to issue to the financial market (stocks and corporate bonds).
- How much dividends to promise on the stock, and how much bond coupons to promise on the bonds.

After the firm has obtained sufficient amounts of financial capital to finance the production plan, it now enters the production cycle. (Here: if the firm cannot obtain all the credit it needs, or the stocks are sold at a price that does not yield enough financial capital, does it adjust the production plan? Do we assume credit rationing on the credit market? Do we assume a perfect financial asset market?)

**Production** Production takes time, defined by the production period. So from the moment the firm's financial policy is in place, it can start production, and the next event is that the produced output is ready.

**Decisions to be taken after production but before selling**

- Pay the wage bill, and the energy bill.
- Distribute the output to the outlet stores.

It is an important modelling decision whether the wage bill is paid before or after production. Here we chose to pay the wage bill after production.

**Decisions to be taken after selling but before a new production cycle starts** The firm needs to decide how much of the total result from sales revenues (the earnings) is allocated to:

- retained earnings,
- debt repayments,
- interest payments,
- dividend payments,

- coupon payments,
- tax payments.

The earnings can be used to pay dividends, bond coupons and to make interest payments and debt repayments. The retained earnings are reinvested into the firm and are used to finance the new production cycle, i.e. to buy new inputs, allocate funds to the wage fund, make investments in R&D, invest in new technologies. In principle, the dividend payments are paid out of free cash flows, which comprise the cash available after all business expenses have been met. Fig. 6.2 shows the financial inflows and outflows of the firm's operations.

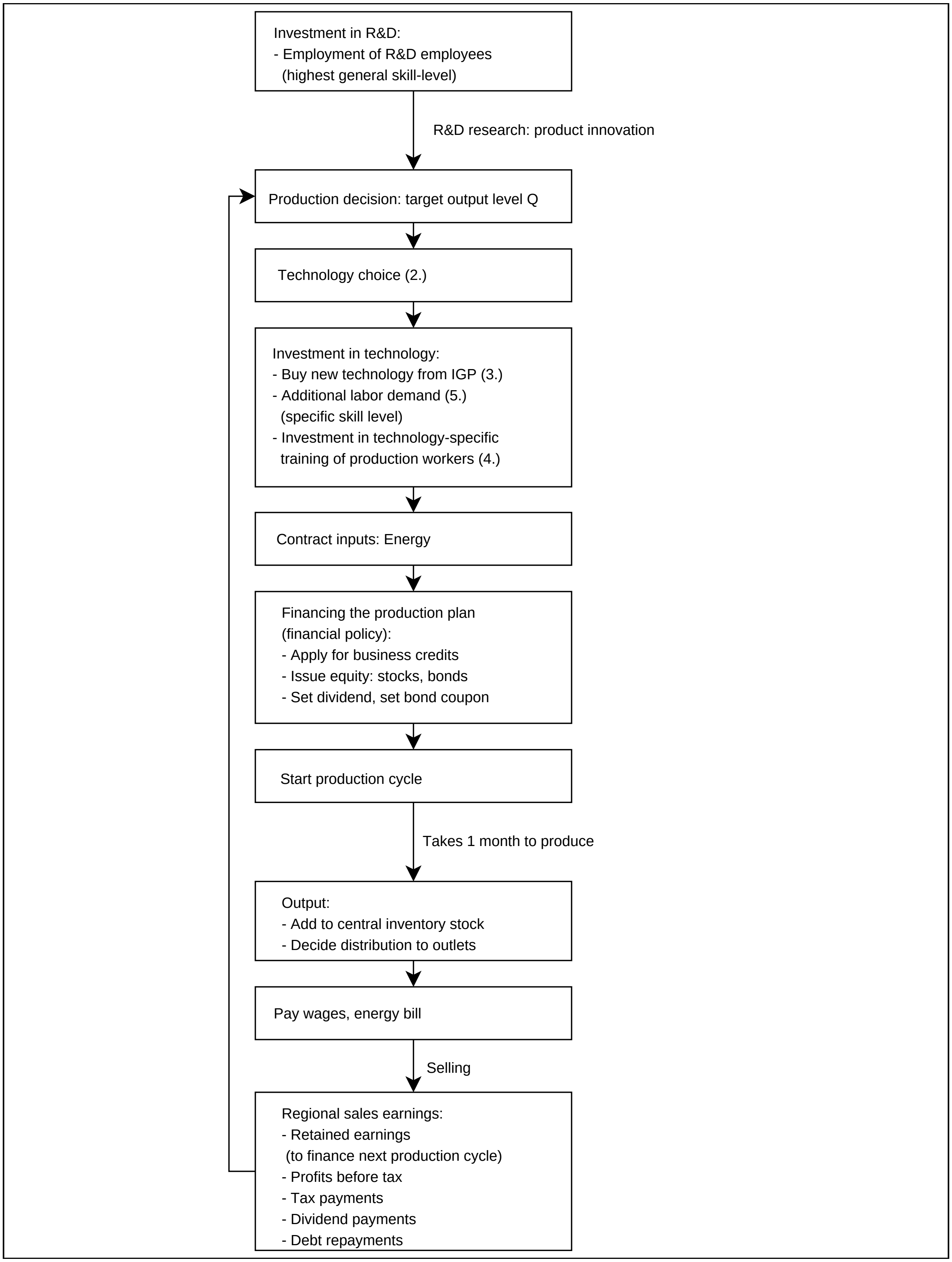


Figure 6.1: Diagram of a firm's financial policy decisions. The figure specifically refers to Consumption Good Producers (CGPs) since it includes the investments in new technologies and on-the-job training for their production workers. For the Investment Goods Producers the decision sequence is the same, only these two steps are removed.

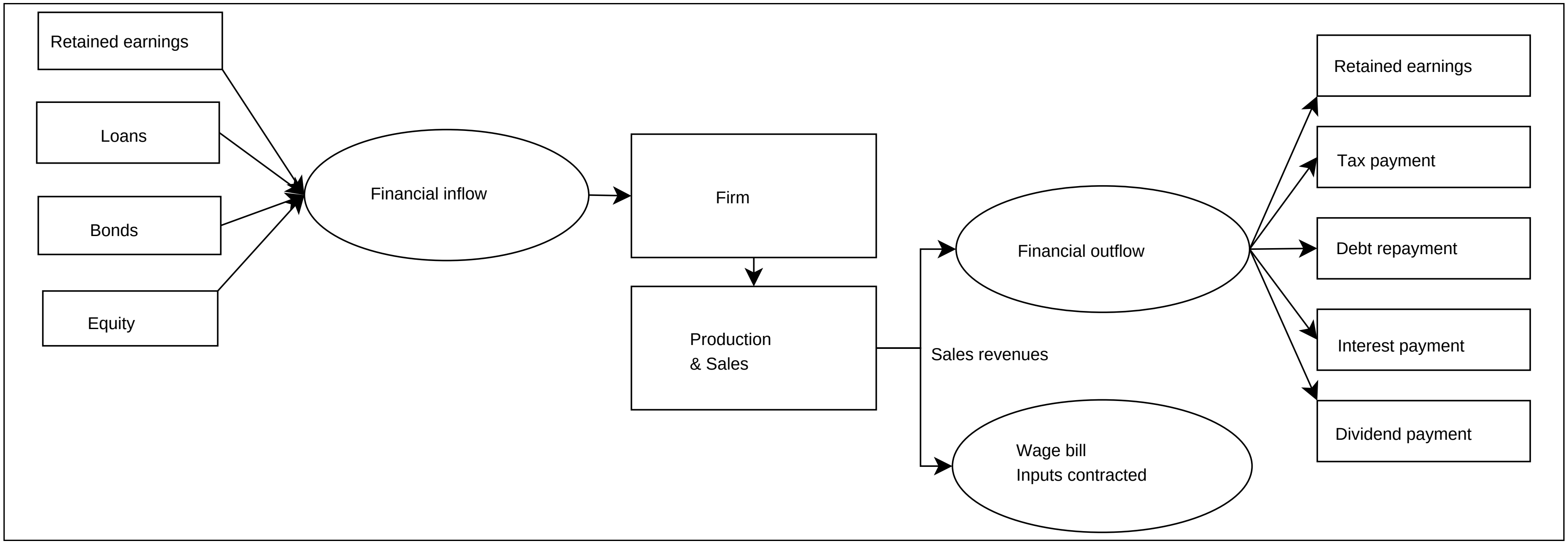


Figure 6.2: Diagram of the firm's financial inflows and outflows.

## 6.4 Financial policy rules

> 'One of the main theories of how firms make their financing decisions is the Pecking Order Theory, which suggests that firms avoid external financing while they have internal financing available and avoid new equity financing while they can engage in new debt financing at reasonably low interest rates. Another major theory is the Trade-Off Theory in which firms are assumed to trade-off the Tax Benefits of debt with the Bankruptcy Costs of debt when making their decisions. One last theory about this decision is the Market Timing Hypothesis which states that firms look for the cheaper type of financing regardless of their current levels of internal resources, debt and equity.'[1]

**Pecking Order Theory** The Pecking Order Theory (Myers 1984) is a positive theory which states that companies prioritize their capital structure according to a hierarchy of financial resources:

1. Internal financing (retained earnings) is preferred as the primary source.
2. Debt, since it is a relatively safe source.
3. Bonds, since these are safer than other securities.
4. Equity, which is used as the 'means of last resort' for financing, since it is risky.

Internal funds are used first, and only if external financing is required, the firm prefers debt over equity. Therefore, 'there is no well-defined target debt-equity mix, because there are two kinds of equity, internal and external, one at the top of the pecking order and one at the bottom.' (Zoppa and McMahon 2002, p. 3)

There is empirical evidence pointing towards some additional explanatory value of the Pecking Order Theory over the Optimal Capital Structure Theory (i.e., the capital structure irrelevance hypothesis of Modigliani and Miller which states that capital structure is irrelevant for the financing needs of the firm). The problem with the Optimal Capital Structure Theory is that it tries to find the optimal combination of financial resources, i.e. and optimal mix of debt and equity. Liesz (2005) reports surveys examining the capital structure decisions made by real businesses, both observed and self-reported, stating that a large majority of firms uses a hierarchy of financial resources instead of targeting an optimal capital structure.

### 6.4.1 Empirical evidence on financial resources

Table 6.1 (reproduced from: Tirole 2006, p. 96, Table 2.2) lists the way in which average nonfinancial enterprises use various sources of financing, as a percentage of the total financing needs.

A first observation from Tables 6.1-6.2 is that if we only consider the European countries, and we restrict ourselves to consider only the following financial resources: retained

[1] Source: Wiki page on Corporate Finance, `http://en.wikipedia.org/wiki/Corporate_finance`.

Table 6.1: Average financing of nonfinancial enterprises, 1970-1985. Source: Mayer (1990). Reproduced from Tirole (2006, p. 96), Table 2.2.

| | Canada | Finland | France | Germany | Italy | Japan | UK | US |
|---|---|---|---|---|---|---|---|---|
| Retained earnings | 54.2 | 42.1 | 44.1 | 55.2 | 38.5 | 33.7 | 72.0 | 66.9 |
| Capital transfers | 0.0 | 0.1 | 1.4 | 6.7 | 5.7 | 0.0 | 2.9 | 0.0 |
| Short-term securities | 1.4 | 2.5 | 0.0 | 0.0 | 0.1 | n.a. | 2.3 | 1.4 |
| Loans | 12.8 | 27.2 | 41.5 | 21.1 | 38.6 | 40.7 | 21.4 | 23.1 |
| Trade credit | 8.6 | 17.2 | 4.7 | 2.2 | 0.0 | 18.3 | 2.8 | 8.4 |
| Bonds | 6.1 | 1.8 | 2.3 | 0.7 | 2.4 | 3.1 | 0.8 | 9.7 |
| Shares | 11.9 | 5.6 | 10.6 | 2.1 | 10.8 | 3.5 | 4.9 | 0.8 |
| Other | 4.1 | 6.9 | 0.0 | 11.9 | 1.6 | 0.7 | 2.2 | -6.1 |
| Statistical adjustment | 0.8 | -3.5 | -4.7 | 0.0 | 2.3 | n.a. | -9.4 | -4.1 |
| Total | 99.9 | 99.9 | 99.9 | 99.9 | 100 | 100 | 99.9 | 100.1 |

Table 6.2: Percentage of total financing needs for European enterprises.

| | Finland | France | Germany | Italy | UK | | |
|---|---|---|---|---|---|---|---|
| Ret. Earnings | 42.1 | 44.1 | 55.2 | 38.5 | 72 | | |
| Loans | 27.2 | 41.5 | 21.1 | 38.6 | 21.4 | | |
| Bonds | 1.8 | 2.3 | 0.7 | 2.4 | 0.8 | | |
| Shares | 5.6 | 10.6 | 2.1 | 10.8 | 4.9 | | |
| Sum | 76.7 | 98.5 | 79.1 | 90.3 | 99.1 | | |
| % of sum | | | | | | mean | factors |
| Ret. Earnings | 0.55 | 0.45 | 0.70 | 0.43 | 0.73 | 0.569 | 0.59 |
| Loans | 0.35 | 0.42 | 0.27 | 0.43 | 0.22 | 0.337 | 1.00 |
| Bonds | 0.02 | 0.02 | 0.01 | 0.03 | 0.01 | 0.018 | 18.67 |
| Shares | 0.07 | 0.11 | 0.03 | 0.12 | 0.05 | 0.075 | 4.48 |
| Total | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | |

earnings, loans, bonds and shares, then we cover almost all of the financial resources being used.

We obtain the following distribution of resources (see the lower part of Table 6.2). On average 56.9% of all financing comes from internal resources (retained earnings), on average 33.7% comes from bank loans, 1.8% comes from bonds, and 7.5% comes from shares. If we consider the per country distribution, the bank loans are a predominant factor, exceeding any financing through shares by a factor of 4. For bonds this is an even larger multiplication factor of 18. The total size of bank loans are roughly 60% of the total size of retained earnings (33.7% versus 56.9%, on average).

In conclusion, it seems that bond financing is negligible with a mean of 1.8% and stock financing is also rather small, with a mean of 7.5% of total financial resources used. Equity financing thus seems to be a minor source of financing for most European enterprises. For the most part, firms use retained earnings and bank loans.

**Proposed financial policy rule** A proposed behavioral rule would be to simply use the empirical data above and input it as the behavioral rule of the firms in EURACE. This automatically means a geographical distribution of the way firms are financing their production. To have some variability in the firm population, we can consider the proportions of each financing source to be random variables. For simplicity, the variance is taken to be one tenth of the mean. For example, for France the fraction of internal financing as a percentage of the total financing needs is assumed to be normally distributed $x \sim N(\mu_{FR}, \sigma^2_{FR})$ with mean $\mu_{FR} = 44.1\%$ and variance $\sigma^2_{FR} = 4.41\%$. The fraction of loans as a percentage of the total financing needs is normally distributed with $x \sim N(41.5, 4.15)$, etc. This approach may seem ad hoc, but it is the closest we can get to the empirical data without needing to make strong assumptions about the underlying decision-making process. We just take the data as it is and use it as an input to the simulations.

So according to Table 6.2, the ratio of loans obtained as a percentage of total financing needs would lead to a distribution of a parameter $\theta^{loans}$ for each European country. For France: $\theta^{loans}_{FR} \sim N(\mu, \sigma^2)$ with mean $\mu = 0.42$ and variance $\sigma^2 = 0.042$. For Germany: $\theta^{loans}_{GE} \sim N(0.27, 0.027)$. For Italy: $\theta^{loans}_{IT} \sim N(0.43, 0.043)$. And similarly for the issuing of bonds and shares.

**Market timing: the equity-over-debt ratio** We now turn to the external financing. How much of external financing should be coming from debt and how much from equity? It is a well-documented fact that an issue of new equity will result, on average, in a permanent fall of the stock's price of 3%. The stock market price generally rises after a bank loan agreement is announced (Tirole 2006, p. 100).

The ratio of equity over debt is positively correlated to the business cycle, due to the following features:

- There is a counter-cyclical demand for short-term credit. During a boom firms can issue new equity when stock market prices are high. During recessions firms turn to bank loans, or they buy back shares to take advantage of the low market prices.
- Equity issues are more frequent in upswings than in downswings of the business cycle. This has to do with the market timing: firms issue shares when stock prices are up, and repurchase them when the share value is low.
- A drop in the stock market price immediately after an issue of new equity is smaller in an expansion than in a contraction.

Thus, the distribution of external sources for financing is triggered by the business cycle. (This has not been modelled so far).

**Working capital and short term financing** To finance short-term production the firm needs working capital. Working capital management relates to the upcoming one year period, and it entails managing the firm's short-term assets and short-term liabilities. It thus serves to provide the firm with sufficient cash flow to continue its current operations.

To make decisions regarding the short-term financing of production, the firm uses two measures of profitability:

- Return on capital (ROC): a percentage, determined by the relevant income over the previous 12 months, divided by the capital employed.
- Return on equity (ROE): a percentage, determined by the stock returns for the firm's shareholders over the previous 12 months, divided by the capital employed.
- Debt-to-equity ratio (DER): a firm which has a high profitability has a low debt-to-equity ratio.

(This has not been modelled so far).

## 6.5 Firm payout policy

We now turn to the question how the firm sets its dividends, and more generally, the firm's payout policy. According to textbook corporate finance theory (i.e. the Modigliani-Miller theorem) it does not matter how the firm pays out dividends, either in cash or in stock, and the dividend policy is irrelevant/indeterminate. However, in reality it does make a difference whether a corporation makes its payments in cash dividends or via a share buyback. In case shareholders need to pay tax on dividends, it is more profitable for them if the firm performs a buyback, since it increases the value of the outstanding shares. Another possibility is to pay the dividends in new shares rather than in cash, but this decreases the value of the outstanding shares by diluting it.

### 6.5.1 Empirical findings & stylized facts

In Brav et al. (2004) the authors report results on surveys conducted with 384 financial executives and 23 in-depth interviews on the determinants of dividend and share repurchase decisions. The population of firms was divided into two main categories: 256 publicly owned firms (having outstanding shares) and 128 privately owned firms. The public companies are divided among three classes: dividend paying firms (166), share repurchasing firms (167), and firms that do not currently pay out (77). The main findings are the following.

For dividend paying firms (166 out of 256):

- Dividend decisions are made before or simultaneously with investment decisions. Dividends are not paid out of free cash flows, but sometimes even a bit before the investment decisions are made. Investment opportunities therefore do not affect how much to pay out in dividends. If there are investment opportunities then managers prefer to raise new funds than lowering the pay out.
- Dividend decisions are very conservative. Dividend payments are downward rigid, and are strongly history dependent: retaining the historical level of dividend payments is important, i.e. keeping a constant dividend-to-earnings ratio. Firms that

pay dividends do not want to decrease them. They try to maintain the historic dividend policy. The reason for such dividend conservatism is that there is not much reward in increasing dividends but there is a high penalty for reducing them. Managers therefore only increase dividend payments in response to permanent changes in earnings.

- The decision variable is *changes* in dividends rather than the dividend level. The new dividend level is determined by a constant dividend-to-earnings ratio, and bounded from below by the level of previous dividend pay outs.

For share repurchasing firms (167 out of 256):

- Repurchase decisions are made after the investment decision, and are more flexible/adjustable than the dividend decision.
- The repurchase decision is negatively affected by investment opportunities: if new opportunities arrive the repurchase decision can be postponed.

If dividends are decreased for some reason then payments get priority in the following order (i.e. this is the substitutability of dividends for other payouts):

- pay down debt
- repurchase shares
- increase investments

However, if share repurchases are decreased, then managers do not choose to increase the dividends symmetrically, since this would create a historical precedent and imply an increase in all the future dividend payments due to the downward rigidity of dividend payments.

### 6.5.2 Allocation rules

The firm's payout policy specifies the proportion of earnings that is distributed to shareholders as dividends and share repurchases. How much of total earnings should be paid depends on a number of factors: the firm's growth opportunities, the financial constraints, and it should also depend on the earnings size.

#### Dividend decision

For the dividend payout decision we assume that firms use two rules of thumb:

1. Per share dividends $d_t^f$ should not decrease with respect to the historical level of dividend payments.
2. The manager (CFO) wants to maintain a constant dividend-to-earnings ratio, $D_t^f = \theta^{div} EARN_t^f$, $0 \leq \theta^{div} < 1$.

The dividend-to-earnings ratio $\theta^{div} = D_t^f / EARN_t^f$ is the percentage of earnings that are paid out to investors. We assume that managers target the dividend per share instead of the total dividend payout level. The first rule reflects the empirical observation that dividend payments are conservative: managers do not want to decrease the per share dividend payout. The second rule is less strictly maintained than the first, but gives some guidence on the target level: the total dividend payout should be positively correlated with the size of the earnings. As a general rule, firms with high earnings should pay out more than firms with low earnings. Both rules are derived from survey data and in-depth interviews with actual financial executives (see Brav et al. (2004)).

**Dividend payout rule** The total dividend payout that maintains a constant dividend-to-earnings ratio is equal to $D_t^f = \theta^{div} EARN_t^f$. Per share dividends are bounded from below by historical dividends per share (the D/S-ratio), hence the dividend payout rule is determined by:

$$D_t^f = \max\{D_{t-1}^f, \theta^{div} EARN_t^f\}, \tag{6.1}$$

$$d_t^f = \max\{d_{t-1}^f, \frac{\theta^{div} EARN_t^f}{s_t^f}\}. \tag{6.2}$$

Here $d_t^f$ is the per share dividend, $EARN_t^f$ is net earnings and $s_t^f$ is the total supply of shares.

**Share repurchasing decision**

Repurchasing of shares is triggered by a historically 'low' stock price, i.e. an undervaluation of the stock. The reason for this is twofold: (i) to improve the stock price, (ii) it is relatively cheap when the stock price is low. The main criteria for repurchasing stock are therefore:

1. If the stock is undervalued: the price-to-earnings ratio drops below a critical value.
2. If there is an excess of cash on the balance sheet.
3. If there are fewer profitable investment opportunities.
4. If the earnings per share ratio (EPS) drops below a certain level that is required to achieve a targeted EPS growth level.
5. Offsetting the earnings dilution effect caused by stock option compensation or employee stock plans.
6. To improve the liquidity of the stock.

**Share repurchasing rules**

**Ad.1** Trigger: Price-to-earnings ratio drops below a critical level.
Action: repurchase shares up to the target level:

```
If PE_RATIO < TARGET_PE_RATIO
   REPURCHASE = REPURCHASE_VALUE / PRICE_STOCK
EndIf
```

$$\text{If } \theta^{pe} < \overline{\theta^{pe}} : \Delta s_t^{f,+} = \frac{REP_t^f}{PRICE_t^f}. \tag{6.3}$$

**Ad.4** Trigger: Earnings-per-share drops below a target *growth* level of the E/S-ratio.
Action: Repurchase shares until the E/S-ratio attains the aspired growth level.

```
EPS_RATIO_GROWTH = EPS_RATIO_t / EPS_RATIO_t-1

If EPS_RATIO_GROWTH < TARGET_EPS_RATIO_GROWTH

   TARGET_EPS_RATIO = EPS_RATIO * (1 + TARGET_EPS_RATIO_GROWTH)

   TARGET_SHARES =  EARN / TARGET_EPS_RATIO

   REPURCHASE = SHARES - TARGET_SHARES

EndIf
```

$$\begin{array}{ll} g^{eps} & = \theta_t^{eps}/\theta_{t-1}^{eps} \\ \text{If} & g^{eps} < \overline{g^{eps}} : \\ & \overline{\theta_t^{eps}} = (1 + \overline{g^{eps}})\theta_t^{eps} \\ & \overline{s}_t^f = EARN_t^f/\overline{\theta_t^{eps}} \\ & \Delta s_t^{f,+} = (s_t^f - \overline{s}_t^f). \end{array} \tag{6.4}$$

**Debt repayment decision**

The propensity to pay down debt increases with the firm's debt ratio, due to managers' concern with the firm's credit rating.

**Debt-to-earnings rule** Target: maintain a constant debt-to-earnings ratio:

$$DEBT_t^f = \theta_t^{f,debt} EARN_t^f. \tag{6.5}$$

Trigger: If the D/E-ratio becomes larger than the previous period D/E-ratio, repay debt such that the new D/E-ratio remains constant. The target debt growth level is equal to the realized earnings growth level:

```
If (DEBT_t+1 / EARN_t+1) > (DEBT_t / EARN_t)

    DEBT_t+1 = (EARN_t+1 / EARN_t) * DEBT_t

EndIf
```

$$\begin{aligned} &\text{If} \quad (DEBT^f_{t+1}/EARN^f_{t+1}) > DEBT^f_t/EARN^f_t : \\ &DEBT^f_{t+1} = \frac{EARN^f_{t+1}}{EARN^f_t} DEBT^f_t . \end{aligned} \tag{6.6}$$

### 6.5.3 A behavioral model of firm payout policy

In Section 6.4.1 above we dealt with the structure of the firm's financial policy, i.e. the firm's cash inflow from various sources of financing, consisting of internal financing (retained earnings) and external financing (loans, bonds, and equity). In this section we turn to the structure of the firm's payments, i.e. its payout policy.

#### Pay out priority

The accounting starts from total revenues from sales. Net revenues are defined as total revenues minus costs of sales. After subtracting operational costs and interest payments on loans and bonds from the net revenues, the firm determines its earnings (profits before tax). Net earnings (after-tax profits) are then defined as earnings minus a tax provision.

Empirically, the order of payout priorities is to first pay out dividends from net earnings, then determine the retained earnings for investments, and then to determine the external payments such as share repurchases and debt repayments:

1. total revenues from sales.
2. net revenues (total revenues minus costs of sales).
3. operational costs: pay the wage bill, energy bill, input contracts.
4. interest payments on loans and bonds.
5. earnings (profits or firm income before tax).
6. tax provision.
7. net earnings (profits after tax).
8. dividend payment decision: maintain a constant dividend-to-earnings ratio.
9. retained earnings and investment decision: financed out of retained earnings, possibly using new loans, equity or bonds.
10. external payout decision:
11. share repurchase decision: maintain a constant earnings-per-share ratio.

12. debt repayment decision: maintain a constant debt-to-earnings ratio (first the firm repays its current debt completely, then it applies for new loans).

This subdivision is illustrated in Fig. 6.3 which shows a hierarchical tree of the financial payout categories of the corporation. Some of these payouts are not really decisions to be taken by the firm, but are commitments that are determined exogenously (i.e., taxes and interest payments).

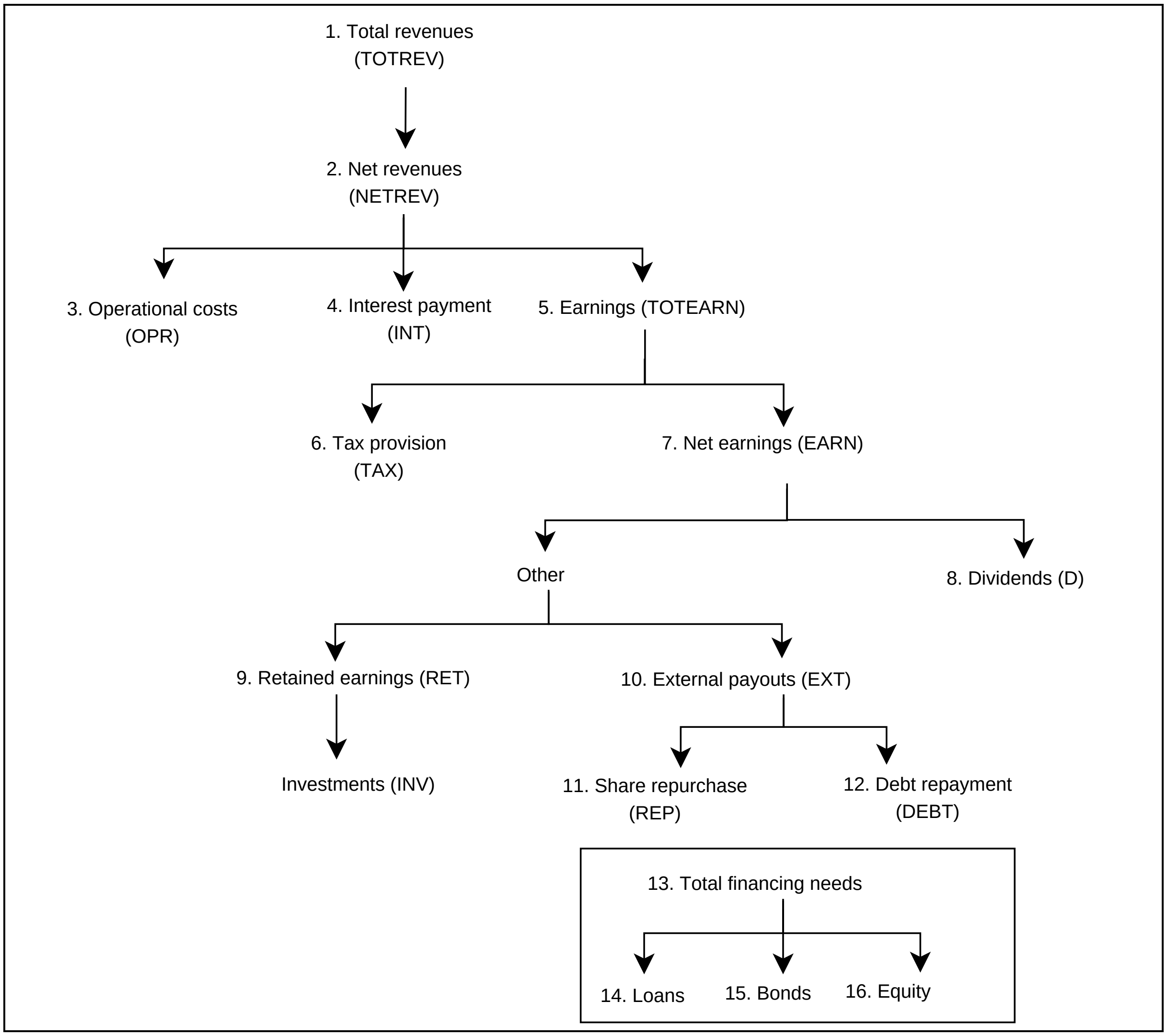


Figure 6.3: Diagram of the hierarchical structure of financial payout categories for a corporation.

#### Decision 1: Total revenues

To determine the total net revenues the firm goes through a sequence of computations (see Fig.6.1). Total net revenues at time $t$ accrue from the total revenues from sales minus the costs of sales over the period $(t-1,t)$:

$$TOTREV_t^f = \sum\nolimits_{\tau=t-1}^{t} REV_\tau^f. \tag{6.7}$$

**Decision 2: Total net revenues**

Net revenues are total revenues minus cost:

$$NETREV_t^f = \sum\nolimits_{\tau=t-1}^{t} REV_\tau^f - COST_\tau^f. \tag{6.8}$$

**Decision 3: Operational costs**

The operational costs consist of paying the wage bill, energy bill and input contracts:

$$OPR_t^f = \sum\nolimits_{\tau=t-1}^{t} w_\tau^f + ENGY_t^f + INPUT_t^f. \tag{6.9}$$

**Decision 4: Interest payment decision**

Interest payments on outstanding debt are made before taxes:

$$INT_t^f = \min\{r_t DEBT_t^f, NETREV_t^f - OPR_t^f\}. \tag{6.10}$$

THE NEXT PART IS UNSURE: If interest payments on outstanding debt cannot be met, does the firm go into bankruptcy?

[If reserved financial resources were insufficient to service the outstanding debt, then new debt must be issued. The target for new loan applications is now:

$$TARGETLOANS_{t+1}^f = \max\{0, r_t DEBT_t^f - INT_t^f\}. \tag{6.11}$$

If the financial resources were sufficient then the remainder becomes the firm's total earnings before taxes.]

**Decision 5: Total earnings**

Total earnings are equal to net revenues minus operational costs and interest payments:

$$TOTEARN_t^f = NETREV_t^f - OPR_t^f - INT_t^f. \tag{6.12}$$

Before-tax profits are equal to earnings, provided that the earnings are positive:

$$PRETAXPROF_t^f = \max\{0, TOTEARN_t^f\}. \tag{6.13}$$

**Decision 6: Tax payments**

Taxes are paid on before-tax profits ($t_c$ is the corporate tax rate):

$$TAX_t^f = t_c PRETAXPROF_t^f. \tag{6.14}$$

If earnings are negative, the firm does not pay any taxes.

#### Decision 7: Net earnings

Net earnings (after-tax profits) are equal to before-tax profits minus tax payments:

$$EARN_t^f = TOTEARN_t^f - TAX_t^f. \tag{6.15}$$

#### Decision 8: Dividend payments

Dividend payments are a constant fraction of net earnings (maintaining a constant dividend-to-earnings ratio, i.e. the D/E-ratio $\theta_t^{f,div}$), provided that this is greater than the historical per share dividend level. Otherwise the dividend policy is conservative and remains at the previous level:

$$D_t^f = \max\{D_{t-1}^f, \theta^{div} EARN_t^f\}, \tag{6.16}$$

$$d_t^f = \max\{d_{t-1}^f, \frac{\theta^{div} EARN_t^f}{s_t^f}\}. \tag{6.17}$$

After dividend payments, the remaining financial resources are used to finance retained earnings and external payouts (share repurchases and debt repayments):

$$X_t^f = EARN_t^f - D_t^f. \tag{6.18}$$

#### Decision 9: Retained earnings and investment decision

Retained earnings are a fraction of net earnings minus dividend payout:

$$RET_t^f = \theta_t^{f,ret}(EARN_t^f - D_t^f). \tag{6.19}$$

After retained earnings, the remaining financial resources for external payouts are given by:

$$X_t^f = (1 - \theta_t^{f,ret})(EARN_t^f - D_t^f). \tag{6.20}$$

Note: This is not so much a decision, but a computation that might be informative to the firm. Note that the total sum of external payouts is not restricted to lie below the remaining financial resources $X_t^f$, since the firm can apply for additional loans, or issue new equity (this will be discussed below, after the total financial needs have been determined).

#### Decision 10: Share repurchase decision

**Price-to-earnings ratio** If the price-to-earnings ratio $\theta_t^{f,pe}$ drops below a critical level $\overline{\theta}^{f,pe}$, the firm buys back shares until the stock price has increased such that the target price-to-earnings ratio is attained:

$$\theta_t^{f,pe} = PRICE_t^f / EARN_t^f. \tag{6.21}$$

$$\text{If } \theta^{pe} < \overline{\theta^{pe}} : \Delta s_t^{f,+} = REP_t^f / PRICE_t^f. \tag{6.22}$$

(How exactly the repurchasing of shares affects the stock price has not yet been modelled so far.)

**Earnings-per-share ratio** If the target is to obtain a certain growth in the earnings-per-share ratio, then the firm buys back shares such that a certain aspiration growth level is attained. The target growth level is given by: $\overline{\theta^{f,eps}}_t = (1+\bar{g})\theta^{f,eps}_{t-1}$. The earnings-per-share ratio is given by:

$$\theta^{f,eps}_t = EARN^f_t / s^f_t. \tag{6.23}$$

The target number of shares is therefore:

$$\bar{s}^f_t = EARN^f_t / \overline{\theta^{f,eps}}_t. \tag{6.24}$$

The firm performs a share repurchase to obtain the target growth level in the earnings-per-share ratio:

$$REP^f_t = PRICE^f_t(s^f_{t-1} - \bar{s}^f_t), \text{ if } \bar{s}^f_t < s^f_{t-1}. \tag{6.25}$$

where $REP^f_t$ is the total value of the share repurchase.

### Decision 11: Debt repayment decision

Before applying for new loans, the firm first repays its current debt in full to the bank at which it had obtained this loan. So the debt repayment is simply given by:

$$REPAY^f_t = DEBT^f_t. \tag{6.26}$$

After the share repurchase and the debt repayments, the firm's remaining financial resources are:

$$X^f_t = NETEARN^f_t - D^f_t - RET^f_t - REP^f_t - REPAY^f_t. \tag{6.27}$$

### Decision 12: Determination of the total financial needs

Now that we have determined how firms set their payout policy, we can compute the total financing needs. This serves as an entry point to determine the financial resources that the firm needs, in addition to its net earnings. The total financing needs are equal to the sum of all payouts as illustrated in Fig. 6.3:

$$NEEDS^f_t = OPR^f_t + INT^f_t + TAX^f_t + D^f_t + RET^f_t + REP^f_t + REPAY^f_t. \tag{6.28}$$

The additional financial resources the firm needs to obtain, by applying for new loans and issuing of shares and bonds, are given by the difference between the total net revenues and the total financing needs:

$$ADD^f_t = NETREV^f_t - NEEDS^f_t. \tag{6.29}$$

### Decision 13: Determination of the level of new loans and new equity

The maximum debt is computed by a constant debt-to-earnings ratio. The target debt is a fraction of the net earnings:

$$MAXDEBT^f_{t+1} = \theta^{f,debt}_t EARN^f_t. \tag{6.30}$$

Here $\theta_t^{f,debt}$ is the predetermined debt-to-earnings ratio (it is a parameter that can be set by the firm). If the additional financing needs are below the maximum level of debt, then the firm can simply apply for new loans by setting the target for loan applications equal to $ADD_t^f$ (and not issuing any new equity):

$$\begin{aligned}&\text{If } (ADD_t^f \leq MAXDEBT_t^f): \\ &TARGETLOANS_t^f = ADD_t^f \\ &TARGETEQUITY_t^f = 0.\end{aligned} \tag{6.31}$$

Otherwise, if additional financing needs are larger than the maximum debt level, then the firm needs to obtain additional financing through equity issuing. Loans are then set to the maximum debt level, and the remainder is financed through new equity:

$$\begin{aligned}&\text{If } (ADD_t^f > MAXDEBT_t^f): \\ &TARGETLOANS_t^f = ADD_t^f, \\ &TARGETEQUITY_t^f = ADD_t^f - MAXDEBT_t^f.\end{aligned} \tag{6.32}$$

If new equity is issued, the value of $TARGETEQUITY_t^f$ must now be split up into two separate decisions: $TARGETSTOCKS_t^f$ and $TARGETBONDS_t^f$. (This has not been modelled so far.)

**Determination of payout parameters from empirical data**

We can use the empirical data as reported in Section 6.4.1 to close the model. In Table 6.2, we have that the retained earnings, loans, bonds, and equity sum up to the total financial needs of the firm. The empirical data then gives us the distribution among these sources such that the total financial needs are met.

The payout parameters have been determined according to several fundamental financial ratios, following as closely as possible the financial practice reported in Brav et al. (2004):

- $\theta_t^{f,div}$: the target is to maintain a constant dividend-per-share level (D/S-ratio), or a constant dividend-to-earnings ratio (D/E-ratio).
- $\theta_t^{f,ret}$: follows after the dividend payments have been determined.
- $\theta_t^{f,debt}$: the target is to maintain a constant debt-to-earnings ratio (D/E-ratio).

Of course, the values of the parameters $(\theta_t^{f,div}, \theta_t^{f,ret}, \theta_t^{f,debt})$ have to come from somewhere. As a starting point, we can consider them to be uniformly distributed on the unit interval. This provides heterogeneity in the firms financial policies. As proposed in Section 6.4.1 above, a better alternative would be to derive these parameters from the empirical data.

**Decision 9b: Retained earnings determined from empirical data**

After the total financing needs have been computed by the firms in a EURACE simulation, they can use this percentage to compute their retained earnings and then subtract this amount from the net earnings (after tax profit), effectively setting the parameter $\theta_t^{f,ret}$. This means we can 'endogenise' $\theta_t^{f,ret}$ by calibrating it to empirical data.

According to Table 6.2, the ratio of retained earnings as a percentage of the total financing needs for an average firm in each European country (France, Italy, Germany, UK), is empirically given by:[2]

- For France: $\nu_{FR}^{ret} \sim N(\mu, \sigma^2)$ with mean $\mu = 0.45$ and variance $\sigma^2 = 0.045$.
- For Germany: $\nu_{GE}^{ret} \sim N(0.70, 0.070)$.
- For Italy: $\nu_{IT}^{ret} \sim N(0.43, 0.043)$.
- For UK: $\nu_{UK}^{ret} \sim N(0.73, 0.073)$.

Retained earnings can now be set as a percentage of the total financing needs, instead of as a fraction of net earnings:

$$RET_t^f = \nu_t^{f,ret} NEEDS_t^f. \tag{6.33}$$

**Decision 13b: Loans determined from empirical data**

How much short-term debt and long-term debt should the firm obtain? This is connected to debt seniority, which means that more senior debt gets a higher priority when it needs to be serviced. The ratio of loans as a percentage of the total financing needs is given by:

- For France: $\nu_{FR}^{loans} \sim N(\mu, \sigma^2)$ with mean $\mu = 0.42$ and variance $\sigma^2 = 0.042$.
- For Germany: $\nu_{GE}^{loans} \sim N(0.27, 0.027)$.
- For Italy: $\nu_{IT}^{loans} \sim N(0.43, 0.043)$.
- For UK: $\nu_{UK}^{loans} \sim N(0.22, 0.022)$.

Loans can now be determined by:

$$TARGETLOANS_t^f = \nu_t^{f,loans} NEEDS_t^f. \tag{6.34}$$

**Decision 13b: Bonds issuing determined from empirical data**

How much corporate bonds should the firm issue? The ratio of new bonds as a percentage of the total financing needs is given by:

- For France: $\nu_{FR}^{bonds} \sim N(0.02, 0.002)$.
- For Germany: $\nu_{GE}^{bonds} \sim N(0.01, 0.001)$.
- For Italy: $\nu_{IT}^{bonds} \sim N(0.03, 0.003)$.
- For UK: $\nu_{UK}^{bonds} \sim N(0.01, 0.001)$.

New bonds can now be determined by:

$$TARGETBONDS_t^f = \nu_t^{f,bonds} NEEDS_t^f. \tag{6.35}$$

[2] Finland has been deliberately left out, since the financial resources considered here (retained earnings, loans, bonds and equity) only account for 76.7% of the average Finnish firm's financial resources.

#### Decision 13b: Equity issuing determined from empirical data

How much equity should the firm issue? The ratio of new equity as a percentage of the total financing needs is given by:

- For France: $\nu_{FR}^{equity} \sim N(0.11, 0.011)$.
- For Germany: $\nu_{GE}^{equity} \sim N(0.03, 0.003)$.
- For Italy: $\nu_{IT}^{equity} \sim N(0.12, 0.012)$.
- For UK: $\nu_{UK}^{equity} \sim N(0.05, 0.005)$.

New equity can now be determined by:

$$TARGETSTOCKSf_t = \nu_t^{f,equity} NEEDS_t^f. \tag{6.36}$$

### 6.5.4 Glossary of accounting terminology

Below is a list of accounting terms that are used to model the behavioral rules of firms.[3]

**Dividends** Distribution of earnings to owners of a corporation in cash, other assets of the corporation, or the corporation's capital stock.

**Earnings** Earnings are before-tax profits, calculated as net revenues minus operational costs and interest payments.

**Net earnings** Net earnings are after-tax profits.

**Net revenues** Revenues from sales of products or services, less the costs of conducting the sales.

**Retained earnings** Accumulated undistributed earnings of a company retained for future needs or for future distribution to its owners.

**Revenues** Sales of products, merchandise, and services; and earnings from interest, dividend, rents.

**Operating Cycle** Period of time between the acquisition of goods and services involved in the manufacturing process and the final cash realization resulting from sales and subsequent collections.

**Debt-to-earnings ratio** Measure of performance calculated by dividing the debt of a company by the net earnings. The Debt/Earn-ratio is defined as $\theta^{f,debt}$:

```
DebtEarn_RATIO = DEBT/EARN
```

[3]Source: Online Accounting Terminology Guide of the New York Society of CPAs, `http://www.nysscpa.org/prof_library/guide.htm`.

**Dividend per share** Measure of performance calculated by dividing the total dividend payout of a company by the average number of shares outstanding during a period. The D/S-ratio is defined as $\theta^{f,dps}$:

```
DPS_RATIO = DIV/SHARES
```

**Dividend-to-earnings ratio** Measure of performance calculated by dividing the total dividend payout of a company by the net earnings. The Div/Earn-ratio is defined as $\theta^{f,div}$:

```
DivEarn_RATIO = DIV/EARN
```

**Earnings Per Share (EPS)** Measure of performance calculated by dividing the net earnings of a company by the average number of shares outstanding during a period. The EPS-ratio is defined as $\theta^{f,eps}$:

```
EPS_RATIO = EARN/SHARES
```

**Price-to-earnings ratio** Measure of performance calculated by dividing the price of the stock of a company by the net earnings. The P/E-ratio is defined as $\theta^{f,pe}$:

```
PE_RATIO = PRICE_STOCK/EARN
```

### Summary and parameter count

Firms have been modelled using 7 parameters:

$$(\theta_t^{f,ret}, \theta_t^{f,debt}, \theta_t^{f,div}; \nu^{ret}, \nu^{loans}, \nu^{bonds}, \nu^{equity}). \tag{6.37}$$

The first three are related to the cash outflow (i.e., to the payout policy), while the last three are related to the cash inflow (i.e., the financial policy). The $\nu$-parameters can be derived directly from the empirical data, while the $\theta$-parameters give the proportions of net earnings that are allocated respectively to retained earnings, debt payments and dividends. Using empirical surveys on real-world managerial decision-making, we have been able to link these parameters to some common financial ratios, such as dividend-per-share, dividend-to-earnings, earnings-per-share, price-to-earnings, and debt-to-earnings ratios (given by $\theta^{dps}, \theta^{div}, \theta^{eps}, \theta^{pe}, \theta^{debt}$).

### Open questions

- How to relate the outflow $\theta$-parameters to the inflow $\nu$-parameters?

The following questions should be answered from empirical data on firm behavior:

- Which fraction of after-tax profits (or out of earnings before tax?) does a firm pay out in dividends? This is the dividend ratio.
- Which fraction of earnings does it use to pay off old debt?
- Which fraction of earnings is used to pay interest on outstanding debt?

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